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Evening Lecture and Concert

Shaping Musical Performance by Genealogical Trees of Vector Fields

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In contrast to the quasi-digital notation of music in European standard scores, performance involves infinitesimal deformations in time, pitch, durations, loudness, articulation and tuning. How can this shaping process–which Theodor W. Adorno termed "micrologic of infinite precision"–be controlled? Generalizing the well-known tempo curve, it is seen that special vector fields, the performance fields, adequately conceptualize Adorno's infinitesimals of micrologic. However, developing performance fields is not a one-time action, it must be built in a genealogical way from coarse to more and more refined performances. Performance grammars have to describe this continuous refinement process of shaping artistic performance. We describe and illustrate the genealogical stemma theory of performance, its implementation in the analysis and performance software RUBATO and present examples of performances which have been generated by such software on MIDI grand pianos.

A. Dynamical Systems

1.1 Celestial Mechanics

Organizer : Kenneth Meyer

Key note lecture

Horseshoes and the Problem of the Gaps

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A normally hyperbolic invariant annulus with transverse homoclinic points exists in the Newtonian five-body problem. In an attempt to understand the implications of this we show that there is actually an invariant Cantor set of annuli nearby. The dynamics on this set is a skew-product over a horseshoe map. Such a map can be viewed as a "random" map of the annulus. By making use of this randomness, one can avoid the problem of gaps between invariant circles on the original annulus which can be barriers to Arnold diffusion.

Invited lectures

Secular System in Souriau Variables

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The idea of using the "Laplace Vector" and the angular momentum vector as orbital elements comes from the fundamental paper of Lagrange "Théorie des variations séculaires des éléments des planètes" (1781).

I present here a work of my student K. Abdullah showing how a slight modification of this approach leads to the appearance of surprising properties of the secular part of the perturbating function.

Regularizable and non–regularizable collisions in n–body problems

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Collisions appear as singularities in the n–body problem. A binary collision occurs when only two masses collide. If all the masses coincide at the same point we get a total collision. Between these two cases one can get different types of collisions, for instance, triple collision (TC), simultaneous binary collisions (SBC), simultaneous binary and triple collisions, etc.

If a solution of Newton equations of motion is defined on some maximal interval $[0, t_c)$ with $t_c < \infty$, it is said that it has a singularity at $t = t_c$. Moroever if the masses approach a limiting position as $t \to t_c$, it can be proved that the limiting position must be a collision point. It is said that the solution ends at a collision.

The extension of a collision solution for $t \geq t_c$ has been studied in several cases. If the extension is possible in some sense it is said that the collision is regularizable in that sense. In that case one can also look for the smoothness of the regularization. If the collision is non–regularizable, it is interesting to describe which are the dynamical consequences. If small changes in the initial conditions lead to substantial changes in the solutions, how one can classify the possible transitions from approaching the collision to escaping from it?

We consider a geometrical approach, searching for continuity with respect to initial conditions for orbits passing close to a collision. After surveying known results, some recent progress, dealing with finite order of smoothness in the SBC case ([1],[2]) and with the classificacion of passages near TC ([3],[4]), is presented.

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On the homology of the integral manifolds in celestial mechanics

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The integral manifolds of the N-body problem are the level sets of energy, angular momemtum, linear momentum and center of mass. They are invariant under the flow, and their topology influences the possible dynamics of the system. Understanding their structure, and how that structure depends on the energy and angualar momentum parameters, is fundamental to understanding the global structure of the N-body problem. Recently, we have made progress in describing the topology of the integral manifolds, and detecting changes in that topology, by analyzing the homology of the manifolds. This talk will survey these results.

Melnikov Techniques in the Sitnikov Problem

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Using Melnikov techniques we get two different families of periodic orbits in the Sitnikov problem. The first family is obtained using a generalization of the Hartman -Grobman theorem that we prove here; and the second one is gotten using the clasical horseshoe of Smale. Both families are obtained analytically. These results improve a previous one of H. Dankowicz and P. Holmes.

Existence and Stability of Relative Equilibria in the n-Body Problem

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Special periodic solutions of the Newtonian n-body problem consisting of configurations of bodies rotating rigidly about their centers of mass are called *relative equilibria*. These solutions become fixed points in a rotating coordinate frame, hence the name. One important conjecture in the field, attributed to Moeckel, asserts that the only linearly stable relative equilibria are those containing a dominant mass, ie. one mass which is much larger than all the others. For example, placing n equal masses at the vertices of a regular n-gon with an additional body of mass m at the center yields a relative equilibrium for all values of m. If m is sufficiently large, then this configuration, referred to as the $1+n$-gon, is linearly stable.

We provide some results in support of Moeckel's dominant mass conjecture. One of these states that any relative equilibrium of n equal masses is not spectrally stable provided $n \geq 24,306$. We also consider a limiting problem with n "big" masses and p "small" masses, with the small masses being order ϵ [1]. This problem splits into two parts, where the n big bodies form a relative equilibrium of the n-body problem and the limiting positions ($\epsilon = 0$) of the p small bodies are critical points of a two-dimensional potential function. Assuming that the small masses do not coalesce in the limit, a case studied by Moeckel in [2], we derive sufficient conditions on the limiting positions of the bodies to insure that the entire family is linearly stable for ϵ sufficiently small. We then apply these conditions to the $1+n$-gon family showing that it is possible to thicken the ring around the central body and maintain its stability.

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The Stability of the Thomson Heptagon in an Ideal Fluid

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In his Adams prize essay of 1882 J. J. Thomson showed that in a uniformly rotating coordinate system a regular polygon configuration of N vortices is linearly stable for $N < 7$ and unstable for $N > 7$. The case of the heptagon can not be decided with the help of the linear terms. G. J. Mertz showed in 1978 with methods from fluid mechanics that the heptagon configuration is stable. By using methods common in celestial mechanics we can recover the result of Mertz and prove in addition that the heptagon configuration is locally Liapunov stable except for the rotational symmetry.

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Contributed talks

Barrier to transport in point vortex dynamics

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We study the dynamics of a system of identical point vortices and of test particles transported by the velocity field created by the vortices. Of particular interest is to determine how the specific vortex dynamics (periodic, quasi-periodic or chaotic) affect the transport and diffusion properties of the test particles; and the process of vortex cluster formation and possible trapping of particles at the interior of each vortex cluster.

For the case of four identical point vortices (*i.e.*, vortices with identical circulation) we have found that, by conservation laws, the vortex mouvement is cofined to a finite region of radius $R_v = \sqrt{L/k}$. The particle phase space is therefore partitioned into an inner region, where the vortex motion takes place, and outer region, from which the vortices are excluded.

We have found that in case of *quasi-periodic* vortex motion, there is a sharp barrier to transport between the central chaotic region, punctured with regular islands, and the outer region where the trajectories are on tori. Mixing and diffusion are therefore only possible in the central region which exhibits in this case a lobe structure, with several islands in addition to the ones encircling the vortices. The existence of such a sharp barrier to transport between regular and chaotic region is well understood by applying the *frequency analysis method* of Laskar[1], and corresponds to the existence of an invariant surface of quasi-periodic orbites.

Finally, for the subspace of symmetrical vortex configurations, the formation of vortex clusters can be explain by the existence of a bifurcation energy beyond which the system of four vortices becomes equivalent to a system of two or three vortex clusters.

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Positive solution II-nd part of 16-th Hilbert's problems

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1. Nonlocal (first) integral of polynomial vector field on the plane. Notations. Denote by $\mathbf{v}$ polinomial of degree n vector field on the plane; by P_j $(0 \le j \le n)$ different points on the plane; by t_j the time on the phase curve going through the point P_j, so $x(t_j)$ would be integral curve of vector field $\mathbf{v}$; by $f_j(x)$ $(0 \le j \le n)$ smooth function on phase plane, excluding probably some isolated phase curve of vector field $\mathbf{v}$. **Definition.**Iff there exist some diffeomorphisms $g_j(t)$ $(0 \le j \le n)$ such, that

$$H(P_j, t) = \sum_{j=1}^{n} f_j(x(t_j)) \, |_{t_j = g_j(t)} \equiv Const, \tag{1}$$

when we call the function $H(P_j, t)$ the (correspondingly, global) non-local first integral of vector field $\mathbf{v}$ (iff in (1) the time goes the whole of line $\mathbf{R}$). We call $x(g_j(t))$ $(0 \le j \le n)$ as support of non-local first integral.

Theorem. Let vector field $\mathbf{v}$ has convex periodic phase curve. Then in exterior of these circle there exist global non-local first integral of $\mathbf{v}$, iff there exist n circles of $\mathbf{v}$ in the above exterior.

2. Reduction to convex case. *Lemma.* Let us polynomial vector field $\mathbf{v}$ has k periodical integral curves. Then there exists one parameter family of vector fields $\mathbf{v}_\varepsilon$ with $deg\mathbf{v}_\varepsilon = deg\mathbf{v} + 4$ so, that $\mathbf{v}_0$ orbitally equivalent to $\mathbf{v}$ and for small value of parameter the vector field $\mathbf{v}_\varepsilon$ has at least k periodical integral curves. Moreover in its interior there is the smallest convex periodical integral curve.

3. Upper bound. For nesting periodical orbits of polynomial vector field the number of limit circles isn't greater than $n(n+1)/2$ for $n \ge 4$ (correspondingly, 6 for $n = 2, 9$ for $n = 3$).

Proof (see [1]) For above constructions it is needed $(n+1)$ circles. It is needed to remove critical values of global nonlocal integral by help of suitable number of limit circles ($n(n+1)/2$ for $n \geq 4$).

4. Bogdanov-Taken's bifurcation (see [2]) deals with normal form of degree 2. Appearing limit circles are convex. From above follows that it's number isn't greater than 6 (realy 1, see(2)).

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Periodic solutions of the spatial elliptic restricted three body problem

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We show the existence of periodic solutions in the three-dimensional elliptic restricted three body problem when the masses of the two primaries are equal. These solutions are perturbations of circular orbits of the Kepler problem when the massless body is far away from the primaries and its orbital plane is perpendicular to that of the primaries. The orbits are doubly symmetric and exist for a discrete sequence of values of the mean motion.

We scale the equations of motion so that the infinitesimal body is at distance unity from the primaries which are close to each other and move very fast. The distance between the primaries is taken as a small parameter but then the perturbation of the Kepler problem fails to be analytic. We then use a modified version of Poincaré's method of analytic continuation.

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2nπ-Periodic Solutions of the Sitnikov Problem

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In this paper we give criteria for the detection and generation of $2n\pi$-periodic solutions of the Sitnikov problem. The Sitnikov problem is the simplest case of the spatial restricted problem of three bodies and is governed by

$$\ddot{z} + z/(z^2 + r(t)^2)^{3/2} = 0, \tag{1}$$

where $r(t)$ is a periodic function, $r(t) = r(t + T)$. In the normalized coordinate system we have $T = 2\pi$ and $2r(t) = (1 - e\cos E)$ with $0 \leq e < 1$ and

$$\cos E = -e/2 + \sum_{n=1}^{\infty} (1/n)[J_{n-1}(ne) - J_{n+1}(ne)] \cos nt. \tag{2}$$

This problem admits no classical integrals of motion. There are many attempts in dealing with this problem among which Moser's contribution has significant importance [1]. The existence of chaotic solutions near the escaping orbits were studied by Moser. Dankowicz and Holmes [2] reconsider the problem using Melnikov's theory and prove the existence of horseshoes. In our previous work [3], we dealt with Sitnikov's problem near the 3:2 resonance and studied its geometrical features when $|z| \ll |r(t)|$. In this study we present a method by which we can isolate $2n\pi$-periodic solutions. we use the following "integrable" equations

$$\ddot{x}_k + x_k/(x_k^2 + M_k^2)^{3/2} = 0, \quad k = 1, 2, \tag{3}$$

where M_1 and M_2 are the maximum and minimum values of $r(t)$ for $t \in [0, 2\pi[$, respectively. We show that at certain circumstances, the solution of (1) lies between x_1 and x_2. This allows us to express the evolution of the system in a "convergent" Hamiltonian of the form

$$\mathcal{H} = \frac{1}{2}\dot{u}^2 + \sum_{n=0}^{\infty} \Phi_n(t) u^n, \tag{4}$$

where Φ_n are 2π-periodic functions of t. The new dependent variable $u(t)$ is defined in terms of $x_1(t)$ and $r(t)$. Having $u(t) = u(t + 2\pi)$ determined from (4), one could be able to construct $2n\pi$-periodic solutions of (1).

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Parabolic fixed points and stability criteria for nonlinear Hill's equation

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We discuss the stability of parabolic fixed points of area-preserving mappings and obtain a new proof of a criterion due to Simó. These results are employed to discuss the stability of the equilibrium of the periodic differential equation

$$\ddot{x} + a(t)x + c(t)x^{2n+1} + \ldots = 0 \ (n \geq 1),$$

when the variational equation at $x = 0$ is unstable-parabolic. In this class of equations the first Lyapunov's Method does not apply but in many cases the stability can be characterized in terms of the variational equation. We assume that $c(t)$ does not identically vanish and does not change sign. We associate two numbers $\sigma,\ \nu \in \{1, -1\}$ to the variational equation and prove that $x = 0$ is stable if $\sigma\nu c(t) \geq 0$ and unstable if $\sigma\nu c(t) \leq 0$. As a consequence, we obtain a necesary and sufficient condition for the stability of the equilibrium of the restricted three-body problem.

This is joint work with Rafael Ortega.

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Study of Complex Instability Using Normal Forms

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We consider a Hamiltonian H of three degrees of freedom and a family of periodic orbits with a transition from stability to complex instability, such that there is an irrational collision of the Floquet eigenvalues of opposite sign. We analyze the local dynamics around the transition studying the Hamiltonian from an analytical point of view using and *ad hoc* normal form. Carrying out a finite number of steps of this normal form process, and skipping the non integrable remainder, the existence of a bifurcating family of two-dimensional tori is derived. This family of tori may unfold on the unstable side of the family of periodic orbits, and then they turn out to be stable (*direct* Hamiltonian Hopf bifurcation), or they may appear on the stable side, being now unstable (*inverse* bifurcation). The existence of a direct or inverse bifurcation depends intrinsically on the Hamiltonian.

Our next objective is to show that, when the above reminder is added, most part (in the sense of the measure theory), of these 2D invariant tori persist in the complete Hamiltonian H.

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1.2 Homoclinic Tangencies

Organizers : Ale Jan Homburg, Floris Takens

Key note lecture

Recent developments in chaotic dynamics

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I will report on some recent results in the theory of dynamical systems with complex behaviour, especially on the properties of attractors, robustness and (non)hyperbolicity, statistical description of the dynamics, and stability.

Invited lectures

Periodic and strange attractors near saddle-focus homoclinic orbits

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We discuss dynamics near Shil'nikov saddle-focus homoclinic orbits in three dimensional vector fields. The existence of periodic and strange attractors is investigated in families for which each member has a homoclinic orbit (so not in unfoldings). We consider how often, in the sense of measure, periodic and strange attractors occur. We also discuss the fate of typical orbits, and establish that despite the possible existence of attractors, a large proportion of points lies outside the basin of an attractor. This is joint work with Sergey Gonchenko and Oleg Sten'kin.

The Lorenz attractor exists

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The following non-linear system of differential equations, also known as the *Lorenz equations*,

$$\begin{aligned} \dot{x}_1 &= -\sigma x_1 + \sigma x_2 \\ \dot{x}_2 &= \varrho x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 &= -\beta x_3 + x_1 x_2, \end{aligned} \tag{1}$$

was introduced 1963 by Edward Lorenz, see [1]. Numerical simulations for an open neighbourhood of the classical parameter values $\sigma = 10, \beta = 8/3$ and $\varrho = 28$ suggest that almost all points in phase space tend to a strange attractor $\mathcal{A}$ - *the Lorenz attractor.* In [2], we prove that the Lorenz equations indeed support a strange attractor, as conjectured in [1]. We also prove that the attractor is robust, i.e., it persists under small perturbations of the coefficients in the underlying differential equations:

Theorem: *For the classical parameter values, the Lorenz equations support a robust strange attractor $\mathcal{A}$. Furthermore, the flow admits a unique SRB measure μ_X with* $\mathrm{supp}(\mu_X) = \mathcal{A}$.

The proof can be broken down into two main sections: one global part, which involves rigorous numerical computations, and one local part, which is based on normal form theory. The novelty of the method of proof lies in that, rather than producing a traditional mathematical proof, we construct an algorithm which, if successfully executed, proves the existence of the strange attractor.

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Arbitrarily complicated dynamics near homoclinic tangencies

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We prove that any multidimensional dynamical systems with non-hyperbolic homoclinic and heteroclinic cycles exhibits all dynamical phenomena possible in the corresponding dimension, in the following sense. Given a C^r-diffeomorphism f in R^n with a heteroclinic (homoclinic) cycle C composed of a finite number of periodic and connecting orbits there exists an algorithm to evaluate an effective dimension d of the problem: the maximal possible number of zero Lyapunov exponents for periodic orbits which can be born in a small neighbourhood of the cycle C at small C^r-smooth perturbations of f. For the case $d < n$ the following result is established:

Theorem. *For any d-dimensional diffeomorphism g and for any ε there exists a diffeomorphism f^* arbitrarily close to f in C^r-topology such that f^* has an absorbing domain D (i.e., $(f^*)^N D \subset D$ for some positive N) in a small neighbourhood of the cycle C, and in D there is an attractive d-dimensional smooth invariant manifold M such that $(f^*)^N|_M$ is C^r-conjugate to g.*

Roughly speaking, small perturbation of any system with a heteroclinic cycle of effective dimension d can produce arbitrary d-dimensional dynamics.

Contributed talks

Control of averaging chaos in quasiperiodically excited system

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Suppression of chaos in periodically forced nonlinear oscillators has received much effort from analytical, numerical and experimental point of view [1,2,3]. In this work suppression of chaos in the averaged system of a nonlinear one-degree- of-freedom oscillator subjected to external and parametric excitations having incommensurate frequencies is reported [4]. An adapted Bogolioubov-Mitropolski method [5] is first developed to reduce the original quasiperiodically driven system to a periodically driven one by averaging in the rapidly varying oscillations [6]. The chaotic threshold is then derived on the slowly varying oscillations using the Melnikov method [7]. The possibility to realize a suitable system control of chaos by introducing a third harmonic parametric component into a nonlinear term of the original quasiperiodically forced system is investigated by acting on the induced effective averaged equations.

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Homoclinic tangencies and Ω-moduli

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As is well known, multidimensional dynamical systems with structurally unstable Poincare homoclinic orbits and heteroclinic cycles possess continuous invariants (moduli) of the topological or Ω-equivalence (conjugacy) (the last is the topological equivalence (conjugacy) on the set of nonwandering orbits). For some classes of such systems the principal Ω-moduli were found and the existence of infinitely many moduli was established (see, for example, [1]–[4]). The importance of calculation of principal Ω-moduli is explained by the fact that, by definition, these invariants represent such parameters whose arbitrary changes lead to bifurcations of nonwandering orbits (periodic, homoclinic etc.). Thus, Ω-moduli can be considered as substantive control parameters along with the splitting parameters which are natural for such type bifurcation problems.

In the present report we state some results related to the following problems:
1) Determination of principal Ω-moduli in the cases of multidimensional diffeomorphisms with structurally unstable homoclinic orbits or heteroclinic cycles. A special attention is given for the cases where weakest eigenvalues of saddles are complex conjugate. Also we consider questions on existence of moduli of local Ω-conjugacy in the case of symplectic maps.
2) The study of bifurcations for families whose control parameters are a) principal Ω-moduli ; b) both principal Ω-moduli and splitting parameters.

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Asymptotics of Homoclinic bifurcation in a Three-Dimensional systemt

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Using perturbation methods, we predict critical parameter values corresponding to homoclinic bifurcation in three-dimensional systems. The multiple scales perturbation method [1] is first performed to construct a higher-order approximation of the periodic solution [2]. A criterion based on the collision between the periodic orbit and the equilibrium involved in the bifurcation is then applied. This criterion developed initially to predict homoclinic bifurcations in planar autonomous system [3,4,5] is adapted here to derive a critical value of the homoclinic bifurcation in two specific three-dimensional systems. Period- doubling bifurcations are also investigated using the method of harmonic balance and the stability analysis of the bifurcating periodic solution. A numerical study [6,7] showed a good agreement of the critical parameter value with this analytical prediction.

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Dynamics near non-transversal homoclinic points

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We consider discrete dynamical systems $x(n+1) = f(x(n), \lambda)$ having for $\lambda = 0$ a homoclinic orbit $\Gamma := \{f^n(q,0), n \in \mathbb{Z}\}$ asymptotic to a hyperbolic fixed point p. $f : \mathbb{R}^k \times \mathbb{R}^m \to \mathbb{R}^k$ smooth. We are interested in the dynamics in a neighborhood of Γ. To investigate this dynamics we will use Lin's method. In the theory of smooth dynamical systems (vector fields) this method is proved to be a powerful tool for investigating the dynamics near homoclinic orbits or even heteroclinic cycles. Here we carry forward the ideas of Lin's method to discrete systems. We apply the results to study the dynamics near non-transversal homoclinic points:

If the stable and unstable manifolds of p have a quadratic tangency in q we give a "Poincaré-map" involving two different transversal homoclinic points which is conjugated to shift dynamics. We consider the dynamics of this map in dependence on the parameter λ.

By means of the canonical Poincaré-map we study the dynamics near homoclinic points in periodically forced systems. In particular we study homoclinic points and subharmonic solutions.

Resonant homoclinic flip bifurcations: a numerical investigation

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Three-parameter unfoldings of resonant orbit flip and inclination flip homoclinic orbits have recently been studied in [2]. The key idea is to look at a sphere in parameter space around the central singularity and glue respective codimension-two unfoldings of homoclinic flip bifurcations together. Several cases are possible, and near some resonant flip bifurcations one can find homoclinic-doubling cascades as first found in [1].

We explain some of these unfoldings and show results of a careful numerical study of the model proposed by Sandstede in [3]. Two particular unfoldings of a resonant orbit flip and a resonant inclination flip do indeed occur as proposed in [2] when the sphere around the central singularity is taken small enough. This also confirms the conjecture that the parameter space has cone structure close to the central codimension-three point. When the sphere is taken larger extra interesting codimension-three bifurcations occur.

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On the structure and bifurcations in a generic unfolding of a 2 d.o.f Hamiltonian system with the contour with two saddle-foci

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Let (M, Ω) be a C^∞ four-dimensional symplectic manifold and X_{H_μ}, $\mu \in \mathbb{R}$, be a generic smooth one-parameter unfolding of a smooth Hamiltonian vector field with Hamiltonian H_0 such that X_{H_0} has two singular points p_1, p_2, both of saddle-focus type that belong to the same level set of Hamiltonian and their stable and unstable manifolds intersect each other transversely along two heteroclinic orbits forming a contour Γ. Without loss of generality one may suppose the equilibrium p_1 does not change with μ and p_2 does depend on μ, with $f(\mu) = H(p_2) - H(p_1)$.

Theorem. There are positive constants μ_0, c_0 small enough and a neighborhood U of Γ such that for X_{H_μ}, $|\mu| \le \mu_0$, in $U \cap \{H_\mu = c\}$, $|c| \le c_0$: 1. There exist invariant hyperbolic subsets $\mathcal{H}_{\mu,c}$ such that the Poincare map $P_{\mu,c}$ generated by

$\mathcal{H}_{\mu,c}$ is conjugated to a symbolic system with countable alphabets for $\mu = 0,\ c = 0$, $\mu \ne 0,\ c = 0$ or $\mu \ne 0,\ c = f(\mu)$. If $\mu \ne 0,\ c \ne f(\mu)$ then the alphabet is finite and the number of states grows with the logarithmic asymptotics. 2. There are two sequences

$\{\mu_k^{(1)}\},\ \{\mu_k^{(2)}\},\ \ \mu_k^{(i)} \in (-\mu_0,\ \mu_0)$, such that $X_{H_\mu},\ \mu = \mu_k^{(i)}$, possesses a nontransverse homoclinic orbit to p_i with quadratic tangency. After every passage of μ through $\mu_k^{(i)}$ the number of 1-circuit homoclinic orbits to p_i increases by 2. 3. For $\mu = c = 0$ the set $\mathcal{H}_{\mu,c}$ exhausts all orbits lying entirely in U, in particular, all homoclinic and heteroclinic to $p_i,\ i = 1, 2$, orbits.

Corollary 1. The vector field X_{H_0} in the level $H_0 = 0$ near Γ has:

i) a countable set of transverse heteroclinic contours of any circuitness;

ii) each saddle-focus p_i has a countable set of transverse homoclinic orbits of any circuitness;

iii) for each integer $N > 0$ there is a countable set of saddle periodic orbits of circuitness N, every such orbit γ possesses a countable set of heteroclinic orbits with $p_i,\ i = 1, 2,\ \gamma \to p_i$, and $p_i \to \gamma$, where the arrows point out the direction of increasing time.

Corollary 2. For $\mu \neq 0,\ c = 0$ the same assertions as in the items ii), iii) of Corollary 1 hold for p_1 and periodic orbits in the level $H_\mu = 0$. For $\mu \neq 0,\ c = f(\mu)$ the same is valid for the saddle-focus $p_2(\mu)$.

Remark. The contours described exist in the Hamiltonian system derived from the stationary generalised Swift-Hohenberg equation when its two parameters belong to some parabola.

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On singularities robustly accumulated by periodic orbits

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Let $\mathcal{G}^1$ be the set of three-dimensional vector fields X such that all the singularities and periodic orbits of every vector field Y C^1-close to X are hyperbolic. A singularity σ of X is *robustly accumulated by periodic orbits* if for every Y C^1 close to X, the continuation σ_Y of σ belongs to the closure of the periodic orbits of Y. An example of such a kind of singularity is that of the geometric Lorenz attractor L (cf. [1]). Observe that L is a partially hyperbolic attractor with volume expanding central direction.

In this talk we outline the proof of the following result : For *generic* $X \in \mathcal{G}^1$, if σ is a singularity of X robustly accumulated by periodic orbits, then σ belongs to a partially hyperbolic attractor with volume expanding central direction for either X or $-X$. An example of a vector field in $\mathcal{G}^1$ which has neither attractors nor repellers and exhibits a singularity robustly accumulated by periodic orbits is given.

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Impossibility of complete bifurcation description for some classes of systems with simple dynamics

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The main problem of the bifurcation theory is to describe changes of phase portraits of dynamical systems under continuous change of parameters. This description implies construction of the bifurcation diagram, i.e. splitting the space of parameters onto classes of the equivalent systems. It is well known, that many bifurcations of dynamical systems admit the complete bifurcation description by so-called versal families, which, in some sense, contain the complete information on the structure of the bifurcation set for any family of general position. it is known also that majority of dynamical systems with chaotic behaviour of trajectories do not admit the complete bifurcation analysis. It is explained, first of all, by the existence of small perturbations which lead to appearance of infinitely-degenerate orbits. In contrast to the chaotic systems, it seems natural, that systems with simple dynamics must admit the complete bifurcation analysis. But it is not so.

In the present report, an example of the codimension two system with very simple dynamics is considered. Small perturbations of this system can lead to appearance of no more than one periodic orbit. Nevertheless, the complete bifurcation analysis of such system is impossible in framework of any finite parametric family of dynamical systems.

More exactly, bifurcations of a heteroclinic contour composed by a homoclinic orbit to a saddle-focus and a heteroclinic orbit to this equilibrium point are considered. The case of simple dynamics is selected (no more than one periodic orbit is born at bifurcations in a small neighborhood of the contour). In spite of simplicity of dynamic behavior, the structure of the bifurcation set corresponding to multi-circuit heteroclinic orbits is rather complicated. It is demonstrated that there is no versal families of dynamical systems which describe completely the structure of the bifurcation set. Moreover, in any neighborhood of any family with such contour there exists a continuum set of the families which are not equivalent to each other.

A Method for Computing the Homoclinic Orbits of the Lorenz System at $\sigma = 10, \mathrm{b} = 8/3$ and $\rho = 13.926\ldots$

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We present an iterative technique to analytically approximate the homoclinic loops of the Lorenz system for $\sigma = 10, b = 8/3$ and $\rho = \rho_H = 13.926\ldots$. First, the local structure of the homoclinic solution for $t \longrightarrow 0\pm$ and $t \longrightarrow \pm\infty$ is analyzed. Then, global approximants are used to match the local expansions. The matching procedure resembles the one used in Padé approximations. The accuracy of the approximation is improved iteratively, with each iteration providing estimates for the initial conditions of the homoclinic orbit, the valus of ρ_H, and three undetermined constants in the local expansions. Within three iterations the error in ρ_H falls to the order of 0.1%. Comparisons with numerical integrations are made, and a discussion on ways to extend the technique to other types of homoclinic or heteroclinic orbits, and to improve its accuracy, is given. A detailed presentation of the aforementioned analysis can be found in [1].

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1.3 Singular Perturbations

Organizers : Peter Szmolyan, David Terman

Key note lecture

Geometric Singular Perturbation Theory

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The geometric approach to singular perturbation problems [1] has been substantially refined and extended during the last decade. For problems which exhibit layer behaviour this approach has been very successful, see the survey [2] for the basic theory and references to numerous applications. By now, singularly perturbed ordinary differential equations can be seen as dynamical systems of considerable complexity but still amenable to analysis.

However, at points where normal hyperbolicity fails, e.g. fold points or points of self-intersection of the critical manifold, the well developed geometric theory does not apply. These situations are abundant in applications, we just mention relaxation oscillations [3] and turning point problems [4]. A well known related phenomenon are canard solutions which were first analysed by methods from nonstandard analysis [5] . Only recently, a pioneering geometric analysis of canard cycles in van der Pol's equation based on blow-up methods was given [6].

In this talk we survey results [7,8,9,10] aiming at a comprehensive geometric treatment of singular perturbation problems with nonhyperbolic points. We show how blow-up techniques lead to problems which can be analyzed in detail by more conventional methods from the theory of dynamical systems. Several examples, including relaxation oscillations and canard solutions, are discussed. This approach provides also insight into the scaling- and matching-techniques used in classical matched asymptotic expansions.

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Invited lectures

Geometric singular perturbation theory beyond normal hyperbolicity

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Geometric singular perturbation theory has traditionally dealt only with perturbation problems near normally hyperbolic manifolds of singularities. In the talk will be shown how blow up techniques can permit enlarging the applicability to non-normally hyperbolic points. The method will be presented on well chosen examples in the plane and in 3-space, emphasizing problems concerning bifurcations of limit cycles.

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Geometric singular perturbation theory in infinite dimensions

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Geometric singular perturbation theory is a powerful tool for the study of dynamical systems with two different time scales. However, its basic techniques, originally established by N. Fenichel, are strictly finite-dimensional in nature. This talk surveys recent infinite-dimensional developments in the framework of two examples. First, Shilnikov-type manifolds homoclinic to finite-dimensional slow manifolds are described for coupled nonlinear Schrődinger equations. Second, the existence of an infinite-dimensional slow manifold comtaining the global attractor of the Maxwell-Bloch equations is discussed.

Asymptotic Method of Differential Inequalities

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Some recent achievements of singular perturbations theory which are based on asymptotic method of differential inequalities (AMDI) are presented.

The main idea of AMDI is to construct upper and lower solutions by using formal asymptotics. This approach was successfully applied to different classes of problems for nonlinear singularly perturbed equations:

1) Elliptic problems, including systems with "fast" and slow equations.

2) Parabolic problems with solutions periodic in time.

3) Integro-differential equations (ODE and PDE cases).

4) Investigations of stability in Ljapunov sense of steady-state and periodic solutions of the corresponding parabolic equations.

For these classes of problems solutions with internal layers and the case of exchange of stabilities were considered. Theorems of existence of solutions, estimates of asymptotics were stated. More detailed consideration of some of the presented problems can be found in papers [1-3].

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Captures into resonance and scattering on resonance: an example from charged particles dynamics

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Dynamics of a relativistic charged particle in a uniform magnetic field and an electrostatic plane wave is studied. For small amplitude high frequency wave the system is reduced to a two degrees of freedom Hamiltonian system with slow and fast variables. In this system, the phenomena of capture into resonance and scattering on the resonance are described. These phenomena determine the long-term dynamics of the particle.

Gevrey Asymptotics in Singular Perturbations of ODE

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For analytical singularly perturbed ordinary differential equations, I present two very general theorems on the existence of solutions having Gevrey asymptotic expansions and an idea of their proof (work with M. Canalis–Durand, J.-P. Ramis and Y. Sibuya).

Three applications are given:

- the proof of Wasow's conjecture on the nature of turning points (thesis of C. Stenger),
- a theorem on resonance in the sense of Ackerberg - O'Malley (work with A. Fruchard),
- a generalisation of a theorem of Y. Sibuya on the convergence of formal solutions in the theory of singular perturbations.

The theorems were chosen to show the power of *analytical* methods in the theory of singular perturbations.

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Propagating Activity Patterns in Neuronal Networks

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Synaptically coupled neuronal networks can exhibit a wide variety of propagating wave activity. These waves depend on many network properties including whether the coupling is excitatory or inhibitory and the architecture of neuronal connections. In this talk, I will discuss how geometric singular perturbation methods have been used to classify which waves exist for a given network and how to compute their wave speed.

Contributed talks

Bifurcations, scaling laws and hysteresis in singularly perturbed systems

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Dynamical systems depending on a slowly varying parameter can be described by a singularly perturbed differential equation of the form $\varepsilon\dot{x} = f(x,t)$. We are interested in the qualitative behaviour of solutions near a bifurcation point of the associated one-parameter family of dynamical systems $\dot{x} = f(x,\lambda)$.

Bifurcations with a single zero eigenvalue lead to the appearance of scaling laws: certain solutions of the singularly perturbed equation track an equilibrium branch of $f(x,\lambda)$ at a distance of order $\varepsilon^\alpha|t|^\beta$. When $f(x,t)$ is periodic, this can produce hysteresis cycles whose area behaves as ε^μ, where μ is usually a rational number. We present a method based on Newton's polygon to compute the exponents α, β and μ [1,2].

The passage through a Hopf bifurcation leads to the well-known bifurcation delay [3,4]. We construct a scalar feedback control which suppresses this delay by transforming the Hopf bifurcation into a bifurcation with double zero eigenvalue, described by a cubic Liénard equation [5]. We present a result on exchange of stabilities for the associated slowly time-dependent equation [6].

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Exponential Sensitivity in some singularly perturbed boundary value problems

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We consider a singularly perturbed boundary value problem with Dirichlet conditions and study the sensitivity of the internal layers solutions with respect to small changes in the boundary data. Our approach exploits the existence of smooth invariant manifolds and their asymptotic expansions in the small parameter of perturbation. Using this approach we show that the phenomenon is extremely sensitive since the shock layers are only obtained by exponentially small perturbations of the boundary data.
Examples considered include certain equations associated with metastable internal layer motion.

Integral Manifolds in the Theory of Singularly Perturbed Differential-Difference Equations

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We consider the following system with time lag

$$\begin{aligned} \frac{dx}{dt} &= f(t, x, y, \varepsilon), \\ \varepsilon\frac{dy}{dt} &= C(t)y + D(t)y_\Delta + g(t, x, x_\Delta, y, y_\Delta, \varepsilon), \end{aligned} \tag{1}$$

where $t \in \mathbb{R}$, $x, x_\Delta \in \mathbb{R}^n$, $y, y_\Delta \in \Omega_\rho = \{y \in \mathbb{R}^m, |y| < \rho\}$, $\varepsilon > 0$ $(x_\Delta = x(t - \varepsilon\Delta), y_\Delta = y(t - \varepsilon\Delta))$.

In this paper we investigate conditions of existence for an integral manifold of system (1) in form

$$y = h(t, x, \varepsilon).$$

The flow on this manifold is governed by the m-dimentional system

$$\frac{dx}{dt} = f(t, x, h\,(t, x, \varepsilon)\,, \varepsilon). \tag{2}$$

Moreover, the stability property problem for system (1) is equivalent to the stability problem for reduced system (2) [1].

Method of integral manifolds is applied to study linear singularly perturbed system [2]

$$\begin{aligned} \frac{dx}{dt} &= A(t)x + B(t)y, \\ \varepsilon\frac{dy}{dt} &= C(t)y + D(t)y_\Delta + F(t)x + G(t)x_\Delta. \end{aligned} \tag{3}$$

The use of this method permits us to solve a problem of decomposition of singularly perturbed differential-difference system (3) into two independent equations.

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The period function of non homogeneous equations

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We study the period function $T(c)$ of a center of non homogeneous differential system:

$$u'' + g(t, u, u') = 0, \tag{1}$$

where the function g satisfies certain conditions, ensuring the existence of a center at the origin O.

We are interested in the existence of solutions of equation (1), and more precisely in the case where $g(t, u, u') = f(u) + \epsilon h(t, u, u')$. So that the homogeneous equation $u'' + f(u)$ has a center and a monotonic period function depending on the energy. Some sufficient conditions for the monotonicity of $T(c)$ or for the isochronocity of a center in the homogeneous case may be extended in the perturbed case.

We also examine the differential equation of Lienard type $u'' + a(t)u' + f(u) = 0$, and the monotonicity of its period function.

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Invariant manifolds approach to optimal control of nonlinear singularly perturbed systems

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Consider the following nonlinear affine system:

$$\dot{x}_1 = f_1(x_1, x_2) + B_1(x_1, x_2)u, \quad \epsilon\dot{x}_2 = f_2(x_1, x_2) + B_2(x_1, x_2)u, \tag{1}$$

with the functional

$$J = \int_0^\infty [k'(x_1, x_2)k(x_1, x_2) + u'R(x_1, x_2)u]dt, \tag{2}$$

where $x_1(t) \in R^{n_1}$ and $x_2(t) \in R^{n_2}$ are the state vectors, $x = \mathrm{col}\{x_1, x_2\}$, $u(t) \in R^m$ is the control input, and $z \in R^s$ is the output to be controlled, $\epsilon > 0$ is a small parameter.

The functions f_i, B_i, R and k are differentiable with respect to x a sufficient number of times. We assume also that $f_i(0,0) = 0, k(0,0) = 0$ and $R = R' > 0$.

The system (1)-(2) has a non-standard singularly perturbed form in the sense that it is nonlinear in both, the slow variable x_1 and the fast variable x_2. In the standard form the system is linear in x_2.

We are looking for a nonlinear state-feedback

$$u = \beta(x), \quad \beta(0) = 0, \tag{3}$$

that minimizes the cost (2), where $x(0) = x_0$. For each $\epsilon > 0$ the control law (3) is locally optimal on $\Omega \subset R^{n_1} \times R^{n_2}$ if there exists Ω_1, $0 \in \Omega \subset \Omega_1$, such that the closed-loop trajectories for initial data in Ω remain in Ω_1 and for any initial condition $x_0 \in \Omega$ and any control $u(t)$ for which

$$(i)\ x(t) \in \Omega_1,\ t \geq 0; \quad (ii)\ J(x_0, u) < \infty; \quad (iii) \lim_{t\to\infty} x(t) = 0$$

we have $J(x_0, u_0) \leq J(x_0, u)$.

It is known that the optimal controller for such problem can be designed by solving a Hamilton-Jacobi partial differential equation. This equation is solvable iff there exists a special invariant manifold of the corresponding Hamiltonian system. We obtain exact slow-fast decomposition of the Hamiltonian system and of the special invariant manifold into the slow and the fast ones. On the basis of this decomposition we construct high-order asymptotic approximations of the optimal state-feedback, optimal trajectory and optimal open-loop control.

Multiplicative solution asymptotics for singularly perturbed parabolic boundary value problems

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In this talk we consider the boundary value problem

$$\frac{\partial u}{\partial t} = L_h u, \quad t > 0, \quad x \in D; \tag{1}$$

$$u\Big|_{t=0} = 0, \quad u\Big|_{x\in\partial D} = \Psi(x), \tag{2}$$

where $u = u(x, t, h)$,

$$L_h = h\langle \mathbf{A}(x)\nabla, \nabla\rangle + \langle \mathbf{b}(x), \nabla\rangle + c(x)$$

is an operator uniformly parabolic on the set D_0, $\mathbf{A}(x)$ is a symmetric $(n \times n)$ matrix, $\mathbf{b}(x) = (b_1(x), b_2(x), \ldots, b_n(x))$ is a vector-valued function, the components of the matrix $\mathbf{A}(x)$ and of the vector $\mathbf{b}(x)$ are infinitely differentiable on D_0; the function $c(x) \leq 0$ for $x \in D_0$, and $c(x) \in C^\infty(D_0)$; D_0 is a bounded domain such that $(D \cup \partial D) \subset \mathbb{R}^n$; the function $\Psi(x) \geq \delta > 0$ for $x \in \partial D$, $\Psi(x) \in C^\infty(\partial D)$. The boundary ∂D of the domain D is a class C^∞; $h \in (0,1]$ is a small parameter, $\langle\cdot,\cdot\rangle$ is the inner product in $\mathbb{R}^n_x$, $\nabla = (\partial/\partial x_1, \partial/\partial x_2, \ldots, \partial/\partial x_n)$ is the gradient operator.

We construct the global exponential asymptotics of the solution of problem (1),(2) as $h \to +0$ for a finite time interval. We investigate following three cases:

$$(i)\ \langle \mathbf{b}(x), \mathbf{n}(x)\rangle > 0; \quad (ii)\ \langle \mathbf{b}(x), \mathbf{n}(x)\rangle < 0; \quad (iii)\ \langle \mathbf{b}(x), \mathbf{n}(x)\rangle = 0,$$

where $x \in \partial D$ and $\mathbf{n}(x)$ is the unit outward normal vector to ∂D at the point x.

We also construct the global asymptotics of the solution of problem (1), (2) in the case in which the equation coeffients of L_h depend on x and t.

Part of these results are joint work with V.P. Maslov and S.I. Chernykh.

Limit Cycles of Polynomial Dynamical Systems

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Two-dimensional polynomial dynamical systems are considered. The main problem of the qualitative theory of such systems is Hilbert's Sixteenth Problem on the maximum number and relative position of limit cycles. There are three local bifurcations of limit cycles: 1) Andronov – Hopf bifurcation (from a singular point of the center or focus type); 2) separatrix cycle bifurcation (from a homoclinic or heteroclinic orbit); 3) multiple limit cycle bifurcation. The analysis of each of these bifurcations individually does not yield a complete solution of the Problem even in the simplest (quadratic) case of nonlinear polynomial systems. Using Erugin's ideas on the qualitative investigation on the whole, Duff's field-rotation parameters and Perko's termination principle, we connect all local bifurcations of limit cycles and develop a global bifurcation theory of polynomial dynamical systems.

In particular, we give a sketch of proof of the theorem stating that for quadratic systems four is the maximum number of limit cycles and (3:1) is only possible their distribution. The proof is carried out by contradiction. On the first step we show the nonexistence of four limit cycles surrounding a singular point. We consider a canonical system containing three field-rotation parameters and suppose that it has four limit cycles around the origin; then we get into some three-dimensional domain of the field-rotation parameters being restricted by some conditions on the rest two parameters corresponding to the definite case of singular points in the phase plane. This three-parameter domain of four limit cycles is bounded by three fold bifurcation surfaces forming a swallow-tail bifurcation surface of multiplicity-four limit cycles. It can be shown that the corresponding maximal one-parameter family of multiplicity-four limit cycles cannot be cyclic and terminates either at the origin or on some separatrix cycle surrounding the origin. Since we know absolutely precisely at least the cyclicity of the singular point (Bautin's result) which is equal to three, we have got a contradiction with the termination principle stating that the multiplicity of limit cycles cannot be higher than the multiplicity (cyclicity) of the singular point in which they terminate. Since we know the concrete properties of all three field-rotation parameters in the canonical system and, besides, we are able to control simultaneously bifurcations of limit cycles around different singular points, the proof of the theorem can be completed.

In a similar way cubic and more general polynomial systems can be considered. Thus, generalizing the obtained results, we develop a global bifurcation theory of planar polynomial systems. We discuss also how to apply these results for higher-dimensional dynamical systems and how, in particular, to construct a three-dimensional system with a strange attractor on the base of a planar quadratic system with two unstable foci and an invariant straight line.

On WKB asymptotic expansions of high frequency vibrations in stiff problems

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Stiff problems are differential equation problems with very different values of the coefficients in different regions of the domain. Such problems model the vibrations of a system consisting of two (or more) materials, one of them very stiff respect to the other. Stiff problems originated in J.L. Lions [1]. We study the asymptotic behavior of the eigenvalues and eigenfunctions of the corresponding spectral problem.

Let Ω be a bounded domain of $\mathbb{R}^n$, $n = 1, 2, \ldots$, divided in two parts Ω_0 and Ω_1 by the interface Γ: $\Omega = \Omega_0 \cup \Omega_1 \cup \Gamma$. Consider the spectral boundary value problem

$$L_0(x, \partial_x)u^\varepsilon - \lambda^\varepsilon u^\varepsilon = 0 \quad \text{in } \Omega_0 \quad \text{and} \quad \varepsilon L_1(x, \partial_x)u^\varepsilon - \lambda^\varepsilon u^\varepsilon = 0 \quad \text{in } \Omega_1$$

with the Dirichlet boundary conditions on $\partial\Omega$ and the standard conjugate conditions on Γ. Here L_0 and L_1 are elliptic operators. The asymptotic behavior as $\varepsilon \to 0$ of eigenvalues λ_k^ε and eigenfunctions u_k^ε has been widely studied (see [2] and the references given there). The study of so called *low frequencies* $\lambda_k^\varepsilon = O(\varepsilon)$ leads us to a Dirichlet eigenvalue problem posed in the less stiff structure Ω_1 and does not provide a good insight on the vibration problems over all Ω.

From a physical viewpoint there is the different kind of proper vibrations (so called *high frequency vibrations*) which correspond to sequences of eigenvalues $\lambda_{k(\varepsilon)}^\varepsilon$ with $k(\varepsilon) \to \infty$. Such vibrations are generated by the stiffer structure Ω_0 and have strongly oscillatory character in Ω_1 (see [3]). Constructing of asymptotic expansions of the high frequency vibrations has been an open problem. Use a WKB-technique we find for some operators L_i and domains Ω_i the full asymptotic expansions of such vibrations in the form

$$u^\varepsilon(x) \sim \sum \varepsilon^k v_k(x) \quad \text{if } x \in \Omega_0, \quad u^\varepsilon(x) \sim \sum \varepsilon^k f_k(x, \varepsilon^{-1}S(x)) \quad \text{if } x \in \Omega_1$$

where S is a solution of the corresponding Hamilton-Jacobi equation. The asymptotics has a discrete character with respect to a small parameter ε. The sequences $\varepsilon_s \to 0$ on which expansions are constructed and justified are given by formula.

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Radiation Reaction and Center Manifolds

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We consider a charged particle subject to self-generated electric and magnetic fields and under the influence of external fields, the classical Abraham-Lorentz model. An important issue for such models is to derive an "effective equation" for the particle position $q(t) \in \mathbb{R}^3$, i.e., an ODE for q alone, with the contributions from Maxwell's equations for the fields incorporated. By formal calculations it can be seen that this equation is of type

$$(m_0 + \varepsilon^{-1} m_e)\ddot{q} = F(q, \dot{q}) + aq^{(3)}, \tag{1}$$

with $a > 0$, m_0 and m_e the rest mass and the effective mass of the particle, respectively; F denotes the external forces. This is the famous Lorentz-Dirac equation which (due to the term with the third derivative) has undesirable "run-away"-solutions, i.e., solutions that go off to infinity exponentially fast. There have been a lot of attempts to single out those initial data that lead to physically relevant solutions of (1) and to interpret them as approximations of the full problem, but no satisfactory answer has been know.

In this talk we outline a complete solution to this problem (and a rigorous derivation of (1)). It will be shown that the physically relevant solutions are exactly those which build up a normally hyperbolic manifold for (1) as a singular perturbation problem. In addition, those solutions are good approximations of the solutions to the full Abraham-Lorentz equations. The true "effective equation" then is the corresponding equation on the center-like manifold, and this equation has only physically reasonable solutions. (Joint work with H. Spohn.)

A perturbation method based on integrating vectors

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In this paper a perturbation method based on integrating vectors and multiple scales will be presented for initial value problems for regularly or singularly perturbed ordinary differential equations. Asymptotic approximations of first integrals will be constructed which are valid on long time-scales. In some cases approximations of first integrals can be obtained that are valid for all times. Several examples are treated to show how this perturbation method can be applied.

On the asymptotic behaviour of solutions of singular initial value problems

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The following singular initial value problem is under consideration

$$\alpha(t)x'(t) = F(t, x(t), x'(t)), \tag{1}$$

$$x(0) = 0, \tag{2}$$

where $\alpha : (0, \tau) \longrightarrow R^{n\times n}$ - a matrix with continuous elements,

$$\alpha(t) = \text{diag}(\alpha_1(t), \ldots, \alpha_n(t)),$$

$\alpha_i(t) \to 0, \quad t \to +0, \quad i \in \{1, \ldots, n\},$ F is a continuous function in some domain. For any $\rho \in (0, \tau)$ ρ – solution of the problem (1), (2) is said to be a continuously differentiable function $x : (0, \rho) \longrightarrow R^n$ which is satisfield (1) when $t \in (0, \rho)$ and also $x(t) \to 0$ when $t \to +0$. Sufficient conditions are given wherein the problem (1), (2) has a nonempty set of ρ - solutions (where $\rho > 0$ is small sufficiently) with needed asymptotic properties when $t \to +0$. In particular, special cases $\alpha_i(t) = \beta(t)$, $i \in \{1, \ldots, n\}$ and $\alpha_i(t) = \beta_1(t),\ i \in \{1, \ldots, n_1\},\ \alpha_j(t) = \beta_2(t),\ j \in \{n_1 + 1, \ldots, n\}$ are under consideration. New efficient algorithms have been designed. All theorem's conditions are effective. The investigation methods can be used for solving of a wide range of problems of local analysis.

1.4 Stochastic Systems

Organizers : Ludwig Arnold, Peter Kloeden

Key note lecture

White Noise Eliminates Instability

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For deterministic systems with nontrivial limit behaviour the concept of a global attractor has turned out to be useful. A global attractor is a compact invariant subset of the state space, which attracts every bounded set.
When taking additional random influences into account, the concept of a global attractor has to be extended. One possible approach is to allow the attractor to be a random

set. A compact random set is said to be a *random attractor* if it is invariant in a suitable sense, and if it attracts every bounded deterministic set. It turns out that random attractors exist for several classes of systems, among them random perturbations of systems from mathematical physics.
When considering random influences of small intensity, the question of the relation between the attractor of the unperturbed system and the family of random attractors of the randomly perturbed systems arises. It turns out that, with the intensity of the perturbation tending to zero, the random attractor stays in arbitrarily small neighbourhoods of the unperturbed attractor almost surely.
However, in general the random attractor does *not* converge to the unperturbed attractor. Examples show that random perturbations may cause the random attractor to become *smaller* than the unperturbed attractor, where 'smaller' not only refers to set inclusion, but also to (Hausdorff) dimension.
A possible explanation of this phenomenon is the fact that the perturbed system becomes transient in a vicinity of unstable regions of the attractor of the unperturbed system, even if the intensity of the random perturbation is very small.

Invited lectures

Bifurcation theory for stochastic differential equations

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In many physical systems which can be modelled by Itô stochastic differential equations, the equilibrium and stability properties of the system change as certain parameters in the system vary. Stationary probability measures may appear or disappear; (almost surely) stable fixed points may become (almost surely) unstable; pairs of trajectories may diverge (almost surely) instead of converging. Very often these phenomena are associated with the change of sign of a relevant Lyapunov exponent. In this talk we shall give some examples and results concerning such systems.

Random homoclinic dynamics

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Transversal random homoclinic points play an important role for the description of smooth random dynamical systems exhibiting chaotic behaviour. In discrete time their existence gives rise to an invariant random homoclinic set on which the dynamics can be modeled by a random subshift of finite type. Following [1] we present a corresponding random version of the Smale Birkhoff Theorem which pays tribute to this phenomenon.

In continuous time a random Melnikov method can be used to have a recourse to the result in discrete time. We describe a method from [2] where criteria are provided in terms of simple zero-crossings of a Melnikov process for a randomly perturbed differential equation on $\mathbb{R}^p$ of the form

$$\dot{x} = f(x) + \epsilon g(\theta(t)\omega, x, \epsilon)$$

where f is a bounded C^3 function such that this equation possesses a hyperbolic fixed point x_0 for $\epsilon = 0$ and a homoclinic solution $\zeta : \mathbb{R} \to \mathbb{R}^p$ corresponding to x_0, i.e. $\lim_{t\to\pm\infty} \zeta(t) = x_0$, $(\theta(t))_{t\in\mathbb{R}}$ is an ergodic metric dynamical system on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with Ω being a topological space and $t \mapsto \theta(t)\omega$ being continuous for all $\omega \in \Omega$, $g : \Omega \times \mathbb{R}^p \times (-\epsilon_0, \epsilon_0) \to \mathbb{R}^p$ for some $\epsilon_0 \in \mathbb{R}_+$ is a measurable, uniformly bounded function which is C^3 for each fixed $\omega \in \Omega$ and continuous for each fixed (x, ϵ). In case of $p = 2$ and the unperturbed system being Hamiltonian, the Melnikov process can be nicely presented as

$$t \mapsto M(\theta(t)\omega) = \int_{-\infty}^{\infty} f(\zeta(s-t)) \wedge g(\theta(s)\omega, \zeta(s-t), 0)ds$$

and hence consists just of given quantities such that it can be, in principle, computed. Steinkamp's Melnikov criterion requires that the number of simple zero-crossings of the Melnikov process has positive expectation. If this is satisfied, then a Poincaré section can be constructed that leads to a random dynamical system in discrete time with a transversal random homoclinic point.

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Cohomology of flows of stochastic and random differential equations

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Suppose a stochastic differential equation in $d-$dimensional Euclidean space possesses a global forward flow. We investigate conditions under which this flow is conjugate to the flow of a non-autonomous random ordinary differential equation, i.e. one can be transformed into the other via a random diffeomorphism of $d-$dimensional space. Viewing a stochastic differential equation in this form which appears closer to the setting of ergodic theory, can be an advantage when dealing with asymptotic properties of the system.

Lyapunov's Second Method for Stochastic Differential Equations

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During the past century Lyapunov's second method has gained increasing significance and has given decisive impetus for modern development of stability theory of dynamic systems. A manifest advantage of this method is that it does not require the knowledge of solutions of equations and thus has exhibited a great power in applications. On the other hand, since Itô introduced his stochastic calculus about 50 years ago, the theory of stochastic differential equations has been developed very quickly. Naturally, Lyapunov's second method has been developed to deal with stochastic stability by many authors (cf. [1–11]). There are several books available expounding the main ideas of Lyapunov's second method for stochastic differential equations e.g. Arnold [1], Has'minskii [5], Kushner [7]. In recent years, the original ideas have been refined, generalized and extended in several directions. Many new concepts have been introduced. Especially, some new ideas and approaches which might provide an exciting prospect of further advancement are still in the initial stages of investigation. In this talk, some of these aspects will be discussed.

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The Conley Index for Random Dynamical Systems

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The Conley index provides a topological tool in the qualitative study of dynamical systems. We will extend this method to random dynamical systems, which serve as a model for dynamics influenced or perturbed by probabilistic noise.

Criteria for the existence of random invariant sets and invariant measures under the presence of noise are given. In particular we will present some results on stochastic stability of invariant objects.

On the rate of dispersion of stochastic flows

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Suppose that $\phi_t(x)$, $x \in \mathbb{R}^d$, $t \geq 0$ is a stochastic flow of homeomorphisms on $\mathbb{R}^d$. We study the growth of the diameter of the image $\phi_t(B)$ of the unit ball $B \subset \mathbb{R}^d$ as $t \to \infty$. Using *chaining* we show that under suitable Lipschitz and boundedness conditions on the local characteristics of the flow, $\phi_t(B)$ grows at most linearly in t. In the second part of the talk we discuss the special case of an *isotropic Brownian* flow in more detail. We show that if the top Lyapunov exponent is nonnegative, then the diameter of $\phi_t(B)$ (and even that of $\phi_t(S)$ for any nontrivial bounded connected subset S of $\mathbb{R}^d$) actually grows linearly in t i.e. $\liminf \frac{1}{t} diam\,(\phi_t(B)) > 0$ almost surely. If – on the other hand – the top Lyapunov exponent is negative, then the following dichotomy holds: with probability one $\phi_t(B)$ either grows linearly or shrinks to a point as $t \to \infty$ and both events happen with strictly positive probability.

The talk is based on joint work with M. Cranston (Rochester) and D. Steinsaltz (Berkeley). The problem is motivated by applications in oceanography (the temporal dispersion of an oil slick on the surface of the ocean).

Contributed talks

The comparison of the stability of two statistical parameter estimators under complex disturbances in the data

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In the report, we make a comparison of the orthogonal regression (OR) estimator and the least squares (LS) estimator of the parameters of the autoregressive system:

$$x_t^* = \sum_{i=1}^{n} a_i x_{t-i}^* + \sum_{i=0}^{n} b_i u_{t-i}^*, \quad t = n+1, \ldots, N, \quad (N > n+1).$$

The both "proper" and "non-proper" disturbances are considered, namely, those under which the estimators do not possess the estimate consistency, asymptotic unbiasedness, etc. We prove the theorem about the LS and the OR estimators identity as functions of data in the first order term of the Taylor-series expansion with respect to the "true" non-perturbed data value. Then, by Monte-Carlo analysis, we investigate the region of the estimates' linear dependence on the data under small disturbances. By the above theorem, in this region the LS and the OR estimators are identical. To implement more thorough Monte-Carlo simulations, the terms up to third order in the Taylor-series expansions for both the LS and the OR estimation criteria were analytically achieved. It was found that under circumstances considered as quite general, with non-proper disturbances, the OR estimator yields less mean square deviation, than the LS estimator.

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New Aspects of Non-Integrability of Multi-Dimensional Systems Based on Symbolic Dynamics

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Some essentially new approaches and results in the problem of analytic non-integrability of multi-dimensional systems are presented. Traditionally, the analytic non-integrability means the absence of non-trivial analytic (or even meromorphic) first integrals, i.e., functions constant along each trajectory of the system under consideration. The problem of obtaining conditions that guarantee these non-integrability properties is the classical one and goes back to H. Poincaré. Results related to lower-dimensional systems (two-dimensional mappings or three-dimensional flows, in particular, Hamiltonian systems with two degrees of freedom) are well-known. Nevertheless, the number of rigorous results related to multi-dimensional systems is very limited.

We will describe and discuss conditions that guarantee the absence of any meromorphic first integral in the multi-dimensional case, regardless of a finite dimension in question. This is the strongest analytic non-integrability and not any Hamiltonian nature (which is of no importance) is assumed here. The conditions obtained persist under small perturbations and can be constructively verified for concrete dynamical systems in mechanics and physics. Some relevant results were published in [1,2].

Firstly, it is well-known that under some simple conditions the presence of transversally intersecting separatrices of hyperbolic periodic solutions implies the existence of a set of the so-called "quasi-random" motions. These motions are described by methods of symbolic dynamics. In the case of a two-dimensional mapping or a three-dimensional flow, the presence of quasi-random motions leads to the analytic non-integrability. The corresponding proof by V. M. Alekseev is not already valid for higher dimensions. Specific multi-dimensional conditions will be described that guarantee the non-integrability. The cause of the latter phenomenon is that the set of quasi-random motions is "rough enough" and does not lie on any smooth regular submanifold of positive codimension.

Secondly, we consider branching of solutions in the complex domain from the point of view of symbolic dynamics methods. This allows us to reformulate the above non-integrability conditions to the present case. The conditions obtained are easily applicable for a wide class of dynamical systems arising in mechanics and physics, in particular, those where other non-integrability approaches are not applicable or very hardly applicable. Even in the lower-dimensional case, the conditions obtained are simpler and more general than the known ones.

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Lyapunov Exponents of Linear Stochastic Jump Diffusions

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Under the uniformly elliptic condition or the hypoelliptic condition as in the diffusion case, see [1,2], the linear stochastic jump diffusion process projected on the unit sphere is a Fellor process and has a unique invariant measure using the relation of the transition probability between diffusions and jump diffusions in [3]. The deterministic Lyapunov exponent has an integral expression on the sphere with respect to the ergodic invariant measure so that the almost sure exponential stability of the linear stochastic jump systems depends on the its sign. The critical case of zero Lyapunov exponent will be discussed and examples will be treated as appropriated.

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Unified Conditions for Parameter Identifiability in AR, ARMA, ARMAX, EIV stochastic linear systems

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We consider the class of parameterized stochastic systems which includes AR, ARMA, ARMAX, EIV, and AR(I)MAX model structures. Data samples are supposed to consist of records of finite length, with the sample size depending on the number of records. It is shown that under natural assumptions about distributions of initial values, input variables, and disturbances the systems have parameter identifiability conditions expressible by restrictions on ranks of the real matrices of the system coefficients. For the "proper" parameterizations, derived conditions are not only sufficient but necessary. The results remain valid for the systems with data formed by samples of continuous records with a length tending to infinity.

The system is described by the equations

$$[\gamma_0(\theta) + \cdots + \gamma_p(\theta)s^p]\,\tilde{w}(t) = [\mu_0(\theta) + \cdots + \mu_p(\theta)s^p]\,\varepsilon_1(t)$$
$$w(t) = \tilde{w}(t) + \varepsilon_2(t), \qquad t \in \{1, \ldots, N-p\}.$$

Input-output signals are joined together in the vector variable $\tilde{w}(\cdot) \in R^{(r+m)}$, s is a backward shift operator, $(s\tilde{w}(t) \doteq \tilde{w}(t+1))$, and $\varepsilon_1(\cdot)$, $\varepsilon_2(\cdot)$ are unmeasured disturbances. Then, $\gamma_i(\theta) \in R^{r\times(r+m)}$, $\mu_i(\theta) \in R^{r\times l}$ are parameterized matrix coefficients of the system, and $\theta \in \Omega \subset R^v$ is a fixed parameter vector. The system does not undergo the controllability and stability restrictions.

Let us introduce *the measured trajectory* $z \doteq [w(1); \ldots; w(N)] \in R^{N(r+m)}$. In the paper, we investigate identifiability of the parameter value θ given the set $\{z_1, \ldots, z_L\}$ of measured trajectories. Consistency of the θ estimates is considered in the limit $L \to \infty$.

Let $\mathbf{P}_\theta$ be the distribution of z caused by given distributions of $\varepsilon_1(\cdot)$, $\varepsilon_2(\cdot)$, $u(\cdot)$, and w_0. The system is called *globally identifiable at* θ, if $\mathbf{P}_\theta = \mathbf{P}_\xi$, $\xi \in \Omega$ implies $\xi = \theta$. The system is called *locally identifiable at* θ, if there exists a neighborhood $V(\theta) \subset \Omega$ such that $\mathbf{P}_\theta = \mathbf{P}_\xi$, $\xi \in V(\theta)$ implies $\xi = \theta$ [1].

The problem statement: *It is needed to define restrictions on the parameterization $\theta \leftrightarrow \varphi_\theta$, under which the system is locally (globally) identifiable at $\theta \in \Omega$.*

Theorem. *The system is globally (locally) identifiable at θ, if and only if the equation*

$$\rho(s)\,[\gamma_\theta(s), \mu_\theta(s)q] = [\gamma_\xi(s), \mu_\xi(s)]$$

with respect to unimodular $\rho(s) \in R^{r\times r}[s]$, orthogonal $q \in R^{l\times l}$, and $\xi \in \Omega$ ($\xi \in V(\theta) \subset \Omega$), implies $\rho(s) \equiv I$, $q = I$, $\xi = \theta$.

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1.5 Symmetry

Organizers : Michael J. Field, Martin Golubitsky

Key note lecture

Bifurcation with Euclidean symmetry: Universality, Reduction and Ginzburg-Landau equations

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I will consider mathematical issues concerning Ginzburg-Landau theory, by which I mean the validity, universality, and structure, of reduced equations near criticality in spatially extended systems. The extraction of Ginzburg-Landau equations (variously known as amplitude, modulation and envelope equations) is part of this theory. Particular attention is paid to the Euclidean symmetries present in such systems.

In local bifurcation theory, the word ‘universality’ has a precise meaning and refers to those aspects of local bifurcations that are driven by the type of bifurcation and the symmetries that are present. This point of view goes back to Landau’s theory of second order phase transition in crystals, and is made mathematically precise for general problems with compact symmetry group in equivariant bifurcation theory [1]. Spatially extended systems have noncompact, usually Euclidean, symmetry, and the Ginzburg-Landau equations are supposed to be universal (in the above sense) for bifurcations in such systems. In my talk, I will describe complete results on universality for local bifurcations in systems with Euclidean symmetry. Some of these results can be found in [2] and show that the theory is richer than is often realized.

Another feature of Landau theory and equivariant bifurcation theory is reduction, locally, to a minimal system of reduced equations (for example, by center manifold reduction). I will present such a reduction method, preserving ‘essential’ solutions, for systems with Euclidean symmetry. Again, some of these results can be found in [2].

The extraction of Ginzburg-Landau equations corresponds to truncation of the reduced equations at lowest order. This step is problematic even formally. In situations where the extraction of Ginzburg-Landau equations is possible, their (exact, for all time) validity is clarified by the methods described in this talk. In addition, the presence and implications of normal form symmetry in the Ginzburg-Landau equations will be discussed.

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Invited lectures

Drifts for Euclidean Extensions of Dynamical Systems

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(This is joint work with I. Melbourne and M. Nicol.) We consider the behaviour of generic group extensions of ergodic dynamical systems by noncompact groups, in particular by $SE(n)$ the special euclidean group. A natural question of physical interest is whether the norms of solutions of such systems are unbounded or not, and if they are unbounded then at what rate do they typically grow. For extensions of uniformly hyperbolic mappings we show using a central limit theorem argument that there are generically unbounded orbits whose variance grows proportional to time. For group extensions of quasiperiodic and periodic we give conditions that ensure generic boundedness and unboundedness. Some implications of these results on the behaviour of instabilities of reaction diffusion systems on unbounded domains is also considered.

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Bifurcation of relative equilibria in Hamiltonian systems

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The relative equilibria of a symmetric Hamiltonian dynamical system are the critical points of the so-called *augmented Hamiltonian*. In the same spirit as in the work of Krupa [1] in the context of non-conservative systems, the underlying geometric structure of the system is used to decompose the critical point equations and construct a collection of implicitly defined functions and reduced equations describing the set of relative equilibria in a neighborhood of a given relative equilibrium. The analytical and group invariance structure of these reduced equations is exploited to solve the problem in specific cases. In particular, a persistence result of Lerman and Singer [2] is generalised to the framework of Abelian proper actions. The method is illustrated with an example from wave resonance in mechanical systems which was initially studied with a different method by Chossat and Dias [3].

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Stable ergodicity

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We describe recent work on stable ergodicity for diffeomorphisms equivariant with respect a compact Lie group. Some of our results are joint work with Parry (Warwick) [1] and Nicol (UMIST) [3]. One of the techniques we use depends on constructing a symbolic dynamics (finite coding) on the orbit space of a group invariant (partially) hyperbolic set. This is done using a generalization of Markov partitions appropriate for the study of non-free group actions. (In general, dynamics on the orbit space will not be expansive.)

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Animal Gaits and Coupled Oscillators

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The observed spatio-temporal symmetries of animal gaits — that is, the rhythmic patterns of movement of the legs — provide useful clues to CPG architecture. Plausible assumptions about the symmetries of a CPG for quadrupedal locomotion that generates the range of symmetries of observed gaits imply that such a CPG should consist of eight nominally identical subcircuits, or 'cells', arranged in an essentially unique manner. Two cells are assigned to each limb, and it is conjectured that their role is to control the timings of two distinct muscle groups. Analogous CPG architecture is proposed for bipeds, hexapods, and myriapods.

Predictions include the following. Gaits naturally divide into two classes: primary (walk, trot, ...) and secondary (gallops, canter, ...). There is a new primary gait called *jump*. There are restrictions on the wave numbers of primary gaits of myriapods of which the common tripod gait of hexapods is an example. Bipeds perform two distinct gaits with the 'out-of-phase' symmetry of the walk, and two distinct gaits with the 'in-phase' symmetry of the hop. Some evidence suggests that the two walk gaits in humans are the usual walk and run gaits; in the walk the limbs flex at the ankle, and in the run the ankle is more rigid.

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Forced symmetry breaking and pattern formation in weakly anisotropic systems

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Pattern formation in two-dimensional weakly anisotropic systems is considered from the point of view of forced symmetry breaking. In the absence of anisotropy the problem is formulated on an isotropic spatially periodic lattice with a symmetry group G. This group acts on the amplitudes $\mathbf{z}$ via its representation Γ [1]. In the presence of anisotropy the group G acts simultaneously on the (two-dimensional) applied anisotropy vector $\mathbf{s}$, but does so in general with a different representation Γ'. The notion of (Γ, Γ') co-equivariance is introduced to describe dynamical systems which are G-symmetric in the sense that

$$\gamma(g) \cdot \mathbf{f}(\mathbf{z}, \mathbf{s}) = \mathbf{f}(\gamma(g) \cdot \mathbf{z}, \gamma'(g) \cdot \mathbf{s}), \qquad \forall g \in G. \tag{1}$$

Using Poincaré series the most general (Γ, Γ') co-equivariant systems are constructed [2]. The resulting equations for $G = T^2 \dot{+} D_6 \oplus \mathbb{Z}_2$ and $G = T^2 \dot{+} D_6$, corresponding to pattern formation on a hexagonal lattice with and without midplane reflection symmetry, are truncated at third order in both $\mathbf{z}$ and $\mathbf{s}$ and analyzed. The results are applied to Rayleigh-Bénard convection in weak imposed Poiseuille and Couette flows.

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Bifurcation from discrete rotating waves

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My talk will address recent developments in the understanding of bifurcation from periodic solutions in equivariant dynamical systems. The focus will be mainly on the theory for bifurcation from *isolated* periodic solutions in dynamical systems with a compact symmetry group.

Equivariant bifurcation theory is concerned with the bifurcations of equilibria and periodic solutions in differential equations that are equivariant with respect to the action of a (compact) Lie group. A systematic approach to bifurcation from symmetric equilibrium solutions is laid out in [1]. Importantly, bifurcations in equivariant systems are generally different from bifurcations one would expect in non-symmetric systems.

Until recently, a theory for bifurcation from symmetric periodic solutions was developed only to deal with certain special situations. The theory for periodic solutions with purely spatial symmetries was developed by Chossat and Golubitsky [2]. Fiedler [3] studied systematically bifurcation from periodic solutions with discrete spatiotemporal symmetry using return map techniques. However, the latter study was confined to cyclic symmetry groups.

The treatment of bifurcation from periodic solutions with spatiotemporal symmetries using return map techniques was taken up again recently by Lamb [4]. It turns out that an extension of Fiedler's approach involves consideration of *twisted equivariant* maps (called k-symmetric maps in [4,6]). In [5], Lamb and Melbourne developed a systematic theory for generic bifurcation from spatiotemporally symmetric periodic solutions (with compact symmetry groups) that are isolated in phase space, using the return map approach.

My talk will survey some of the main principles and results in studying bifurcation from periodic solutions with spatiotemporal symmetry.

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Translational Symmetry-Breaking for Meandering Spirals

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Mathematical models with Euclidean symmetry successfully describe the meandering behavior of spiral waves (e.g. in Belousov-Zhabotinsky chemical reactions). However, no physical experiment is ever infinite in spatial extent, so the translational symmetry is only approximate. As with any approximation, it is natural to question the circumstances and the extent to which it is valid. For example, in the Belousov-Zhabotinsky reaction, the Euclidean model is extremely successful. However, for spiral waves in cardiac tissue, one expects the presence of blood vessels and other inhomogeneities to render this Euclidean approximation less than ideal. There is thus a range of problems involving spiral waves which are more or less well-modeled with full Euclidean symmetry. It would therefore be interesting to characterize the effects of system symmetry-breaking terms on the meandering behavior of spirals.

In this talk, we investigate the effects on spiral wave dynamics of breaking the translation symmetry while keeping the rotation symmetry. This is accomplished by introducing a perturbation in the five-dimensional center bundle equations (describing Hopf bifurcation from one-armed spiral waves) which is SO(2)-equivariant but not equivariant under translations. This system symmetry-breaking term induces a "bifurcation at infinity" which is analyzed in detail.

Time-Reversible Equivariant Linear Systems

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This talk will describe joint work with Igor Hoveijn (Groningen) and Jeroen Lamb (Warwick/Imperial) on the normal form theory of linear systems of differential equations which are equivariant with respect to actions of half the elements of a compact Lie group and time-reversible with respect to the other half. General and Hamiltonian linear systems will both be considered. At the heart of the theory lies a representation theory result which reduces the classification of all such systems to the study of 10 'universality classes'. For some of these universality classes the normal form theories are isomorphic to Jordan normal form theories of classical Lie algebras, but for the others new classifications are needed.

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Bifurcations from relative periodic solutions

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Relative periodic solutions are ubiquitous in dynamical systems with continuous symmetry. Recently, Sandstede, Scheel and Wulff derived a center bundle theorem, reducing local bifurcation from relative periodic solutions to a finite dimensional problem. Independently, Lamb and Melbourne showed how to systematically study local bifurcation from isolated periodic solutions with discrete spatiotemporal symmetries.

In this talk, we show how the center bundle theorem, when combined with certain group theoretic results, reduces bifurcation from relative periodic solutions to bifurcation from isolated periodic solutions. In this way, we obtain a systematic approach to the study of local bifurcation from relative periodic solutions.

Contributed talks

Symmetries in Exterior Differential Systems

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Using Lie's symmetry approach for integrating Frobenius integrable vector field distributions, we present two symmetry techniques for generating solutions of first order nonlinear PDEs. In the first, we use the idea of Cauchy characteristics to show how one is able to solve the Cauchy problem using symmetries.

In the second, we consider a first order non-linear PDE and derive its corresponding *Vessiot distribution*, whose integrability conditions generate a first order quasilinear PDE. We then show that Lie's symmetry approach can be used to easily find a family of solutions of this quasilinear PDE that can then be used to generate solutions of the original first order non-linear PDE.

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Hopf bifurcations on the FCC lattice

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When a system undergoes a Hopf bifurcation to a pattern forming instability with spatial symmetry group Γ, the Equivariant Hopf Theorem [1] guarantees a unique primary solution branch for every $\mathbb{C}$-axial isotropy subgroup of $\Gamma \times S^1$, where S^1 represents the group of temporal phase shifts. For Γ a wreath product group, the method of Dias [2] can be used to determine all such $\mathbb{C}$-axial subgroups easily. In this paper we analyze three-dimensional pattern forming Hopf bifurcations with the spatial periodicity of the face-centered cubic (FCC) lattice. This is an equivariant Hopf bifurcation with spatial symmetry $\Gamma = T^3 \dot{+} \mathbb{O} \oplus \mathbb{Z}_2$, where T^3 is the 3-torus of translations, $\mathbb{O}$ is a representation of the octahedral group of orientation-preserving symmetries of the cube, and the nontrivial element of $\mathbb{Z}_2$ represents spatial inversion through the origin. We use Dias' method by first extending Γ to the larger group $O(2) \wr S_4$. By relating the two sets of $\mathbb{C}$-axial subgroups, we find all solution branches guaranteed by group theory to be primary. We use the Poincaré series to find the most general possible equivariant system of amplitude equations up to cubic order, which determines the possible bifurcation diagrams for the primary solution branches. Although motivated by reaction–diffusion problems, our results are done in a model-independent fashion, and are applicable to a wide variety of pattern formation problems near onset. This work is an extension of that done for the steady state FCC bifurcations in [3, 4].

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The relations between Painlevé property and Lie symmetries of differential equations

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Second order ordinary differential equations can be classified by investigating the realizations of real low dimensional Lie algebras in terms of vector fields in two dimensions [1]. An equation admitting r-dimensional, r=1, 2, 3 or 8, Lie algebra of point symmetry generators possess a canonical representative for the corresponding differential equation.

For the case $r = 1$ the differential equation can be transformed to an autonomous form. The case $r = 2$ gives two classes of representative equations. For equations having three point symmetries, there are five representatives of equivalance classes. For $r = 8$, a number of linearizability results proved in literature [1].

It is of great interest to compare the Lie classification of second order equations with the Painlevé classification which produces 50 equations [2,3]. The question which arises is this: Is there an overlap between the Lie classification and the Painlevé classification? The first step in answering this question is to determine the symmetries of the Painlevé equations. Alternatively one has to reduce some of the Lie equations to the Painlevé ones.

The aim of this work is to bring some light on this issue, using restricted point transformations.

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Polynomial Infinitesimal Generators of Lie's Symmetries for Polynomial Planar Vector Fields

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In this work we show that if all the intersection points of the projective curves defined by the components of the polynomial vector field submerged in $\mathbb{C}\mathbb{P}^2$ are different, the critical points of the components of the polynomial planar vector field are strong and there exists a polynomial inverse integrating factor then there exists a polynomial infinitesimal generator of Lie's symmetry. The result continues being valid if all the critical points, be strong or not, belong to the algebraic curve defined by the polynomial inverse integrating factor. In these cases the Lie's symmetries are related with the class of funtions which belongs to the inverse integrating factor independently of the class of their first integral.

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Generic bifurcation from a 2-manifold

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Systems with continuous symmetry typically have equilibrium sets that are n-manifolds with $n > 0$. Under symmetry-breaking perturbations such a manifold breaks up into lower-dimensional pieces or to a discrete set of points. We describe the geometry of equilibrium configurations that arise generically in the particularly interesting case when $n = 2$ (for spherical or toroidal symmetry, for example), both when the symmetry is totally broken and when some residual symmetry persists.

On the stability of bifurcating periodic orbits in reversible and symplectic mappings

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We survey some recent results on the bifurcation of periodic points from a fixed point in parametrized families of reversible or symplectic diffeomorphisms; such bifurcation problems arise for example when studying subharmonic branching in reversible or Hamiltonian systems. In some joint works with A. Vanderbauwhede we analyzed some elementary "branching phenomena" which can occur near a fixed point; we provided a "structure-preserving" reduction result which can be used to study such branchings and we also briefly discussed how one can determine the stability of bifurcating periodic orbits. Here, we will fix our attention on this last problem, pointing out the primary role played by normal form theory. As an application we provide the complete analysis of the stability of periodic orbits appearing in the simplest bifurcation scenario for both Hamiltonian and reversible systems.

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Smooth normalization of reversible vector fields

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It is shown that Sternberg linearization theory remains true in the category of reversible vector fields.

Let $R : \mathbb{R}^n \to \mathbb{R}^n$ be a linear involution, *i.e.*, $R^2 = I$. A vector field ξ is said to be *R-reversible* if

$$R_* \xi = -\xi.$$

Two vector fields ξ and η are called *R-equivalent* if there is a diffeomorphism $H : \mathbb{R}^n \to \mathbb{R}^n$ such that $H \circ R = R \circ H$ and

$$H_* \xi = \eta.$$

Theorem. *Let k be a positive integer and A be an R-reversible hyperbolic linear vector field. There exists a number $Q = Q(A, k)$ such that if two smooth vector fields ξ and η are R-reversible, $\xi(0) = \eta(0) = 0$, $A \equiv D\xi(0) = D\eta(0)$, and the vector field $(\xi - \eta)$ is Q-flat at the origin, then ξ and η are locally C^k R-equivalent.*

This solves the problem recently posed by Bonckaert.

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Calculation of the solution of Boussinesq Problem using Lie symmetries

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Firstly, the Lie point symmetries of cylindrically symmetric homogeneous Navier equations are obtained. Using the symmetries the general class of similarity solutions is found. The subclass that also satisfies the non-homogeneous system of the medium subject to a singular force is determined. Substituting the subclass into the non-homogeneous system, a system of ordinary differential equations is obtained. The solution of the system satisfying the boundary conditions of Bousssinesq problem gives the exact solution.

Applications of a new G-Invariant Implicit Function Theorem

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The presentation concerns a generalization [1, Theorem 3.1] of the G-invariant implicit function theorem of E. Dancer [2]. It will be shown how this generalization can be applied to rotating and modulated wave solutions of nonlinear equivariant autonomous or periodic evolution equations (with applications to laser dynamics) and to nonlinear elliptic boundary value problems on bounded symmetric domains (with applications to elastostatics).

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On time-dependent symmetries of evolution equations

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We investigate the symmetry algebra of the generic scalar (1+1)-dimensional evolution equation. Using some auxiliary results, we have found the formulae for the leading terms of commutators of two symmetries and two formal symmetries and have proved several general theorems on the structure of the algebra in question. The generalization of the above results to the case of system of evolution equations is also discussed.

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Hopf-zero Bifurcations of Reversible Vector Fields

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Let X be a (germ of) C^∞ vector field defined on $\mathbb{R}^n, 0$ given by an ordinary differential equation

$$X: \quad \dot{x} = F(x), \quad x \in \mathbb{R}^n, \tag{1}$$

where $F(x)$ is a smooth function, $F(0) = 0$. We say that system (1) is time-reversible if there is a germ of a smooth involution $\phi : \mathbb{R}^n, 0 \to \mathbb{R}^n, 0$ ($\phi \circ \phi = id.$) such that the relation

$$F(\phi(x)) = -\phi'(x) \cdot F(x), \quad x \in \mathbb{R}^n, 0 \tag{2}$$

holds.

A φ time-reversible vector field X on $\mathbb{R}^n$, 0 is called to be of type (n, k) if the dimension of the fixed point set of φ, $S = Fix\{\phi\}$, is equal to k.

The main objective of the paper is to study symmetric critical points of generic one-parameter families of reversible vector fields of type $(3, 1)$ with emphasis on the topological classification of all such systems. We assume that all the systems considered here are ϕ_0 reversible, where $\phi_0(x, y, z) = (-x, -y, z)$. Note that in generic case the eigenvalues of such systems are either $(0, \alpha, -\alpha)$ or $(0, \alpha i, -\alpha i)$, where α is a nonzero real parameter. We focus on systems which have eigenvalues $(0, \alpha i, -\alpha i)$.

The main results of this paper include a classification and qualitative behavior of symmetric singularities occurring generically for 1-parameter families of (3,1)-type vector fields. Also interesting conclusions are obtained from the classification which involves the existence of invariant tori, periodic orbits, homoclinic and heteroclinic orbits for vector fields considered. We also give a complete list of bifurcation diagrams.

On symmetry reduction in reversible and Hamiltonian systems

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We consider reversible systems which are equivariant with respect to a compact continuous group of symmetry operators compatible with the reversibility. We use the approach of M. Krupa to study bifurcations near relative equilibria which are invariant under the reversing symmetry. We show that such bifurcations are completely determined by the bifurcations near an equilibrium of a reversible "normal" vectorfield. We discuss several examples in detail. We illustrate a similar approach for Hamiltonian systems on an example, namely when the symmetry group is S^1. For this Hamiltonian case a general theory along these lines is still lacking, but we hope to present at least some new partial results.

Symmetries and Mode Interactions in the Optimisation of Hydrophone Placement in Acoustic Arrays

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The placement of a number of hydrophones (underwater microphones) in an array is studied with a view to optimising the signal to noise ratio. This leads to an optimisation problem for a performance measure on $\mathbb{R}^{2n}$ for the horizontal array case (where we are optimising over the spatial configuration of n hydrophones). That is, the position of the ith hydrophone is given by $x_i \in \mathbb{R}^2$ and the array configuration is given by $\underline{x} = (x_1, \ldots, x_n)$. The performance measure derived in Haywood (1994) is of the form

$$\nu(\underline{x}) = \frac{1}{k_2 - k_1} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{W(k_2|x_i - x_j|) - W(k_1|x_i - x_j|)}{|x_i - x_j|}.$$

Here $k_i = 2\pi f_i/c$, where $f_1 \leq f \leq f_2$ is the frequency range which the array is designed to search, and the function W is defined as

$$W(u) = \int_0^u J_0(v)^2 dv$$

where J_0 is the Bessel function of order 0.

Both this underlying performance measure, and the solutions observed by simulated annealing (Haywood 1994), exhibit strong symmetry properties. In particular, optimisation of the performance measure can be reduced to considering a set of equations which are $O(2) \times S_n$ equivariant. These equations can be augmented with a parameter λ which allows us to consider a bifurcation from the trivial solution of all the hydrophones lying at the same co-ordinate. By considering the solutions branches we would expect to see in such a system, including simple mode-interactions (using the standard approach of, for example, Golubitsky et. al. 1988) we reduce the problem to optimising a series of functions living on $\mathbb{R}$ and $\mathbb{R}^2$ which provides a systematic sweep across many of the local minima (for a array with n hydrophones we require 2 optimisations of one-dimensional functions and $n/2$ or $(n-1)/2$ optimisations over $\mathbb{R}^2$ depending on whether n is even or odd).

At the very least the results thus obtained matches the results obtained through simulated annealing for finding global minima using an extremely cheap numerical method, and in addition provide a useful classification scheme for possible configurations.

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1.6 Topological Methods and Conley Index

Organizers : Konstantin Mischaikow, Marian Mrozek

Key note lecture

On the Conley index over a base space

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Basing on the papers [1] and [2], we present a recent generalization of the Conley index theory. Let S be an isolated invariant set of a flow ϕ on a locally compact metrizable space X. For a given continuous map $\omega : X \to Z$, where Z is a Hausdorff space, we define the Conley index of S over the base space Z as the so-called fiberwise deforming homotopy type of the adjunction of an isolating block B for S and Z, via the map $\omega|_{B^-} : B^- \to Z$, where B^- is the exit set of B. Usually the index over a base provides more information then the classical Conley index if X is not contractible. The additional information can be applied to problems on continuation of isolating invariant sets. In the case of the suspension flow of a homeomorphism and Z equal to the circle, the new index can replace the discrete-time Conley index.

References

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Invited lectures

On transition matrices

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From the Conley index theory, a transition matrix gives algebraic information about the existence of codimension one connecting orbits between Morse components of a Morse decomposition of an isolated invariant set in a one parameter family of flows. There are basically three different formulations of transition matrices: singular, topological and algebraic transition matrices. In this talk, I shall discuss relations of these formulations and attempt to put them in a unified framework. As an outcome of this new view, I present a generalization of transition matrices for a family of flows with more than one parameters.

Periodic orbits for fourth order conservative systems and a Morse type theory

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For a large class of fourth order Lagrangian systems we can find periodic orbits on prescribed energy manifolds via analogues of area preserving twist-maps. Appropriate generating functions give rise to second order recurrence relations. By studying the associated gradient flow we find a multitude of periodic orbits (infinitely many in most cases). The arguments are based on a Morse type theory via the Conley index, where we use 'braid types' of periodic orbits to define isolating neighborhoods. The results apply for example to the Swift-Hohenberg and the extended Fisher-Kolmogorov equations [1], [2]. This is joint work with J.B. Van den Berg.

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Contributed talks

The implementation of the Allili-Kaczynski algorithm of the construction of chain homomorphism induced by multivalued map

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Our talk presents an implementation of the algorithm for computing chain homomorphisms induced by multivalued representable maps. For simplicity and efficiency of data representation, we use the cubic grid, which does not require computing of subsequent barycentic subdivisions. The theoretical background for the algorithm comes from Allili and Kaczynski. This is a part of a joint project of computing the Conley Index for discrete dynamical systems.

References

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Computer Assisted Proof of the Existence of a Periodic Orbit in the Rössler Equations

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In 1976, Rössler [4] introduced an example of a set of nonlinear differential equations in $\mathbb{R}^3$: $x' = -(y + z)$, $y' = x + by$ and $z' = b + z(x - a)$. Its numerical simulations show a variety of qualitative behaviours as the parameter a is changed, to begin with a stable fixed point through an attracting periodic orbit which experiences a series of period-doubling bifurcations to finally reach a strange attractor. Indeed, for $a = 5.7$ and $b = 0.2$ the existence of chaotic dynamics has been recently proved by Zgliczyński [5].

The values of the parameters discussed here are $a = 2.2$ and $b = 0.2$. For these values an attracting periodic orbit is numerically observed, but as far as we know there has been no strict proof of this fact so far. Our aim is to fill this gap.

In this proof of the existence of the periodic orbit, computer was used to construct heuristically a set N built of a huge number of tiny cubes to be a candidate for an isolating neighbourhood of the hypothetic orbit. For arbitrarily chosen $t > 0$, Lohner's method for strict integration of ordinary differential equations was used to prove that the image of N with respect to the time-t map was entirely included in the interior of N. In such a way an index pair $(N, \emptyset)$ was obtained. Then an algorithm by Kaczyński, Mrozek and Ślusarek [1] was used to compute homology of N, which was the homology of a circle S^1 as expected. According to a theorem by Mrozek [3], the maximal invariant set S contained in N with respect to the discretisation of the flow was also an isolated invariant set for the flow itself. Moreover, it had the same Conley Index. After some more computations performed in order to check additional assumptions, a theorem by McCord, Mischaikow and Mrozek [2] was applied to finally obtain the existence of a periodic orbit in this neighbourhood.

The method used here may make use of other algorithms being recently implemented to compute a discrete Conley Index of an isolating neighbourhood and to use the same theoretical basis to prove the existence of a periodic orbit even if it is not necessarily attracting. Moreover, this method is dimension-independent and thus may be used to prove the existence of a periodic orbit in $\mathbb{R}^n$ also for $n \neq 3$.

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Sensitive dependence on initial conditions using topological tools ?

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In my talk I would like to address the following question: *Can we say something about sensitive dependence on initial conditions, using topological tools only ?*

At first sight the answer to this question is negative, because topological tools like Conley index or fixed point index are tools which do not see what happens in very small portions of the phase space. The silent assumption behind this reasoning is that we use only finite number of sets on which we compute various topological invariants.

In this talk we discuss how to show existence of complicated orbits which are homoclinic to a hyperbolic periodic orbits both in discrete (map) and continuous (ODE) case.

Discrete case: Let x_0 be a hyperbolic fixed point for a homeomorphism $f : \mathbb{R}^n \to \mathbb{R}^n$. We can then construct a sequence of parallelograms N_i, $i \in \mathbb{N} \cup \{0\}$, $x_0 \in N_i$

$$N_{i+1} \subset N_i, \quad \lim_{i->\infty} \operatorname{diam}(N_i) = 0, \quad N_{i+1} \stackrel{f}{\Longrightarrow} N_i, \quad N_i \stackrel{f}{\Longrightarrow} N_{i+1}$$

where the symbol $\Longrightarrow$ denote a covering relation introduced in [1].

Suppose now that we have a loop of covering relations

$$N_0 \stackrel{f}{\Longrightarrow} M_1 \stackrel{f}{\Longrightarrow} M_2 \stackrel{f}{\Longrightarrow} M_2 \ldots \stackrel{f}{\Longrightarrow} M_k \stackrel{f}{\Longrightarrow} N_0 \tag{1}$$

From this we conclude that the exists a point $x_0 \in N_0$ such that $\lim_{i\to\pm\infty} f^i(x) = x_0$ and $f^i(x) \in M_i$ for $i = 1, \ldots, k$, which can correspond to a possibly complicated behavior.

To illustrate the **continuous case** we consider a planar $T = 2\pi/\kappa$-periodic equation

$$z' = (1 + e^{i\kappa t}|z|^2)\overline{z}, \quad z \in \mathbb{C} \tag{2}$$

for $0 < \kappa \leq 0.495$. Here origin is a hyperbolic periodic trajectory γ of period T. In [2] it was shown using two isolating segments U, W that this equation has symbolic dynamics on three symbols, i.e. we have a complicated dynamics. This coresponds to the loop covering relations (1). We construct an isolating segment Z for γ, such that diameter of $\lim_{|t|\to\infty} \operatorname{diam}(Z_t) = 0$ and for some t $U_t = Z_t$. From this we obtain homoclinic orbits to 0 with a complicated dynamics (see [3] for details).

References

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B. Infinite-dimensional Systems

2.1 Delay Equations

Organizers : Sjoerd Verduyn Lunel, Hans-Otto Walther

Key note lecture

Effects of time delays on the dynamics of feedback systems

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Recently there is a lot of attention devoted to feedback systems, i.e., optical feedback lasers, phase-locked frequency synthesizers and wave equations with feedback stabilization at the boundary. In the implementation of a feedback system, it is very likely that time delays will occur. It is therefore of vital importance to understand the sensitivity and robustness of the feedback system with respect to time delays in the loop. In this lecture we discuss a number of illustrative examples and some useful tools, based on the radius of the essential spectrum of the semigroup associated with the system, to study robustness with respect to time delays.

References

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Invited lectures

Chaotic motion in delay equations

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Since the late 1970s, chaotic dynamics in delay equations was observed numerically, and analytical proofs for the existence of such behavior in specific examples were obtained from 1980 on. We give a review of results in this direction up to the present, describing different mechanisms for the creation of erratic motion. Such mechanisms are: Conjugacy to chaotic interval maps, unstable periodic solutions and transversally homoclinic orbits, and Šil'nikov-like saddle connections.

The Global Dynamics of Cyclic Feedback Systems with Delay

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We show that in a cyclic feedback system with delay, no solution which has a finite rate of oscillation (as measured by a discrete Lyapunov function) can approach an equilibrium at a rate faster than exponential. As a consequence of this rather technical result, we obtain information about a variety of dynamical phenomena, including Floquet exponents and subspaces, Morse decompositions, and Morse-Smale properties (transversality) of such systems.

Synchronization and Phase-locked Oscillations in Delayed Neural Networks: Connecting Orbits and Basins of Attraction

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We consider the following system of delay differential equations

$$\dot{x}_1(t) = -\mu x_1(t) + f(x_2(t-\tau)),$$
$$\dot{x}_2(t) = -\mu x_2(t) + f(x_1(t-\tau))$$

which describes the computational performance of a network of two identical neurons with delayed feedback, where x denotes the activation, $\mu > 0$ is the decay rate, $f : \mathbb{R} \to \mathbb{R}$ is the signal function and $\tau > 0$ is the synsptic delay. In many applications such as content-addressable memory, it is very important to give a detailed description of the structures of the global attractor.

The synchronized activities ($x_1 = x_2 = y$) of the network are characterized by the scalar DDE $\dot{y}(t) = -\mu y(t) + f(y(t-\tau))$. The detailed fine structures of a basic attracting invariant set as a 3-dimensional smooth solid spindle inside the global attractor of the scalar equation were presented in the monograph of Krisztin, Walther and Wu [The Fields Inst. Monographs Series, Vol. 11, AMS, 1999], the connection of this spindle with the global attractor was discussed in Krisztin and Walther [preprint, 1999], the singularity of the spindle at the tips was described in Walther [preprint, 1999], and the topological and geometric structures of high dimensional invariant sets are being investigated in Krisztin and Wu [preprint, 1999].

Recent work of Chen and Wu [Advances in Differential Equations, to appear; Physics D, to appear] and Chen, Krisztin and Wu [J. Differential Equations, to appear] shows that the general geometric approach developed in the aforementioned monograph, a discrere Lyapunov function which counts sign changes of elements in the phase space and the Poincare-Bendixson theory developed in Mallet-Paret and Sell [J. Differential Equations, 125, 385–489, 1996] for cyclic systems of DDEs can be effectively applied to describe the dynamics of certain sets of asynchronous orbits. In particular, we obtain when τ passes the first Hopf bifurcation value τ_1 an analog of the above 3-dimensional solid spindle which is separated into the basins of attraction of the two tips by a smooth disk bounded by a phase-locked orbit. We also show that the unstable space of the monodromy operator of the phase-locked orbit is at least 3-dimensional, when τ passes the second Hopf bifurcation value τ_2 where a bifurcation of the aforementioned solid spindle consisting of synchronous orbits, and there are connecting orbits from the synchronous periodic orbit to the phase-locked orbit. This provides a mechanism for desynchronization.

Limiting profiles as square waves or pulses of the phase-locked and synchronized orbits as the delay approaches infinity are also investigated in some recent works of Chen and Wu [preprint, 1999].

Contributed talks

Analytic solutions to partial differential equations with deviating arguments

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We investigate analytic solutions to the following problem

$$D_t u(t,z) = f\left(t, z, \left\{(I^i D_z^j u)(\alpha^{(i,j)}(t,z))\right\}_{i,j=0}^{K}\right) ,$$

$$u(0,z) = \phi(z) , \qquad (t,z) \in G_\eta ,$$

on the Haare pyramid

$$G_\eta = \left\{(t,z) \in \mathbb{C}^2 : z \in \Omega,\ |t| \leq \eta \min\left(\hat{d},\, \mathrm{dist}(z, \partial\Omega)\right)\right\}$$

and Ω is an open subset of the complex plane with nonempty boundary.

The symbols D_t, D_z mean the partial derivatives and

$$(Iw)(t,z) = \int_0^t w(s,z)ds$$

We consider existence and uniqueness of analytic solutions and continuous dependence on known functions with respect to some weighted norms in a space of analytic functions.

Computational Scheme of a center manifold for neutral functional differential equations

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This work addresses a computational algorithm of terms of a center manifold for neutral functional differential equations. The Bogdanov-Takens and the Hopf singularities are considered. Finally, as an illustration of our scheme, we give an example where the second term of a center manifold is determined explicitly.

Real-Analytic Solutions of Functional Equations

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We consider an equation

$$\phi(x) - A(x)\phi(Fx) = \gamma(x), \qquad x \in \mathbf{R}^n,$$

in real-analytic functions $\phi(x)$ with given real-analytic matrix function $A(x)$, vector function $\gamma(x)$ and mapping $F : \mathbf{R}^n \to \mathbf{R}^n$. The relations are studied between local and global solvability of the equation. In particular we state conditions which guarantee that the local solvability implies the global one.

On existence of periodic solutions for a linear difference system with continuous argument under periodic control

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The purpose of this paper is a condition for the system of difference equations with continuous argument under continuous and periodic control $u(t)$

$$x(t+1) = Ax(t) + u(t); \tag{1}$$

$$x(\tau) = \varphi(\tau), \quad \tau \in [0, 1),$$

that must be fulfilled for existence of a periodic solution. Here $u(t)$ is a continuous and T-periodic by t n-vector-function, $t \in R^+$, A is a real nonsingular $n \times n$ matrix, $x(t)$ is a real phase vector, $\varphi(t)$ - is an initial n - vector-function. Solution of this problem is an initial step in stabilization of a periodic motion for the system (1). An extensive literature is devoted to the theory of such kind of systems, for example [1,2]. There has been much interest recently in the construction of general or special solutions for difference systems with continuous argument, see e.g. [3,4]. In particular, existence conditions for a periodic solutions with an integer period of the system (1) were established, for example in [5].

This paper is devoted to existence conditions for periodic solutions of (1) with arbitrary period (both commensurable with the step of the system and incommensurable). The general solution of the homogeneous system

$$x(t+1) = Ax(t), \tag{2}$$

corresponding to (1) is constructed. Existence conditions of a periodic solutions for (2) are developed. It was shown that the system (1) has a T-periodic solution if and only if matrix A has an eigenvalue equal to $2\pi ik/T$, and the initial function $\varphi(\tau)$ in accordance with T has a form: for an integer T - $\varphi(\tau)$ is arbitrary 1-periodic function; for a rational $T = k/m$ (where k and m are mutually disjoint integers) - $\varphi(\tau)$ is $1/m$ - periodic by τ function; for irrational T - φ is arbitrary constant. For the system (1) the sufficient conditions for existence of a periodic solution with integer, rational and irrational period are obtained. The theory of normal form by Poincare is used [6, chap. 5]. The problem of series' convergence arising from the small devisors problem is investigated on the base of [6, p. 108]. Illustration examples are given.

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Neutral Equations with Causal Operators

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The paper is concerned with existence results, both local and global, for functional equations of the form $(Vx)(t) = (Wx)(t)$, in which V and W stand for causal (nonanticipative) operators on various function spaces.

The case of functional differential equations of the for $(d/dt)(Vx)(t) = (Wx)(t)$ is also covered.

Methods of Nonlinear Analysis are used in proving the existence results. Many special types of equations are treated as particular cases of the general equations mentioned above.

Asymptotic properties of functional differential equations

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New results on boundedness/unboundedness of solutions and exponential stability of differential equations with delayed argument and integro-differential equations are proposed.

A) In previous studies in this theme stability of integro-differential equations was achieved through their "ordinary part". As to their integral part, it was required to be small enough. In our approach stability is achieved through the "integral parts". Tests for exponential stability even in the case of "non-stable ordinary part" are proposed.

B)Delay in the following differential equations of a second order

$$\ddot{u}(t) + p(t)u(t - h(t)) = 0 \tag{1}$$

destroys stability. It is shown that for non-decreasing positive bounded coefficient $p(t)$ convergence of the integral $\int_0^\infty h(t)$ is necessary and sufficient for boundedness of all solutions. The case when $p(t)$ tends to infinity has been also considered.

Invariant manifolds for RFDE under discretization

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In our previous work we showed that the unstable manifold around hyperbolic equilibrium of a scalar retarded functional differential equation (RFDE for short) is preserved under the Euler method. That is, if $\mathcal{W}^U$ and $\mathcal{W}^{U_h}$ denote local unstable manifolds of the original and the Euler discretized equation, respectively, then $\|g - g_h\|_{C^0} \to 0$ as $h \to 0$, where g and g_h are functions representing the manifolds locally as graphs and h is the step size.

The aim of the present talk is two-fold. On one hand we generalize the above mentioned result by allowing general RFDE and general numerical method. On the other hand we show the higher order closeness of invariant manifolds, i.e. $\|g - g_h\|_{C^j} \to 0$ as $h \to 0$, where j depends on the order of the numerical method. The proof is based on the Parameterized Contraction Mapping Principle while smoothness follows from the Fibrum Contraction Theorem.

Connections to the Hartman-Grobman Theorem are also discussed. Using it we obtain conjugacy between the solution flow and its numerical approximation on a neighborhood of the hyperbolic equilibrium lying on the global attractor.

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On error estimates for waveform relaxation methods for delay differential equations

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In the talk we present delay dependent error estimates for the iterative processes

$$x'_{k+1}(t) = F(t, x_{k+1}(t), x_k(t), x_k(\cdot)), \quad k = 0, 1, \ldots, \quad t \in J = [0, T],$$
$$x_{k+1}(t) = g(t), \quad t \in [-h, 0]$$

for solving the initial value problems for systems of equations of the form

$$\begin{aligned} x'(t) &= f(t, x(t), x(\cdot)), \qquad t \in J = [0, T], \\ x(t) &= g(t), \quad t \in [-h, 0], \quad h > 0, \end{aligned}$$

where $f(t, x, y(\cdot)) = F(t, x, x, y(\cdot))$. For F we assume the one-sided Lipschitz condition with respect to the second argument

$$(F(t, x, y, z) - F(t, \bar{x}, y, z), x - \bar{x}) \leq m(t)\|x - \bar{x}\|^2$$

and the Lipschitz conditions with respect to the last two arguments

$$\|F(t, x, y, z) - F(t, x, \bar{y}, \bar{z})\| \leq K(t)\|y - \bar{y}\| + L(t)\|z - \bar{z}\|_{\beta(t)}$$

with some continuous nondecreasing function β such that $0 \leq \beta(t) \leq t$, where $\|y\|_t = \max_{-h \leq s \leq t} \|y(s)\|$ for $t \in J$. When x_* is the exact solution of our initial value problem and $u_k(t) = \max_{0 \leq s \leq t} \|x_*(s) - x_k(s)\|$ then we obtain the following results.
If $m(t) < 0$, $K(t) \equiv 0$, $P_*(t) = \max_{0 \leq s \leq t} \left[\frac{L(s)}{-m(s)}\right]$, then

$$u_k(t) \leq u_0(\beta^k(t)) \left(\prod_{i=0}^{k-1} P_*\left(\beta^i(t)\right)\right) H_k(t),$$

where the sequence $\langle H_k \rangle$ of functions depends on m and satisfies the conditions $0 \leq H_k(t) \leq 1$, $\lim_{k \to +\infty} H_k(t) = 0$.

Another result reads: if $m(t) \geq 0$ and $K(t) \equiv 0$, then

$$u_k(t) \leq \left(\prod_{i=0}^{k-1} L(\beta^i(t))\right) \exp\left(\int_0^t m(s)ds\right) u_0(\beta^k(t)) \int_0^t \Phi_{k-1}(1)(s)ds, \quad t \in J,$$

for $k = 1, 2, \dots$, with a suitably defined sequence $\langle \Phi_k(1) \rangle$ depending on β.
We have also the following result: if $\overline{m}(t) = \max(0, m(t))$, then

$$u_k(t) \leq \frac{t^k}{k!} \exp\left(\int_0^t \overline{m}(\tau)d\tau\right) \left(\sum_{i=0}^{k} \left[\binom{k}{i} K^{k-i}(t) L_*^i(t) u_0(\beta^i(t))\right]\right), \quad k = 0, 1, \dots ,$$

where $L_*(t) = \sup_{0 \leq s \leq t} \left[L(s) \exp\left(-\int_{\beta(s)}^s \overline{m}(\tau)d\tau\right)\right]$.

On equations with delay depending on solution

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Consider the following equation

$$\dot{x}(t) + \sum_{i=1}^{m} A_i(t) x(t - (H_i x)(t)) = f(t), \quad t \in [0, \infty), \tag{1}$$

$$x(\xi) = \varphi(\xi), \quad \xi < 0.$$

Assume that the elements of the $n \times n$ matrices A_i, $i = 1, 2, ..., m$, are measurable and essentially bounded on $[0, \infty)$, the function $\varphi : (-\infty, 0] \to \mathbf{R^n}$ is continuous, the function $f : [0, \infty) \to \mathbf{R^n}$ is measurable and essentially bounded. Suppose also that the operations H_i, $i = 1, 2, ..., m$, put into correspondence to each absolutely continuous function $z : [0, \infty) \to \mathbf{R^n}$ scalar measurable essentially bounded functions $H_i z : [0, \infty) \to \mathbf{R}$, $i = 1, 2, ..., m$. An absolutely continuous function $x : [0, \infty) \to \mathbf{R^n}$, is called a solution of equation (1) if it satisfies the equation almost everywhere.

Along with (1) let us consider a linear equation

$$(\mathcal{L}_x y)(t) \equiv \dot{y}(t) + \sum_{i=1}^{m} A_i(t) y(t - (H_i x)(t)) = \psi(t), \quad t \in [0, \infty), \tag{2}$$

$$y(\xi) = 0, \quad \xi < 0.$$

Here $x : [0, \infty) \to \mathbf{R^n}$ is an absolutely continuous function, $\psi : [0, \infty) \to \mathbf{R^n}$ is measurable and essentially bounded.

As a solution of (2) we understand an absolutely continuous function $y : [0, \infty) \to \mathbf{R^n}$, which satisfies (2) almost everywhere.

We suggest an approach for investigation of the nonlinear equation (1). Its' essence is stressed out by the following result.

Let a function $x : [0, \infty) \to \mathbf{R^n}$ *be a solution of (1), then it is also a solution of (2), where*

$$\psi(t) = f(t) - \sum_{i=1}^{m} A_i(t)\tilde{\varphi}(t - (H_i x)(t)), \tag{3}$$

$$\tilde{\varphi} = \begin{cases} \varphi(t), & t < 0, \\ 0, & t \geq 0. \end{cases}$$

Collocation methods for computation of periodic solutions of retarded functional differential equations

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A periodic solution of a differential equation can be computed as the solution of a periodic two-point boundary value problem (BVP). The periodic BVP is a finite-dimensional problem for ordinary differential equations (ODEs) (a point has to be found) and it is an infinite-dimensional problem for functional differential equations (FDEs) (a function segment has to be found). The most successful method for the numerical solution of periodic BVPs for ODEs is the well known collocation method using piecewise polynomials. As far as we know, no prior numerical work on the application of the collocation technique (not based on approximation by truncated Fourier series) for solving periodic BVPs for FDEs exists. Furthermore, the corresponding theoretical results (convergence, asymptotic error of numerical schemes) are non existing. We show that the collocation method with piecewise polynomials is quite efficient for solving periodic BVPs for RFDEs with constant delays. Our numerous numerical experiments show numerical order of convergence of the collocation solution to the exact solution and the asymptotic error of the different collocation schemes we applied. In particular, we show that special interpolators for the approximation of the computed solution in the past recover superconvergence at mesh points which is well known for the ODEs case. Adaptive mesh selection, based on the algorithm used in the AUTO software package for ODEs, allows to compute periodic solutions with complicated profile efficiently.

On nonlinear parabolic functional differential equations

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We shall consider initial-boundary value problems for the equation of the form

$$D_t u + \sum_{|\alpha|\le m} (-1)^{|\alpha|} D_x^\alpha [f_\alpha(t,x,...,D_x^\beta u,...)] +$$
$$\sum_{|\alpha|\le m-1} D_x^\alpha [g_\alpha(t,x,...,D_x^\gamma u,...)] + \sum_{|\alpha|\le m-1} D_x^\alpha [\tilde{g}_\alpha(t,x,u)] +$$
$$\sum_{|\alpha|\le m-1} D_x^\alpha [h_\alpha(t,x,H_1(u),...,H_r(u))] = F, \quad (t,x)\in Q_{T_0} = (0,T_0)\times\Omega$$

where $\Omega \subset R^n$ is a bounded domain with sufficiently smooth boundary, $|\beta| \le m, |\gamma| \le m-1$, H_j are linear continuous operators (defined in some function spaces), functions $f_\alpha, g_\alpha, h_\alpha$ have (usual) polynomial growth in $D_x^\beta u, D_x^\gamma u, H_j(u)$ respectively; g_α, h_α are not continuous in $D_x^\gamma u, H_j(u)$ respectively (but they are locally bounded), functions $\tilde{g}_\alpha$ may be rapidly increasing in u.
We shall formulate an existence theorem on weak solutions. Further, boundedness of $\| u(t) \|_{L^2(\Omega)}$ for $t \in (0,\infty)$ will be shown. Finally, we shall formulate a result on the stabilization of solutions as $t \to \infty$.
An important problem where the conditions of our theorems are satisfied is a climate model, considered by J.I. Diaz, G. Hetzer, J. Hernández, L. Tello. The stabilization result is a joint work with J.I. Diaz and is based on arguments applied by the above mentioned authors.

Oscillation of Linear Systems

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We investigate the notion of oscillation for linear systems with discrete and continuous time. A real valued function oscillates by the definition if it has arbitrarily large zeros. A vector valued function oscillates if its composition with every linear functional oscillates.

Let T be a strongly continuous semigroup and let A denote its generator. We say that a point x oscillates if the function $T(\cdot)x$ oscillates.

We show that if the time parameter is discrete then every point oscillates if the spectrum of A has no real nonnegative values. We apply this result to oscillation of solutions of infinite difference equations.

The situation when the time is continuous is more complicated. If T is a strongly continuous group then if the spectrum of A has no nonnegative real values then every point oscillates. In the case when T is a semigroup even under this assumption not all points have to oscillate, however we show that in this case the oscillation is generic. Moreover, applying the results from [1] we obtain that if the semigroup is generated by the functional differential equation then every solution oscillates iff the generator has no real eigenvalues.

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Posters

Iterative methods for the Darboux problem for partial functional differential equations

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We consider the Darboux problem for the following hyperbolic partial functional differential equation

$$D_{xy}z(x,y) = f\left(x, y, z_{(x,y)}\right), \qquad (x,y) \in E := [0,a] \times [0,b] \tag{1}$$

$$z(x,y) = \phi(x,y), \qquad (x,y) \in E^0 := [-a_0, a] \times [-b_0, b] \setminus (0,a] \times (0,b], \tag{2}$$

where $f : E \times C^1(B,X) \to X$, $\phi : E^0 \to X$ and $B := [-a_0, 0] \times [-b_0, 0]$. The function $z_{(x,y)} : B \to X$ represents the functional dependence in (1) and it is defined by $z_{(x,y)}(t,s) = z(x+t, y+s)$, $(t,s) \in B$. If $X = \mathbb{R}$ then by the linearization of (1) we define two monotone sequences uniformly convergent to the solution of problem (1), (2). The convergence that we get is of the Newton type, which means that the difference between the m-th term of either of the approximating sequences and the solution can be estimated by $A/2^{2^m}$, where A is some constant not dependent on m.

If X is a Banach space we also prove a theorem on the convergence of the Newton method for problem (1), (2). Also in this case we get the convergence of the Newton type.

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Modal stabilization in systems with delay

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Characteristic polynomial of the linear stationary control systems without delay

$$\dot{x}(t) = Ax(t) + bu(t), \quad u(t) = c^T x(t) \tag{1}$$

has n roots. The problem of the spectrum control (modal control) is equivalent to the coefficients of the characteristic polynomial control. For systems with delay

$$\dot{x}(t) = Ax(t) + A_1 x(t-\tau) + bu(t), \quad u(t) = \sum_{j=0}^{m} c_j^T x(t - j\tau), \quad \tau > 0 \tag{2}$$

characteristic quasipolynomial

$$f_n(\lambda, e^{-\lambda\tau}) = \sum_{i=0}^{n}\sum_{j=0}^{i} p_{ij} e^{-j\lambda\tau}\lambda^{n-i}, \quad p_{00} = 1 \tag{3}$$

has countable number of roots. There is no good interconnetion between coefficients and spectrum here. The problem of modal control is set as the coefficients control problem, although the investigation of transient quality depending on coefficients cause difficulties. System stabilization up to power $\gamma > 0$ often is more advisable now. That is calculation of $c_j^T, \quad j = \overline{0, m}$, when quasipolynomial (3) roots hold $\lambda_i < -\gamma$, $i = 1, 2, \ldots$.
Theorem Let (3) be a given quasipolynomial. Then the quasipolynomial

$$\tilde{f}_n(\lambda, e^{-\lambda\tau}) = \sum_{i=0}^{n}\sum_{j=0}^{i} \tilde{p}_{ij} e^{-j\lambda\tau}\lambda^{n-i}, \quad p_{00} = 1 \tag{4}$$

$$\tilde{p}_{ij} = \sum_{s=j}^{i} p_{sj} C_{n-s}^{i-s} (N + 1 + \gamma)^{i-s} e^{-j(N+1+\gamma)\tau}, \quad N = \max_{i=\overline{1,n}}\{\sum_{j=0}^{i} |p_{ij}|\} \tag{5}$$

is stable up to power $\gamma > 0$.

Explicit expressions for coefficients c_j^T, $j = \overline{0, m}$ of equation $u(t)$ is obtained with formulas (4) and (5).

Equilibrium stability of price formation dynamics

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A price formation mathematical model is considered [1]. It is represented by nonlinear system of differential equations with quadratic and linear fractional right side.

$$\dot{x}_i(t) = -\frac{v_i x_i(t)}{x_i(t) + a_i} - \sum_{j=1, j\neq i}^{n} c_{ij}(x_i(t) - x_j(t)) + \frac{d_i x_i(t)}{x_i(t) - b_i} +$$

$$+ r_i x_i(t)[q_i^0 - \sum_{j=1, j\neq i}^{n} \alpha_{ij}(x_i(t) - x_j(t))], \quad i = \overline{1, n}.$$

The system can be given in vector-matrix form

$$\dot{x}(t) = \{Q - V[A + X(t)]^{-1} + D[X(t) - B]^{-1} + X(t)G\}x(t).$$

Here A, B, D, V, $X(t)$ are diagonal matrices. Conditions of solution $x(t) \equiv 0$ (balanced price) stability and lower bound of stability domain are obtained.

Theorem. Let matrix $\tilde{A} = Q - VA^{-1} - DB^{-1}$ is asymptotically stable, that is $Re\lambda_i(\tilde{A}) < 0$, $i = \overline{1, n}$. Then solution $x(t) = 0$ of the system is asymptotically stable and stability domain contains the sphere with radius

$$R = \frac{\Psi}{\sqrt{\Phi(H)}} \min\{[\Psi(H)|A^{-1} + V(A^{-1})^2|]^{-1}, [\Psi(H)|B^{-1}| + |D(B^{-1})^2|]^{-1}, |G|\},$$

$$\Psi(H) = \frac{\lambda_{\min}(C)}{6\lambda_{\max}(H)}, \quad \Phi(H) = \frac{\lambda_{\max}(H)}{\lambda_{\min}(H)}.$$

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Some Properties of Solutions of Nonlinear Difference Equations

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In the paper asymptotic properties of the solutions of nonlinear difference equation are investigated. We deal with the existence of nonoscillatory bounded and unbounded solutions.

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Oscillation of Iterative Functional Equations

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We consider behavior of solutions of iterative functional equations. Sufficient conditions for the oscillation are given.

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Some oscillation criteria for differential equations generated by delay arguments

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In this paper we investigate linear differential equations with delay arguments of the form:

$$(-1)^{\delta}\, L_n x(t) = \sum_{i=1}^{m} q_i(t)\, x(q_i(t)), \qquad (E_\delta)$$

where $n \geq 2, \delta \in \{1, 2, n, n+1\}$ and $L_0 x = x,\ \ L_i x = \frac{1}{p_i}\frac{d}{dt}L_{i-1}x, p_n \equiv 1$.

The functions p_i, g_k, $q_k : \Re_+ \longrightarrow \Re_+ = (0, \infty)$ are continuous with $g_k(t) \leq t$ on $\Re_+$, $lim_{t\to\infty} g_k(t) = \infty$ and $\int^{\infty} p_i(t) = \infty, i = 1, ..., n, k = 1, ..., m$.

The main purpose of this work is to show how the delay arguments g_i influence the oscillatory character of solutions of the above equations. We consider the equations (E_δ) depending on whether n is an even or odd number as well as the value of δ.

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2.2 Geometric Dynamics

Organizer : Hiroshi Matano

Contributed talks

The Einstein Flow, the Sigma-Constant, and Geometrization of Three-Manifolds

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For the problem of the Hamiltonian reduction of Einstein's equations on a $(3+1)$-vacuum spacetime that admits a foliation by constant mean curvature compact spacelike hypersurfaces M that satisfy certain topological restrictions, we introduce a *dimensionless non-local time-dependent* reduced Hamiltonian system $H_{\text{reduced}} : \boldsymbol{R}^{-} \times P_{\text{reduced}} \to \boldsymbol{R}$, where $\boldsymbol{R}^{-} = (-\infty, 0)$, P_{reduced} is the *reduced symplectic manifold*, and $\boldsymbol{R}^{-} \times P_{\text{reduced}}$ is the associated *reduced contact manifold*. The reduced contact variables $(\tau, \gamma, p^{TT}) \in \boldsymbol{R}^{-} \times P_{\text{reduced}}$ are given by the constant mean curvature τ of an expanding spacelike hypersurface, a Riemannian metric γ with constant scalar curvature -1, and a transverse-traceless symmetric 2-contravariant density p^{TT}.

Taking as a temporal coordinate condition $t = \frac{2}{3\tau^2}$, the reduced Hamiltonian is given by $H_{\text{reduced}}(\tau, \gamma, p^{TT}) = -\tau^3 \text{vol}(M, g)$. For compact 3-manifolds of Yamabe type -1, we establish the following properties: (1) $H_{\text{reduced}}(\tau, \gamma, p^{TT})$ is a strictly monotonically decreasing function of t unless $p^{TT} = 0$ and $\gamma = \tilde{\gamma}$ is hyperbolic, at which point $H_{\text{reduced}}(\tau, \tilde{\gamma}, 0)$ is constant in time; (2) for $\tau \in \boldsymbol{R}^{-}$ fixed, $H_{\text{reduced}}(\tau, \gamma, p^{TT})$ has a critical point at $(\tilde{\gamma}, 0)$ which is unique up to isometry and which is a strict local minimum (in the non-isometric directions) of H_{reduced}; and (3) for $\tau \in \boldsymbol{R}^{-}$ fixed, the σ-constant of M is related to the reduced Hamiltonian by

$$\sigma(M) = -\tfrac{2}{3}\left(\inf_{(\gamma, p^{TT}) \in P_{\text{reduced}}} H_{\text{reduced}}(\tau, \gamma, p^{TT})\right)^{2/3} .$$

If M is a compact hyperbolic manifold, then we conjecture that $(\tilde{\gamma}, 0)$ is a global minimum of H_{reduced}. More generally, for any compact M we conjecture that the Einstein flow generically seeks to attain the σ-constant asymptotically insofar as the reduced Hamiltonian is monotonically seeking to decay to its infimum. However, possible obstructions, such as the formation of black holes, may prevent particular solutions from asymptotically approaching $\sigma(M)$. Further applications are discussed.

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Nonlinear curvature driven evolution of plane curves

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We study the evolution of plane curves satisfying the geometric equation $v = \beta(k, \nu)$, where v is normal velocity, k and ν are curvature and the angle of the tangential vector of the plane curve Γ. As a typical example one can consider the function $\beta(k, \nu) = \gamma(\nu)|k|^{m-1}k$ where $\gamma(\nu)$ is a given anisotropy function and $m > 0$. A key tool in our study is to analyze a degenerate parabolic equation for the curvature k

$$\partial_t k = \partial_s^2 \beta(k) + \alpha \partial_s k + k^2 \beta \tag{1}$$

where α is a functional of the curvature and other geometric quantities. The geometric meaning of α is the tangential velocity of the material points belonging to Γ. We propose a nonlocal functional α keeping the ratio of the local length to the total length of the curve Γ constant with respect to time. This choice of a nontrivial tangential velocity α leads to a powerful numerical scheme.

In the case where the function β is regular we establish short time existence of a classical solution $k \in C(I, C^{2+\sigma}(S^1)) \cup C^1(I, C^\sigma(S^1))$ provided that the initial curve is smooth enough. If β is degenerate we are concerned with the fast $(0 < m < 1)$ as well as slow $(1 < m \leq 2)$ diffusion case. In the case of a slow diffusion the initial curve is supposed to have at most $2 + \frac{1}{m-1}$ order contact with its tangents at inflection points. The idea of the proof is to make use of the theory of fully nonlinear parabolic equations developed by Angenent [1] and the Nash-Moser iterative technique for obtaining a-priori bounds for the gradient of $\beta(k)$. It is similar, in technique, to the one used in the paper by Angenent, Sapiro and Tannenbaum [2]. We also present some of numerical simulations that have been obtained by using a nontrivial tangential velocity α.

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2.3 Hyperbolic Conservation Laws

Organizers : Tai-Ping Liu, Gerald Warnecke

Key note lecture

Recent Progresses in Shock Wave Theory

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There have been intensive activities in the study of shock waves in the past decade. Important progresses are made in the study of the effects of dissipations, relaxations and sources, the well-posedness problem for hyperbolic conservation laws, the regularizing properties of nonlinearity and other numerical and analytical issues. We plan to survey these developements, with emphasis on the physical phenomena and the new analytical ideas when applied to physical systems. We will also discuss open problems and speculate on possible future research directions.

Invited lectures

Convergence Rates for Relaxation Methods Approximating Conservation Laws

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This talk discusses the global error estimates for a relaxation scheme approximating scalar conservation laws. In such schemes there are two parameters: mesh size Δx and the relaxation rate ϵ. We prove that the $L^\infty(0,T;L^1(\mathbb{R}))$-norm of the error goes to zero as $\epsilon^{1/2} + (\Delta x)^{1/2}$ when both the discretization parameter ϵ and the discretization parameter Δx go to zero. We show how to do this without resorting to any restriction on the initial error. Our result shows that the initial error does not prevent the convergence of the conserved variable to the entropy solution of conservation laws, and the initial layer persists in "kinetic" variables only for a short time of order ϵ.

Nonclassical shocks for conservation laws

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Diffusive-dispersive approximations to conservation laws give rise to travelling waves that converge to shocks not verifying the classical Lax entropy condition (or Oleinik-Liu condition) and are called nonclassical shocks.

Various physical models, for which the diffusive-dispersive approximation has a physical justification, have a natural entropy. The set of all discontinuities satisfying the condition related to this single entropy are too many to have uniqueness and a selection criterion must be introduced. A kinetic relation, linked to the approximation, is used to find a unique Riemann Solver and possibly a nucleation criteria is introduced. This last appears to be natural in case of phase transition models.

Front-tracking is then used to construct solutions to the Cauchy problem. The main difficulty is the control of the total variation that may increase also in the scalar case. An equivalent functional is introduced that decreases in time along approximate solutions.

After a first result for the cubic scalar case in [1], a method for the general scalar case is described in [2]. Some assumptions are made on the kinetic function while no nucleation criteria is needed.

It is also shown that the Riemann Solvers with good properties coincides with one associated to a kinetic function of the type considered.

Finally the problem of uniqueness is investigated in [3]. It is proved that, as for the classical case, there exists under mild conditions at most one Lipschitz semigroup compatible with the Riemann solver, moreover any admissible solution satisfying a mild regularity is a semigroup trajectory.

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Stability of Viscous and Inviscid Shock Waves

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We are concerned with shock waves $(u^l, u^r; s)$ of multidimensional systems of n conservation laws. Here $u^{l,r}$ denote the left and right constant states, and s is the shock speed. The direction of propagation is *a priori* chosen along the first coordinate axis. We may consider several kinds of stability, under small initial perturbations. With increasing complexity :

- One-D stability, that is stability against perturbations in the sole x_1 direction. As is well-known, it amounts to assume $\Delta \neq 0$, where Δ is a determinant of the $n-m$ outgoing modes with m vectors representing the degrees of freedom of the shock. Here, $m = 1$ for a Lax shock, but $m > 1$ for an undercompressive shock (see [1]). All overcompressive shocks are unstable at this level.

- Multi-D stability. The theory is due to A. Majda [5] for Lax shocks and to Freistühler for undercompressive ones. At the linearized level, the shock may be strongly or weakly stable, or strongly unstable. The latter prevents from local well-posedness. The analysis requires the knowledge of zeros of an explicit analytic function : the *Lopatinski determinant*. Examples show that the sign of Δ may be involve in the nature of the shock : in full gas dynamics, $\Delta < 0$ is equivalent to strong instability.

- Viscous stability, that is asymptotic stability of a viscous profile of the shock, as a travelling wave of an approximate system. The relevance of this approach was recognized by T.-P. Liu [4]. It was shown in [3] that the sign of Δ plays a rôle in the stability, even in one space dimension. This was extended in [7] to multi-D contexts, where we show that the long-wave limit of the Evans function of the profile is nothing but the Lopatinski determinant. This shows that Majda's instability of the shock itself, which is of Hadarmard's type, implies the asymptotic instability of every possible profile.

On one hand, we shall present the latter aspect. On the other hand, we shall explain why the change of sign of Δ, as the shock varies along a given curve, is related to both weak stability and strong instability (see [6]).

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Compressible Navier-Stokes Equations with Density-Dependent Viscosity and Vacuum

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In this talk, we will present some results on the one-dimensional motion of viscous gas connecting to vacuum state when the viscosity is constant or depends on the density. When the viscosity coefficient is constant, the regularity of the solution near vacuum and the behaviour of the interface separating the vacuum and gas are given. When the viscosity coefficient μ is proportional to ρ^θ and $0 < \theta < 1/2$, where ρ is the density, the global existence and the uniqueness of weak solutions are proved.

Discrete Shock Waves and Small Divisors

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We study finite difference approximations to hyperbolic conservation laws, especially in the presence of shock waves. A small divisor problem, which is caused by discretization will be discussed.

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Contributed talks

A connection between scalar conservation laws and infinite linear systems of PDE's

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We demonstrate that solutions of most scalar conservation laws satisfy an infinite linear system of partial differential equations. This observation allows to prove an existence theorem for such infinite systems and, vice versa, to estimate the solution of the original scalar conservation law. The Hopf's equation is included in such a class of conservation laws.

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Admissibility of traveling waves in scalar balance laws

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We consider hyperbolic balance laws

$$u_t + f(u)_x = g(u), \quad f \in C^2,\ g \in C^1,$$

and their viscous approximations

$$u_t + f(u)_x = \varepsilon u_{xx} + g(u), \quad \varepsilon \ll 1.$$

Assuming that the flux is convex and that g possesses only simple zeroes, Mascia has found several types of heteroclinic traveling waves, i.e. solutions of the form $u(x,t) = u(x-st)$ that tend to constant states $u_{\pm\infty}$ as $\xi := x - st$ tends to $\pm\infty$. All these traveling waves are *entropy* traveling waves in the sense that their discontinuities satisfy an entropy condition. We discuss the question, which types of traveling waves can be obtained as limits of traveling waves of the viscous problem in the following sense: A traveling wave u_0 of the hyperbolic balance law with speed s is called *admissible* if sequences $s_n \to s$ and $\varepsilon_n \searrow 0$ exist such that the viscous problem has a traveling wave solution u_n with

$$\lim_{n\to\infty} \|u_n - u_0\|_{L^1} = 0.$$

This leads to the study of a singularly perturbed o.d.e. where the wave speed s acts as an additional parameter.
We show that not all of the entropy traveling waves are admissible.

Shocks with oscillatory tails in systems of viscous hyperbolic balance laws

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Searching for viscous shock profiles of the Riemann problem, we consider systems of hyperbolic balance laws of the form

$$u_t + f(u)_x \;=\; \epsilon^{-1} g(u) + \epsilon\,\delta\, u_{xx}$$

with $u = (u_0, u_1, \cdots, u_N) \in \mathbb{R}^{N+1}$, $\delta > 0$ fixed, and with real time t and space x. Traveling wave solutions $u = u\left(\frac{x-st}{\epsilon}\right)$ with limiting states $\lim_{\tau\to\pm\infty} u(\tau) = u^{\pm}$ give rise, for $\epsilon \searrow 0$, to shock solutions of the Riemann problem.

Therefore we investigate the second order traveling wave equation

$$\begin{aligned} \dot u &= v \\ \delta\,\dot v &= -g(u) + (A(u) - s)v \end{aligned}$$

and look for heteroclinic orbits between equilibria $u^{\pm}$ of the reaction term: $g(u^{\pm}) = 0$. The asymptotic behavior of viscous profiles $u(\tau)$ for $\tau \to \pm\infty$ depends on the linearization at $u = u^{\pm}$.

A special structure of the source term, e.g. the mixture of conservation laws with balance laws, can lead to lines or, generally, surfaces of equilibria. Investigating the change of stability along these curves of equilibria, it turns out that heteroclinic connections can have oscillating tails. This is possible even for examples with gradient flux terms where the source terms $\dot u = g(u)$ *alone* do not support oscillatory behavior. The interaction of flux and reaction, in contrast, is able to produce purely imaginary eigenvalues of the linearization at $u^{\pm}$.

Near Hopf-like bifurcation points, continua of heteroclinic orbits exist. They correspond to weak shock solutions with oscillating tails at both sides. Besides this, a fixed left state u^- can connect to an whole interval of right states u^+, with fixed traveling speed s.

The results are based on normal form theory and a spherical blow-up construction inside the center manifold near the line of equilibria.

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Stability of a Self-Similar 3-Dimensional Gas Flow

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We are concerned with inviscid gas flow in three space dimension. In the presence of symmetries, the flow becomes self-similar and the system may be reduced to ordinary differential equations. We are interested in steady flow past an infinite cone and its stability with respect to perturbation of the cone. Such a flow has a cylindrical symmetry. Let u and v represent the axial and radial components of velocity. With the additional assumption that the flow is isentropic, the compressible Euler equations are reduced to

$$(\rho u)_x + (\rho v)_y = \frac{-1}{y}(\rho v) \tag{1}$$

$$(\rho u^2 + P)_x + (\rho uv)_y = \frac{-1}{y}(\rho uv) \tag{2}$$

$$(\rho uv)_x + (\rho v^2 + P)_y = \frac{-1}{y}(\rho v^2) \tag{3}$$

$$P = P(\rho). \tag{4}$$

When a uniform supersonic flow hits the obstacle, which is an infinite cone, and the angle of opening at the vertex is sufficiently small, the conical flow can be constructed by studying self-similar solutions. The flow is deflected by an attached shock front beginning at the vertex and is continued so that the state of the air is constant on each concentric cone behind the shock cone and is parallel to the obstacle cone. Since the flow is isentropic and irrotational, the equations (1)-(4) are reduced to a system of ordinary differential equations. Such self-similar flow is suggested by Busemann [1], who gave a graphical method for obtaining them.

We consider a more realistic case when the obstacle is a perturbation of the infinite cone. The shock front and the flow behind it are conical until the expansion wave and the shock wave coming from the bendings of the obstacle interact with the conical flow and the shock front. The flow then becomes rotational and, in general, contains infinitely many interacting shock waves. The main questions are: With all these wave interactions, does a solution exist globally? Is it stable with respect to the perturbation, for finite x and also asymptotically as $x \to \infty$? We answer these questions affirmatively and show that the long-range behavior of the flow is self-similar corresponding to an infinite cone with the asymptotic angle of the perturbation. In particular, the flow between the leading shock and the obstacle tends to be irrotational and isentropic. In fact, there is a boundary layer of high concentration of vorticity and entropy variation. The width of the layer tends to zero as $x \to \infty$. This can occur, of course, only for inviscid flow; for the viscous flow the vorticity would propagate into the flow. Nevertheless, experimental evidences show that the inviscid flow still accurately represents the actual flow, Courant-Friedrichs [2]. The flow pattern eventually tends to a self-similar solution corresponding to the conical flow for the infinite cone without any deflections.

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Cauchy Problem for the Compressible Euler Equation with Damping

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We study the Cauchy problem for the system of compressible adiabatic flow through porous media in the one space dimension and the related diffusive problem, namely

$$\begin{cases} v_t - u_x = 0 \\ u_t + p(v,s)_x = -u, \\ s_t = 0, \\ (v,u,s)(x,0) = (v_0(x), u_0(x), s_0(x)), \\ (v_0,u_0,s_0)(x) \to (v_\pm, u_\pm, s_\pm), \ as\ x \to \pm\infty, \end{cases} \tag{1}$$

and

$$\begin{cases} \tilde{v}_t = -p(\tilde{v},s)_{xx} \\ \tilde{u} = -p(\tilde{v},s)_x \\ s_t = 0, \\ \tilde{v}(x,0) = \tilde{v}_0(x), \ s(x,0) = s_0(x), \\ \tilde{v}_0(\pm\infty) = v_\pm, \ s_0(\pm\infty) = s_\pm. \end{cases} \tag{2}$$

In this paper, we consider the following two cases

Case1: $(v_\pm, s_\pm)$ satisfy $p(v_-, s_-) = p(v_+, s_+) = \bar{p}$;

Case 2: $s_- = s_+ = \bar{s}$.

By introducing the new approach which combines the usual energy methods with special L^1-estimate and with the use of weighted norms, we can solve the problem (2) in detail. The global existence and large time behavior for the classical solutions for both two cases are proved. The asymptotic states for the solutions are given by stationary solution for case 1 and by similarity solution for case 2 respectively. Thanks to our new approach, the almost optimal convergence rates are obtained.

Then, we establish the global existence and large time behavior for the smooth solutions to the problem (1) in both cases by comparing the solutions of (1) to those of

(2) via and energy estimate methods. It is also shown that the problem (1) can be approximated very well time-asymptotically by (2) which obtained from (1) by Darcy's law. The optimal decay rates for the difference between the solutions of (1) and those of (2) are then proved by weighted energy estimates.

Our results for case 1 generalized the results in [1] for more general cases with in addition the decay rates. The results for case 2 strongly improved the results in [2] by removing the key technique condition in [2](referred in [2] as condition V) and giving the optimal decay rates.

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2.4 Hyperbolic Wave Equations

Organizers : Frank Duzaar, Joseph Grotowski

Key note lecture

The notion of hyperbolicity for Euler-Lagrange equations

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The lecture shall begin with a critical review of existing notions of hyperbolicity, i.e. Leray's notion of strict hyperbolicity and Friedrich's notion of symmetric hyperbolicity. We shall then introduce a new notion of hyperbolicity appropriate to systems of Euler-Lagrange equations. This notion is intimately connected with the notion of ellipticity in the calculus of variations (the Legendre-Hadamard condition). We shall show how the basic properties, such as the domain of dependence theorem, follow directly from the new notion, which overcomes the difficulties associated with singularities of the characteristic variety encountered by earlier approaches. Finally we shall indicate how the new approach provides a basis for the solution of problems such as the problem of global nonlinear stability of a given solution.

Invited lectures

Quasilinear Wave Equations and Strichartz Estimates

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In this paper, our aim is the proof of local wellposedness for quasilinear wave equations for initial data less regular than what is required by energy method. This implies to prove Strichartz type estimates for wave operators whose coefficients are only Lipschitz.

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Solutions of the Einstein equations with prescribed singularity structure

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The singularity theorems of Hawking and Penrose are one of the best known theoretical results in general relativity. They assert the existence of spacetime singularities under general circumstances but say nothing about the structure of these singularities. The proofs of these theorems involve differential geometry and ordinary differential equations and do not use the character of the Einstein equations as a quaslinear hyperbolic system. It seems unavoidable that the theory of hyperbolic equations must be used in order to obtain more precise information on this matter. One approach to this problem is to attempt to construct solutions of the Einstein equations with a prescribed type of singularity. Heuristic considerations indicate that the singularities could have a complicated oscillatory structure in general and this suggests difficulties in constructing them. The same heuristics indicates, however, that in the presence of a scalar field things simplify dramatically, the oscillatory behaviour being replaced by uniform blow-up.

Motivated by these considerations, Lars Andersson and the author have studied the Einstein-scalar field system from this point of view. The unknowns are a four-dimensional Lorentzian metric g and a real-valued function ϕ (the scalar field) and the equations are $\mathrm{Ric}(g) = \nabla\phi \otimes \nabla\phi$, where $\mathrm{Ric}(g)$ is the Ricci curvature of the metric g. It was shown that there exists a large class of solutions of these equations with singularities of blow-up type which can be described in considerable detail. The solution of the system of partial differential equations is described approximately near the blow-up hypersurface by a system of ordinary differential equations. (This is known in the relativity literature as the velocity dominated approximation.) The solutions constructed depend on a certain class of free data. These are required to be analytic but are otherwise general.

The analytical part of the proof is similar to that used in a more special problem in [1], the fundamental idea going back to Baouendi and Goulaouic [2].

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Contributed talks

Bounded Solutions and Blow-up in Nonlinear Wave Equations

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We consider a Klein-Gordon equation with linear dissipation and a nonlinear source term

$$u_{tt} - \Delta u + mu + \delta u_t = f(u),$$

in a bounded domain $\Omega \subset \Re^n$, with $m > -\lambda =$ first eingenvalue of $-\Delta$ in $H_0^1(\Omega), \delta > 0, f(u) = \mu u|u|^{r-2}, \mu > 0, r > 2$, and with homogeneous Dirichlet boundary condition

$$u = 0, \quad in \;\; \partial\Omega,$$

and initial conditions

$$u(0) = u_0, \quad u_t(0) = v_0.$$

We give necessary and sufficient conditions for globality and boundedness, and for nonglobality and blow-up of solutions. Also, we characterize those solutions which tend to the nonzero equilibria as well as to the zero equilibrium as $t \to \infty$. We improve previous results of Cazenave [1], Ikehata [2] and Goergiev and Todorova [3].

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Asymptotic behaviour of solutions to a class of semilinear hyperbolc systems.

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In this talk the long time asymptotic behavior of solutions of semilinear symmetric hyperbolic system including Maxwell's equations and the scalar wave-equation in an arbitraty spatial domain are investigated. The possibly nonlinear damping term may vanish on a certain subset of the domain. Decay in the weak topology for suitable initial-states is shown. In the case that the nonlinear damping is in addition montone, also strong local L^q-convergence is shown.

These results are applied to the initial-boundary-value-problem for Maxwell's equations in an exterior domain. It is shown that the solution behaves like a free-space-solution (in whole space with constant coefficients) provided that the initial-state is orthogonal to all stationary states. Here a L^p-regularity-theorem for Maxwell's equations and some commutator-estimates are used.

On the Attractors of Hamiltonian Nonlinear Wave Equations

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We consider the asymptotics in long-time limits, $t \to \pm\infty$, of finite energy solutions to Hamiltonian nonlinear wave equations in the whole space. For "generic" equations the solutions converge to an attractor which is the set of all stationary states. The convergence holds in the Fréchet topology defined by local energy seminorms. The convergence is established for general 1D nonlinear wave equations with nonlinear term concentrated at a finite segment [1], for 3D scalar wave equation coupled to a particle [2], and for 3D Maxwell-Lorentz system with a charge [3]. In [4] the soliton-like asymptotics is established for finite energy solutions to the translation-invariant system from [2]. In [5] the stability of the soliton manifold and the adiabatic effective dynamics are proved for the soliton-like solutions to the system from [2] with slowly varying potential. The related physical problems are discussed in [6].

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Nonlocal problem with integral conditions for hyperbolic equation

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Consider the equation

$$Lu \equiv u_{xy} + A(x,y)u_x + B(x,y)u_y + C(x,y)u = f(x,y)$$

in the rectangle $D = \{(x,y):\ 0 < x < a,\ 0 < y < b\}$.

Nonlocal integral conditions arise when one studies certain problems of plasma physics [1], heat conduction [2], dynamics of ground waters [3]. Motivated by this, we study the following problem:

$$Lu = f,\ \int_0^\alpha u(x,y)dx = \psi(y),\ \int_0^\beta u(x,y)dy = \phi(x),$$

where $\phi(x), \psi(y)$ are given, $0 < \alpha < a,\ 0 < \beta < b$. If $\alpha = a,\ \beta = b$ the existence and uniqueness of generalized solution are proved [4].

Introducing a new unknown function $w(x,y)$, we reduce the equation $Lu = f$ to a loaded equation $\bar{L}w = F$, while the integral conditions take the form

$$\int_0^a w(x,y)dx = 0,\ \int_0^b w(x,y)dy = 0.$$

The existence and uniqueness of the solution in Sobolev space $H^1(D)$ are proved. The proof is based on a-priori estimates and a complete continuity property of the operator, generated by the problem.

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Semilinear wave equations on pseudo-Riemannian manifolds: stability and motion of non-topological solitons.

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The starting point is the equation:

$$\frac{\partial^2 \phi}{\partial t^2} - \Delta\phi + m^2\phi - \beta(|\phi|)\phi = 0 \qquad (*)$$

for a function $\phi : \mathbb{R}^{1+n} \to \mathbb{C}$. This equation supports a class of solitary wave solutions, called *non-topological solitons* ([2]), of the form

$$\phi(t,x) = e^{i\omega t} f_\omega(x), \qquad (**)$$

where f_ω is the solution of a constrained minimisation problem ([3]); in the case $\beta(f) = |f|^{p-1}$ the function f_ω can also be characterised as an "optimiser" of the Gagliardo-Nirenberg inequality ([4]). The stability properties of these solutions have been investigated in both the physics and mathematics literature (see [1],[2] and references therein); an interesting feature is the $\omega-$dependence of the stability conditions. A *modulational* approach to stability theory is developed, which gives stronger results than those obtained previously; in particular, the solution to the Cauchy problem for (*), with initial data close to a Lorentz transform of (**), remains uniformly close to the corresponding point on an explicitly determined curve in the space of Poincare transforms of (**). This stability holds for an approriate range of ω, which may, in the case $\beta(f) = |f|^{p-1}$, be determined sharply from linear theory, using information gleaned from the Gagliardo-Nirenberg characterisation of f_ω.

The main motivation for the present development of this technique is to provide a basis for an investigation of solitons in the equation

$$L_g\phi + m^2\phi - \beta(|\phi|)\phi = 0 \qquad (C)$$

where L_g is the covariant d'Alembertian determined by a pseudo-Riemannian metric g. It is proved that under the rescaling $g \to \epsilon^{-2} g$, with ϵ small, there exist solutions to (C) in which the solitons (**) are concentrated on time-like geodesics. This generalises a previous one dimensional theorem ([5]).

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On discrete models of the wave equation

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The purpose of this work is to present a generalization of Dezin's results [1] dealing with a discrete model of the Laplace operator for the case of pseudo-Euclidean space. The present work is a direct continuation of [2].

Let $\mathbb{R}^n_1$ be pseudo-Euclidean space of type $(1, n-1)$, i.e. $\mathbb{R}^n$ with the Lorentz metric. If we take $n = 4$ we obtain Minkowski space. We construct a combinatorial model of $\mathbb{R}^n_1$ as the complex $C(\mathbb{R}^n_1)$ with boundary operator ∂. Then the dual complex $K(\mathbb{R}^n_1)$ to $C(\mathbb{R}^n_1)$ can be considered as a space of discrete forms. The coboundary operator d^c on $K(\mathbb{R}^n_1)$ is a discrete analog of the exterior differential d. The definition of d^c does not depends on a metric and coincides with the one for the combinatorial model of Euclidean space [1]. But to define a discrete analog of the Hodge star operator $*$ we must take into account a metric. We define a discrete analog of the codifferential δ as the formal adjoint operator (denote δ^c) to d^c. Note that δ^c depends from a metric too. The combinatorial Laplacian is defined by

$$\Delta^c = d^c\delta^c + \delta^c d^c$$

Clearly, the generalized Laplacian with respect to the Lorentz metric on $K(\mathbb{R}^n_1)$ is a discrete analog of the wave operator. The Cauchy problem and boundary value problems for this discrete model of the wave equation are studied.

Within our intrinsic approach, we may write in terms of the generalized Laplacian a discrete model of the Klein-Gordon equation. Likewise, using the combinatorial model of Minkowski space we can describe a discrete model of the Maxwell equations. An important generalization of these equations leads immediately to a discrete model of the so-called Yang-Mills equations of the quantum field theory.

It should be possible to consider one more discrete model in which "time" is the continuous parameter. The latter fact means that a discrete model of the wave equation is constructed on the complex $K(\mathbb{R}^{n-1})$ with the Euclidean metric and $K(\mathbb{R}^{n-1})$ is depending from the continuous parameter t. In this case a system of the second order ordinary differential equations by t is a discrete analog of the wave equation.

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Global Dynamics of Nonlinear Hyperbolic Equations With Non-Monotone Damping

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Many challenging issues concerning global dynamics emerge from nonlinear hyperbolic equations with a kind of weak dissipation such as localized damping, non-monotone damping, or boundary damping. In this work, we study a nonlinear wave equation

$$u_{tt} + Au + g(u_t) + f(u) = h,$$

where typically A represents $-\nu\Delta$ over a bounded 2-D domain with homogeneous Dirichlet boundary condition, $g(u_t) = -\alpha u_t + \beta u_t^3$ with constants α and $\beta > 0$ is a non-monotonedamping, f is a Caratheodory function (or simply a polynomial) satisfying the usual power growth condition and the asymptotic sign condition, and $h \in L^\infty(0,\infty; L^2(\Omega))$ is a given input. This equation has been used as a model in dynamic analysis and stabilizing control of the galloping vibration of power lines of electric transmission. Due to the difficulty caused by the nonlinear and non-monotone damping, the long-term behavior of solutions remains open to a large extent. The following results are obtained. First, the local existence and uniqueness of weak and strong solutions are established by the Bubnov-Galerkin approach and the regularity analysis is conducted. Second, by an asymptotical bootstrap method in dealing with a Lyapunov-like functional $E(u, u_t)$, it is proved that for any initial data in the energy space the weak solution exists globally and remains bounded based on the differential inequality

$$\frac{d}{dt}E(u,u_t) \le -\frac{\epsilon}{2}E(u,u_t) + C_0\beta\|u_t\|_{L^4}^3(\epsilon\|u\|_{H^1} - C_1) + C_2(\epsilon) + \left(1+\alpha^{-1}\right)\|h\|_\infty.$$

Moreover, it is proved that the solution semiflow is dissipative in terms of the existence of absorbing sets. Third, with some restriction that h is time-invariant and $\beta > c\alpha$ for some constant c, it is shown that thesolution semiflow $S(t)$ is uniformly κ-contracting as it satisfies

$$\|S(t)w_1 - S(t)w_2\| \le \psi(t)\|w_1 - w_2\| + \rho_t(w_1, w_2), \quad t > \tau \ge 0,$$

where $\psi(t) \to 0$ and ρ_t is a precompact pseudometric on the energy space. Finally it is proved that there exists a global attractor for the semiflow generated by this nonlinear hyperbolic equation.

Posters

The global Cauchy problem for the critical non-linear wave equation in variable coefficients

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In this work, we study the global existence of both smooth and strong solutions of the critical wave equation in variable coefficients in three dimension of space

$$(E)\ \ \square_A u + \mid u \mid^4 u = \partial_t^2 u - div(A(x).\nabla_x u) + \mid u \mid^4 u = 0, \ \ I\!R_t \times I\!R_x^3.$$

where A is a regular function valued in the space of 3×3 positive definite matrices which is the identity outside a compact set of $I\!R^3$.

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2.5 Hysteresis

Organizer : Jürgen Sprekels

Key note lecture

P.D.E.s with hysteresis operators

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Hysteresis appears in several phenomena, in physics, engineering, chemistry, biology, economics, and so on. A systematic investigation of its mathematical properties began only in the 1970s, see [1].

Hysteresis relations can be represented by a *black box,* which transforms an input function u into an output function w. This is formalized through the notion of *hysteresis operator* $\mathcal{F} : u \mapsto w$, acting between spaces of time dependent functions, e.g. $C^0([0,T])$. *Rate independence* appears as the basic property; this means that if $\mathcal{F} : u \mapsto w$ then $\mathcal{F} : u \circ \varphi \mapsto w \circ \varphi$, for any increasing time diffeomorphism $\varphi : [0,T] \to [0,T]$.

For instance, the model studied by Duhem a century ago corresponds to the transformation $u \mapsto w$ defined by the following Cauchy problem:

$$\begin{cases} \frac{dw}{dt} = g_1(u,w)\left(\frac{du}{dt}\right)^+ - g_2(u,w)\left(\frac{du}{dt}\right)^- & \text{in }]0,T[, \\ w(0) = w^0; \end{cases} \tag{1}$$

here g_1 and g_2 are given continuous functions and w^0 is a datum.

The classical hysteresis models due to Prandtl, Ishlinskiĭ, Preisach will also be outlined, and some of their properties (continuity, monotonicity and others) discussed.

We then briefly deal with the following P.D.E., which arises as a simplified model of (scalar) ferromagnetic hysteresis:

$$\frac{\partial}{\partial t}\left[u + \mathcal{F}(u)\right] - \frac{\partial^2 u}{\partial x^2} = f \qquad \text{in }]a,b[\times]0,T[; \tag{2}$$

here $\mathcal{F}$ is a hysteresis operator and f is a given function. Under suitable assumptions, the corresponding initial and boundary value problem is well-posed in appropriate function spaces.

Details and an extended bibliography can be found in [2].

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Invited lectures

Asymptotically stable almost-periodic oscillations in systems with hysteresis nonlinearities

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We consider a perturbed ODE system

$$x' = f(t,x) + \varepsilon g(x, z(t)) \,,$$

where the perturbation includes the hysteretic element (Γ being a hysteresis operator)

$$z(t) = (\Gamma[t_0, z(t_0)]Lx)(t) \,.$$

We assume that the unperturbed system ($\varepsilon = 0$) has an asymptotically stable almost-periodic solution. We present sufficient conditions for the perturbed system to have an asymptotically stable almost-periodic solution. We illustrate the result with an example from feedback control.

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Elastic Contact Problems and the Stop Operator

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Given a closed convex set $K \subset \mathbb{R}^n$, a forcing function $f : [0,\infty) \to \mathbb{R}^n$, and an initial value $u(0) \in K$, we look for a function u such that $u(t) \in K$ for all t, and $u'(t)$ is as close as possible to $f'(t)$, i.e., $f'(t) - u'(t)$ lies in the normal cone of K at the point $u(t)$. It is well known that this problem admits a unique solution. The operator $\mathcal{S}$ mapping $(u(0), f) \in K \times W^{1,p}([0,\infty), \mathbb{R}^n)$ into $u \in W^{1,p}$ is called the stop operator.

It is known that the stop operator is continuous in many function spaces, but it is not Lipschitz continuous in the spaces $W^{1,p}$ for $1 < p < \infty$. The question of Lipschitz continuity in $W^{1,1}$ has been settled only recently. The stop operator is globally Lipschitz continuous if K is a convex polyhedron. If K has $W^{2,\infty}$-smooth boundary, then the stop operator is locally Lipschitzian. There are examples of domains K with $\mathcal{C}^1$-boundaries where $\mathcal{S}$ is not locally Lipschitz continuous.

As an application we utilize the Lipschitz continuity of the stop operator on a polyhedron to prove wellposedness of a two-point initial-boundary value problem modeling the dynamics of a linearly elastic beam with unilateral contact conditions.

Homogenization of Scalar Wave Equation with Hysteresis Operator

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Longitudinal vibrations of the elastoplastic rod are modelled by scalar wave equation

$$\rho\, u_{xx} = (\mathcal{F}[u_x])_x + f\,,$$

with constitutive stress-strain relation of elastoplastic material in the form of Ishlinskii operator

$$\sigma = \mathcal{F}[e] := \eta_\infty e - \int_0^\infty \mathcal{S}_r[e]\,\mathrm{d}\eta(r)$$

where $\mathcal{S}_r$ is the stop operator. The equation is completed by convenient initial and boundary conditions.

The material of the rod is heterogeneous i. e. spatially dependent, which means spatially dependent material data

$$\rho = \rho(x) \qquad \eta = \eta(x,r)\,.$$

Properties of spatially dependent Ishlinskii operator are studied; existence, uniqueness and estimates of the solution to this initial boundary value problem are proved.

Homogenization problem consists in considering a sequence of materials with periodic structure with diminishing period, which means a sequence of problems with periodic functions ρ^ε, η^ε in the constitutive relation having diminishing period $\varepsilon \to 0$

$$\rho^\varepsilon(x) = \rho\left(\frac{x}{\varepsilon}\right), \qquad \eta^\varepsilon(x,r) = \eta\left(\frac{x}{\varepsilon}, r\right),$$

where $\rho(y)$ and $\eta(y,r)$ are functions periodic in y. The corresponding solutions u^ε converge to a function u^0 which is a solution to the so-called homogenized problem. The form of the homogenized problem is derived and convergence $u^\varepsilon \to u^0$ is proved.

Hysteresis operators in modeling of phase transitions

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Among mathematical methods designed for solving problems in phase transitions modeling, the method of hysteresis operators is one of the most recent ones. Its development is connected with a systematic effort of applied mathematicians (see [1,3,4,6]) to set up an appropriate theoretical framework for describing rate-independent hysteresis phenomena independently of their physical nature. For complex physical problems like those arising in phase transitions, the hysteresis theory makes it possible to replace analytically nonsmooth constraints (represented e. g. by rate-independent evolution variational inequalities) by more regular objects (hysteresis operators) in suitable function spaces. It turns out that, at least in some cases, this may lead to a considerable simplification in the analysis of the problem.

The aim of this contribution is to present some extensions to the results published in [5] for the phase-field type system

$$\begin{cases} \mu(\theta)\, w_t + f_1[w] + \theta\, f_2[w] = 0\,, \\ (\theta + F_1[w])_t - \Delta\theta = \psi(x,t,\theta)\,, \end{cases}$$

motivated by the Frémond-Visintin formulation (see [2]) of the relaxed Stefan problem, where the order parameter w and the absolute temperature θ are the unknowns of the problem, μ and ψ are given functions and f_1, f_2, F_1 are hysteresis operators, and to discuss further issues.

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Posters

Monte Carlo study of the mixed spin Ising ferromagnetic model with a ferrimagnetic surface

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The three-dimensional mixed spin 1/2 and spin 1 Ising model with competing surface and bulk exchange interactions is studied. Within the frame work of the Monte Carlo simulation, the phase diagrams are investigated, they exhibit qualitatively interesting features. The effects of the surface and the bulk crystal field interactions on the phase diagrams and in particular for surface ordering, are also examined. It is also shown that such system can exhibit a variety of phase transitions.

2.6 Large Domains

Organizers : Alexander Mielke, Guido Schneider

Key note lecture

The long wave limit for the water wave problem

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I will describe the derivation and justification of approximating equations for long waves moving on the surface of an inviscid, irrotational fluid in an infinitely long canal without surface tension. A variety of equations, such as the Korteweg-de Vries or Boussinesq equations, have been derived to formally approximate the evolution of such waves.

Using recently developed techniques for justifying modulation equations one can can show rigorously that to the order of approximation considered in formal derivations of the KdV or Boussinesq equations the solutions of the water wave problem split up into two wave packets, one moving left and one moving right, where each of these wave packets evolve independently as a solution of the Korteweg-de Vries equation. In particular, this means that to the order of approximation considered, such counterpropagating waves do not interact with each other. In addition to the fact that this approximation can be rigorously justified, it also has the advantage that the approximating equations are well posed and completely integrable. This work was done in collaboration with Guido Schneider.

Invited lectures

Dimension and Entropy for parabolic PDE's in Infinite Domains

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This is a report on work with P. Collet. We define the topological entropy per unit volume in parabolic PDE's such as the complex Ginzburg-Landau equation, and show that it exists, and is bounded by the upper Hausdorff dimension times the maximal expansion rate. We then give a constructive implementation of a bound on the inertial range of such equations.

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Loss of Stability in the Magnetic Benard Problem

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We study the loss of stability of the trivial equilibrium $v_0 = \vartheta_0 = h_0 = p_0 = 0$ of the system

$$\begin{aligned}
\partial_t v &= \nu\Delta v + \nabla p + SB\partial_z h + S(h\nabla)h - (v\nabla)v + g\alpha k\vartheta \ , \\
\partial_t \vartheta &= \chi\Delta\vartheta + (T_0 - T_1)v_3 - (v\nabla)\vartheta \ , \\
\partial_t h &= r\Delta h + B\partial_z v + (h\nabla)v - (v\Delta)h \ , \quad div\ v = div\ h = 0 \ .
\end{aligned}$$

Here $k = (0,0,1)^t$, $v = (v_1, v_2, v_3)$, $h = (h_1, h_2, h_3)$, while $\nu, \chi, r, \alpha, g, S$ are constants related to the physical setting. (x, y, z) ranges over the infinite plate $\Omega = R^2 \times \left(-\frac{1}{2}, \frac{1}{2}\right)$. We assume Dirichlet conditions for v, ϑ and stress-free conditions for h, i.e. $\partial_z h_1 =$

$\partial_z h_2 = h_3 = 0$ on $\partial\Omega$. The above system describes a conducting fluid in Ω, subject to a temperature gradient induced by the temperature T_0 at $z = -\frac{1}{2}$ and $T_1 < T_0$ at $z = \frac{1}{2}$. Moreover the fluid is subject to a constant magnetic field $H_0 = (0, 0, B)$ parallel to the z-axis. This induces a magnetic field h, coupled to v via the above system. It is our aim to discuss the stability of the trivial solution under perurbations which belong to $L^2(\Omega)$, in particular its dependence on B and $T_0 - T_1$. This point of view is justified by recent results, which establish among others its equivalence with a stability notion due to W. Eckhaus.

Rolls and Modulating Pulses in the Swift–Hohenberg equation

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We consider stationary spatially periodic solutions of the Swift–Hohenberg equation $u_t = -(1+\partial_x^2)^2 u + \varepsilon^2 u - u^3$, $u = u(t,x) \in \mathbb{R}, t > 0, x \in \mathbb{R}^d, 0 < \varepsilon \ll 1$. The family of latex stationary rolls is given by $\{u_{\varepsilon,k}(x) = 2\sqrt{(\varepsilon^2 - (1-k^2)^2)/3}\cos(x_1 + \phi) + \mathcal{O}(\varepsilon^2) : k^2 - 1 \in (-\varepsilon, +\varepsilon), \phi \in S^1\}$. We present results on the nonlinear diffusive stability of Eckhaus–stable stationary rolls in two dimensions ($d = 2$) with respect to perturbations in polynomially weighted spaces [5]. The method consists in combining the results from [3] on the linearization around a roll with renormalization theory in Bloch wave space [1,4].

Next we consider a modified, spatially periodic Swift–Hohenberg equation, which can be considered as a model problem for pattern formation over weakly oscillating domains [2]. In the associated modulation equation the S^1 symmetry is broken. We consider the existence and stability of modulating pulse solutions [6]. These solutions consist of a pulse–like envelope modulating a spatially and temporarely oscillating wave train. They are constructed by means of spatial dynamics and center manifold theory.

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Contributed talks

Energy flow in extended gradient systems

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We study a class of systems on unbounded domain whose restriction to a bounded domain is a gradient system. Typical example is the real Ginzburg-Landau equation

$$u_t = u_{xx} + u - u^3, \tag{1}$$

$u(x,t) : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$, for which all the results apply (other examples include some 1D and 2D parabolic PDE's, damped hyperbolic PDE's, as well as lattice dynamics such as gradient dynamics of Frenkel-Kontorova models).

Extended gradient systems. We propose "axioms" of an extended gradient semiflow: relations between the energy density and energy current (satisfied e.g. by (1) on $C_{b,u}(\mathbb{R})$ in sup-norm as well as localised topologies). We deduce: non-existence of periodic orbits (with period $T > 0$), and that for each x non-stationary, the average return time of any orbit to any neighbourhood of x is ∞. If the phase space is compact (e.g. localised topology in (1)), all Borel invariant probability measures are supported on the set of equilibria, and the ω-limit set of each point contains an equilibrium.

Additional spatial structure. If we assume that the semiflow commutes with a group of spatial translations, the induced semiflow on the space of translationally invariant Borel probability measures is gradient. We deduce that, given any translationally invariant measure μ on the phase space, ω-limit set of μ-almost every initial condition consists of equilibria.

Applications. We suggest applications of the results to variational problems, in particular to construction of orbits and invariant measures of Hamiltonian dynamical systems.

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2.7 Lattice Dynamical Systems

Organizer : Shui-Nee Chow

Key note lecture

Traveling Waves in Lattice Dynamical Systems

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We discuss recent results in the theory of lattice differential equations. Such equations yield continuous-time infinite-dimensional dynamical systems which possess a discrete spatial structure modeled on a lattice. The systems we consider, generally over a higher-dimensional lattice such as $\mathbb{Z}^d \subseteq \mathbb{R}^d$, are the simplest nontrivial ones which incorporate both local nonlinear dynamics and short-range interactions. Of particular interest are traveling wave solutions connecting equilibria which may either be spatially homogeneous or exhibit regular patterns. Also of interest are the effects of anisotropy of the lattice, as well as imperfections in the lattice.

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Invited lectures

Pinning and propagation for waves for bistable lattice systems with long range interaction

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We study an Ising-like model for phase transitions, incorporating long-range interactions. The dynamics of the state is according to an l_2 gradient flow for the Helmholtz free energy. We construct traveling and stationary waves and give criteria for propagation and pinning of a wave. Some results concerning uniqueness and stability of the wave are given.

Dimer automata and edge processes

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We study a class of lattice dynamical systems where the geometric space is a grid (lattice) or, more generally, a graph, the local state space is finite (in most obvious examples the states are "0" and "1", i.e., white and black), the action is local, stochastic, and asynchronous. The motivation for these systems is the following: Deterministic synchronous cellular automata have been proposed as ultimate discretizations of systems modeling interactions and spatial spread. But these systems cannot accomodate the exchange of the contents of two neighboring cells nor a random walk.

To get a true discrete analogue of reaction diffusion equations on lattices we have defined "dimer automata", i.e., lattice dynamical systems where the deterministic local function selects a cell according to a Poisson process and evaluates two cells at a time. These systems can be classified and their dynamical behavior can be characterized in terms of a parameter space. For these systems mean field approximations can be derived which yield a close connection to differential equations and which reproduce experimentally sampled data with astonishing accuracy.

This concept has been generalized in two ways. 1. The regular grid is replaced by an arbitrary graph (topological properties of the graph play some role). 2. The deterministic local function is replaced by a convex combination (in stochastic sense) of such functions.

The resulting class of "edge processes" forms a polyhedron in some parameter space the extremal points of which are (a subset of seven) of the deterministic "dimer systems" mentioned above. The qualitative behavior of the edge processes can be characterized in terms of the theory of stochastic processes. The theory of these systems is put in relation to interacting particle systems in general and to geometric processes studied by T.E.Harris in the seventies.

Finally some findings on two-species systems on lattices are presented.

(joint work with Birgitt Schönfisch, Christoph Bandt, Frank Kriese)

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Cellular Neural Networks: Pattern and Waves

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In this talk I would like to report some results about the patterns and waves of Cellular Neural Networks (CNN). In section 1 we study the stationary solutions of CNN by using the method of iteration map. When the map is one-dimensional, the spatial entropy of stable stationary solutions can be obtained explicitly. When the map is two dimensional, then the Smale's horseshoe is also constructed. In section 2 we consider the travelling wave solutions of lattice dynamical system and prove the existence and multiplicity of monotone travelling wave solutions for some lattice differential equations by using monotone iteration method. In section 3 we illustrate the applications of these results to CNN.

Discrete Breathers

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Discrete breathers are time-periodic spatially localised vibrations of networks of oscillators. Several developments on this topic will be reported, including:

1. Existence of discrete breathers in systems with Euclidean invariance;
2. An effective Hamiltonian for approximate dynamics of generalised multi-breathers [1];
3. Numerical evidence for an almost continuous connection from discrete breathers to Anderson modes [2].

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Contributed talks

Travelling Wave Solutions for some Quasilinear Parabolic Equations

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We study the existence and uniqueness of travelling wave solutions for some quasilinear reaction-diffusion equations and their related discretized systems. These equations arise in various applications such as Mathematical Biology.

Differential equations describing the evolution of densities in particle systems

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The main objects of consideration are nearest neighbour interacting particle systems η_t on homogeneous directed graphs $G = (V, E)$. In this framework homogenity means that for each edge there exists at least one automorphism of the graph mapping this edge to a fixed "root" edge. For each edge we fix one of these automorphisms. By ordering the set of edges we get a sequence of automorphisms.

For each configuration η of the interacting particle system we define distributions of configurations on finite subgraphs $G' \subset G$ as the spatial mean with respect to the fixed sequence of automorphisms. For the subgraph $G' = \{v\}$ consisting of one vertex $v \in V$ this distribution can be interpreted as the density of particles in the configuration η.

We show that existence of all finite distributions for a starting configuration η is sufficient for almost sure existence of all finite distributions for the configuration η_t, $t \geq 0$.

Now we describe the time evolution of the density of particles by a differential equation. Here the time change depends only on the parameters of the interacting particle system and the density of edge markings (that is the density of configurations on the next level subgraph G' consisting of one edge in E).

Some interesting consequences of this relation are presented, as for example the estimation of the parameter for the contact process.

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Complete Stability for a Class of Lattice Dynamical Systems

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We consider a class of lattice dynamical systems originated from cellular neural networks with symmetric feedback templates. In the vector field of this class the state vector and the output vector is related through a sigmoidal nonlinear function. For two types of sigmoidal functions Liapunov functions have been constructed in the literature. Complete stability is studied for these systems using LaSalle's invariant principles on the Liapunov functions. The purpose of this presentation is twofold. The first one is to construct Liapunov functions for other kinds of sigmoidal functions. For example, for a type which is a mixture of the two types described in the literature. The second one is to consider the necessity for condition of isolation of equilibrium in concluding the complete stability. We shall show that for these sigmoidal functions, including the previously studied ones, the condition that every equilibrium is isolated is not necessary.

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Periodic motions of non-autonomous Toda Lattices

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We consider an infinite lattice of particles with a periodic external force and nearest neighbor interaction between particles

$$x_i'' + cx_i' = g_{i-1}(x_i - x_{i-1}) - g_i(x_{i+1} - x_i) + h_i(t), \qquad i \in \mathbb{Z}$$

where h_i are continuous T-periodic functions and $c \geq 0$ is a viscous friction coefficient. If $\overline{h}_i$ denote the mean value of h_i, our aim is to find conditions over such mean values leading to existence and multiplicity of T-periodic solutions.

The strategy of proof has two main steps: first, a related finite system is studied by using a change of variables that enable us a reduction to a simpler subsystem in which classical tools on topological degree can be applied. Second, a simple limiting argument leads to the conclusion, by using some a priori estimates deduced in the previous step.

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2.8 Microstructure

Organizer : Stefan Müller

Key note lecture

How applied analysis can make use of separation of scales: some examples

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Any more complex model in the sciences comes with multiple length and time scales. In many applications, the relevant parameter regime is such that these scales are clearly separated. In this situation, there is hope that the underlying mathematical problem is modular: The contribution of a small scale to the next larger scale can be read off from the solution of an auxiliary problem (which in fortunate cases is explicit).

The standard approach to investigate such a modular structure is by expansion of the solution of the full problem in powers of the (small) ratio of the scales. But this asymptotic analysis may not be applicable in a straightforward way when the description changes from small scale level to large scale level (from a free boundary to density function, say). Moreover, this asymptotic analysis becomes questionable when the solution on a large scale level develops singularities, since it relies on smoothness.

Modern mathematical techniques, mainly developed in variational theory, may help to investigate the modular structure induced by a separation of scales with more rigor than traditional asymptotic analysis. I would like to present a couple of such cases, like thin film micromagnets, Ostwald ripening or the lubrication approximation.

Invited lectures

Branched microstructures: scaling and self-similarity

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We address some properties of a scalar 2D model which has been proposed to describe microstructure in martensitic phase transformations, consisting in minimizing the bulk energy

$$E[u] = \int_0^l \int_0^h u_x^2 + \epsilon |u_{yy}|$$

where $|u_y| = 1$ a.e. and $u(0, \cdot) = 0$. Kohn and Müller[1] proved the existence of the minimizers for $\epsilon > 0$, and obtained bounds on the total energy which suggested self-similarity of the minimizer. Building upon their work, we derive a local upper bound on the energy and on the minimizer itself, and show that the minimizer u is asymptotically self-similar, in the sense that the sequence

$$u^j(x, y) = \theta^{-2j/3} u(\theta^j x, \theta^{2j/3} y)$$

$(0 < \theta < 1)$ has a strongly converging subsequence in $W^{1,2}$.

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Hypersurfaces with mean curvature given by a trace

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We consider smooth, oriented $n-$hypersurfaces $\Sigma_j = \partial E_j$ with interior E_j whose mean curvature is given by the trace of a function in the ambient space $u_j \in W^{1,p}(\mathbb{R}^{n+1})$

$$\vec{\mathbf{H}}_{\Sigma_j} = u_j \ \nu_{E_j} \quad \text{on } \Sigma_j, \tag{1}$$

where ν_{E_j} denotes the inner normal of Σ_j. We investigate (1) when $\Sigma_j \to \Sigma$ weakly as varifolds and prove that Σ is an integral $n-$varifold with bounded first variation which still satisfies (1) for $u_j \to u$, $E_j \to E$. p has to satisfy

$$p > \frac{1}{2}(n+1)$$

and $p \geq \frac{4}{3}$ if $n = 1$. The difficulty is that in the limit several layers can meet at Σ which creates cancellations of the mean curvature.

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A mathematical formulation of rate independent phase transformations using an extremum principle

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A mathematical model for the study of the rate independent mesoscopic behavior of shape memory alloys is presented. The model meets two basic requirements:

1. Nontrivial hysteresis loops occur, i.e. history dependence is reflected,

2. threedimensional microstructures are taken into account.

An abstract framework for rate independent processes in the spirit of the first requirement is presented. Ingrediences are a state space $P \subset X$ where X is a Banach space, a time dependent potential $I : [0,T] \times P \to \mathbb{R}_{\geq}$ and a dissipation functional $\Delta : X \to \mathbb{R}_{\geq}$, which is convex and homogeneous of degree 1. Functions $c : [0,T] \to P$ are denoted as admissible process if for every $t \in [0,T]$, $a \in P$ the stability inequality

$$\Delta(a - c(t)) + I(t,a) - I(t,c(t)) \geq 0$$

is satisfied and the energy inequality

$$I(0,c(0)) - I(T,c(T)) - \int_0^T \{\Delta(\dot{c}) - \partial_t I(t,c(t))\} \, \mathrm{d}t \geq 0$$

holds. Qualitative properties, existence and uniqueness of admissible processes are discussed in simple cases.

In a second step energies describing materials which can undergo phase transformations are relaxed in such a way that they fit into the previously developed framework. Due to the generality of the formulation the existence of admissible processes can be expected in simple cases.

Continuum Limits of Step Flow Models

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We will present various continuum limits in the regime of partial differential equations (PDEs) that arise from the Burton-Cabrera-Frank type step flow models [1]. The motivation and goal is to provide a mathematical model for epitaxial thin film growth and to understand the bridging of various length scales in surface growth phenomena. We concentrate on 1+1 dimensions.

The hierarchy of models we consider can take into the account of the diffusion of terrace adatom, attachment and detachment, edge adatom density, bulk diffusion and multiple species. The PDEs take the form of a coupled diffusion equation for the adatom density and a Hamilton-Jacobi equation for the thin film height profile. Nucleation phenomena is introduced at the top terraces by imposing appropriate boundary conditions for the PDEs.

The equations provide a description of the thin film growth in the mesoscopic scale but they can be potentially linked to the more macroscopic motions laws.

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Contributed talks

Numerical approach to the inverse homogenization problem

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Homogenization theory is a tool for describing effective properties of fine mixtures of materials. It is extensively used in the study of composite materials, in topology optimization and in structural design. We deal with the problem of finding composites with extremal elastic properties, that is, given two materials, finding an optimal way to mix them, in order for the composite to satisfy certain requirements, like maximum bulk modulus or extreme Poisson ratio. This is called "inverse homogenization problem", and has been treated in several papers; see [3], [4], [5]. In order to solve this problem, attention is restricted to periodic mixtures only, and then one looks for an optimal arrangement of the two materials in the periodicity cell.

In the existing literature a "black and white squares" approach is used, that is, the cell is divided in squares, a number is associated to each square, zero for one material, one for the other material. Then those numbers are allowed to take any value in $[0,1]$ by using an artificial interpolation law between the two materials; after applying the optimization procedure a penalty method is applied to get rid of the intermediate densities. Then one has to find a way to eliminate the "checkerboard effect".

Our approach is different; it generalizes to the elastic framework the methods described in [1] and [2]. We use classical shape optimization techniques in the periodicity cell, employing a variable mesh. We simply move the boundary between the zones filled with the two materials in such a way to optimize the elastic properties of the composite. Consequently, we have no need to use an artificial interpolation law between the two base materials, or to penalize the intermediate densities, neither we have to deal with the checkerboard effect.

Our method has the disadvantage that one has to decide from the beginning the number of inclusions to be optimized. The algorithm cannot create any new inclusion or destroy an existing one.

Several numerical examples are presented.

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Quasiconvexity Conditions and Stability of Classes of Lipschitz Mappings

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Let E_k^m be the space of symmetric k-linear maps from $\mathbb{R}^N$ into $\mathbb{R}^m$. For a Borel function $f : E_k^m \to \mathbb{R}$ let

$$f^{qc}(v) := \{\int_{[0,1]^N} f(v + D^k w(x)\, dx : w \in C^\infty(\mathbb{R}^N;\mathbb{R}^m),\ w(y+z) = w(y),\ y \in \mathbb{R}^N,\ z \in \mathbb{Z}^N\}$$

denote the *quasiconvex envelop* of f at $v \in E_k^m$ [1]. Here $D^k w(x)$ stands for the differential of order k of a mapping w at $x \in \mathbb{R}^N$. A set $K \subset E_k^m$ is called *quasiconvex* if $K = \{v \in E_k^m : \operatorname{dist}^{qc}(v, K) = 0\}$.

Let K be a compact set in E_k^m. We consider the class of all continuous $W_{loc}^{k,\infty}$-solutions $u : U \to \mathbb{R}^m$ of the relation

$$D^k u \in K \text{ a.e. in } U, \tag{1}$$

where U is a domain (open connected set) in $\mathbb{R}^N$. For a quasiconvex compact set K this class is stable in the C^{k-1}-norm in the following sense.

Theorem. *Let K be a quasiconvex compact set in E_k^m. There exists a function $\alpha = \alpha_K : [0, +\infty) \times (0, 1) \to [0, +\infty)$ such that* (i) $\lim_{\varepsilon \to 0} \alpha(\varepsilon, \rho) = \alpha(0, \rho) = 0$ *if* $0 < \rho < 1$; (ii) *if V is a domain in $\mathbb{R}^N$ and $v : V \to \mathbb{R}^m$ is a continuous mapping of the class $W_{loc}^{k,\infty}$ with $\|\operatorname{dist}(v(\cdot), K)\|_{L^\infty(V)} < \infty$, then for every bounded domain U with* $\operatorname{diam}_{inn} U < \infty$ *whose neighborhood $\{x \in \mathbb{R}^N,\ \operatorname{dist}(x, U) < \frac{1-\rho}{\rho} \operatorname{diam} U\}$ lies in V there is a continuous $W_{loc}^{k,\infty}$-solution $u : U \to \mathbb{R}^m$ of* (1) *satisfying the inequality*

$$\sum_{j=1}^{k} (\operatorname{diam}_{inn} U)^{j-k} \|D^j(v - u)\|_{C(U)} \le \alpha(\|\operatorname{dist}(v(\cdot), K)\|_{L^\infty(V)}, \rho) \operatorname{diam} U.$$

By $\operatorname{diam}_{inn} U$ *we denote the diameter of a domain U calculated in its intrinsic metric.*

One application of the theorem is the study of ω-stability in the C-norm for classes of Lipschitz mappings. The concept of ω-stability was proposed by A. P. Kopylov [2] as a generalization of the stability property of the class I_N of isometric mappings $u : U \subset \mathbb{R}^n \to \mathbb{R}^n$. The stability of I_N in this context was proved by F. John [3]. Note that in [3] the class I_N was studied in connection with an analysis of linearizations of elasticity problems.

The research was supported by the INTAS grant 97–10170 and by the RFBR grant 99–01–0051.

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Spectral problems in a periodic thick multi-structure with concentrated masses

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Convergence theorems and asymptotic estimates (as $\varepsilon \to 0$) are proved for eigenvalues and eigenfunctions of the main boundary-value problems for the Laplace operator in a plane thick periodic junction Ω_ε with concentrated masses on thin channels. This junction consists of the junction's body

$$\Omega_0 = \{x \in \mathbb{R}^2 : 0 < x_1 < a,\ 0 < x_2 < \gamma(x_1)\},$$

and a large number N of the thin channels

$$G_\varepsilon^j = \{x \in \mathbb{R}^2 : |x_1 - \varepsilon(j+1/2)| < \varepsilon h/2,\ x_2 \in (-1,0]\}, \quad j = 0,1,\dots,N-1.$$

Here $\gamma \in C^\infty([0,a])$, $0 < \gamma_0 = \min_{x_1\in[0,a]} \gamma(x_1)$; h is a fixed number from the interval $(0,1)$; N is a large positive integer, therefore, $\varepsilon = a/N$ is a small discrete parameter which characterizes the difference between the thin channels and their thickness.

The density ρ_ε of the junction is order $O(\varepsilon^{-\alpha})$, $\alpha \geq 0$, on the thin channels (the concentrated masses if $\alpha > 0$), and $O(1)$ outside of them. Spectral problems with different boundary conditions (Dirichlet, Neumann) on the boundary Υ_ε of the thin channels are considered. So, for the spectral problem

$$\begin{array}{rcll} -\Delta_x\, u(\varepsilon,x) &=& \lambda(\varepsilon)\ \rho_\varepsilon(x) u(\varepsilon,x), & x \in \Omega_\varepsilon, \\ \partial_\nu\, u(\varepsilon,x) &=& 0, & x \in \partial\Omega_\varepsilon \cap \{x : x_2 \geq 0\}, \\ u(\varepsilon,x) &=& 0, & x \in \Upsilon_\varepsilon = \partial\Omega_\varepsilon \cap \{x : x_2 < 0\}, \end{array} \qquad (1)$$

with the Dirichlet conditions on Υ_ε, there are three qualitatively different cases in the asymptotic behavior of the eigenvalues and the eigenfunctions of problem (1): $0 \leq \alpha < 2$, $\alpha = 2$, $\alpha > 2$.

In the first case, the energy of the free vibrations is concentrated in the junction's body, and the eigenvalues of problem (1) have the following asymptotics

$$\lambda_n(\varepsilon) = \mu_n + O(\varepsilon) \quad \text{as} \quad \varepsilon \to 0,$$

where μ_n is an eigenvalue of some mixed boundary-value problem for the Laplace operator in the junction's body Ω_0.

In the second case, the energy is concentrated both in the junction's body and in the thin channels. The eigenvalues tend to the corresponding eigenvalues of some discontinuous self-adjoint operator-function. Furthermore, they are split into a countable family of series, the limits of the eigenvalues from each series belong to a finite interval and these intervals are mutually disjoint.

In the third case, the energy of the free vibrations is concentrated in the thin channels; all the eigenvalues tend to zero, and have the asymptotics

$$\lambda_n(\varepsilon) = \varepsilon^{\alpha-2}\Lambda_0 + \varepsilon^\alpha \beta_n + O(\varepsilon^{\alpha+1}) \quad \text{as} \quad \varepsilon \to 0.$$

Existence and relaxation results in the class of anti-plane shear deformations

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In this talk we discuss existence and relaxation results for the problem

$$J(u) = \int_\Omega L(x, u(x), Du(x))dx \to \min, u\Big|_{\partial\Omega} = f \qquad (1)$$

in the class of Sobolev functions $u : \Omega \to \mathbf{R}$.

Standard assumptions in Nonlinear Elasticity lead to the requirement

$$L \geq \alpha |Du|^p + \gamma, \alpha > 0, p > n. \tag{2}$$

It turns out that under this requirement the relaxation result holds [1].

Theorem *Assume that $L : \Omega \times \mathbf{R} \times \mathbf{R}^n \to \mathbf{R} \cup \{\infty\}$ satisfies the requirement (2) and assume that it is bounded in a neighborhood of each point $(x, u, v) \in \Omega \times \mathbf{R} \times \mathbf{R}^n$, where it takes finite value.*

Then for each function $u_0 \in W^{1,p}(\Omega)$ with $\tilde{J}(u_0) < \infty$, where $\tilde{J}$ is the integral functional with the integrand obtained by convexification of L with respect to Du, there exists a sequence u_k such that u_k converges weakly in $W^{1,p}$ to u_0, $u_k\big|_{\partial\Omega} = u_0\big|_{\partial\Omega}$, and $J(u_k) \to \tilde{J}(u_0)$.

This theorem shows that to establish existence in (1) one still has to follow the standard scheme to find a solution of the relaxed problem along which the values of the original and the relaxed integrands coincide.

In the homogeneous case $L = L(Du)$ it is easy to show that the requirement *for each $v \in \mathbf{R}^n$ either $\partial L(v) \neq \emptyset$ or there exist $v_1, \ldots, v_q \in \mathbf{R}^n$ such that v belongs to the interior of the convex hull of $\{v_1, \ldots, v_q\}$ and $\cap_{i=1}^q \partial L(v_i) \neq \emptyset$* is both necessary and sufficient for all boundary value problems (1) to have a solution, see [2].

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Constrained Young measures

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Several problems envolving non linear effects induced by homogenization have been studied since the importance of the subject was pointed out by L. Tartar. At the same time he introduced new appropriated methods crucial for the future development on the subject, like mixing the "old" tool of Young measures with the new tool of compensated compactness (see [4]) or like developping the new tool of H-measures (see [5]).

We inscribe our contribution in the effort to describe such nonlinear effects by answering the following simplified question proposed in [6]:

Given two sequences, u_n on $\omega_1 \times \omega_2$ and v_n on ω_2 such that the constrain $v_n = \frac{1}{|\omega_1|} \int_{\omega_1} u_n(x, y) dx$ holds, what relationship do we have between their associated Young measures? Is the condition sufficient?

Under appropriate hypotheses our main result gives necessary and sufficient conditions for the above problem. Namely, we prove that the following statements are equivalent:

1) ν and μ are probability measures on $[\alpha, \beta]$ such that $\mu \prec \nu$;

2) there exists a sequence (u_n), $u_n : \omega_1 \times \omega_2 \to [\alpha, \beta]$ which gives rise to Young measure $\nu \otimes dx \otimes dy$ and the sequence (v_n) defined by $v_n(y) := \frac{1}{|\omega_1|} \int_{\omega_1} u_n(x,y)dx$ gives rise to Young measure $\mu \otimes dy$.

The order relation $\prec$ on the set of probability measures on $[\alpha, \beta]$, is a version of that introduced by G. H. Hardy, J. E. Littlewood and G. E. Pólya in [1] and is defined by: $\mu \prec \nu$ if and only if $\int_\alpha^\beta \phi(z)d\mu(z) \leq \int_\alpha^\beta \phi(z)d\nu(z)$, for all continuous and convex functions ϕ.

The characterizations of the order relation $\prec$ in terms of doubly stochastic operators and rearrangements (J. V. Ryff [2] and [3]) are employed.

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Posters

Time Dependent Young Measure Solutions for an Elasticity Equation with Diffusion

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A system of a nonconvex elasticity equation possibly coupled with a nonconvex diffusion equation is studied. The aim of this system is to describe crystals consisting of different chemical ingredients which can produce microstructures.
In particular we consider the initial boundary value problem:

$$\begin{aligned} u_{tt}(x,t) - \operatorname{div} S(\nabla u(x,t), c(x,t)) &= 0, \\ c_t(x,t) - \operatorname{div} K(\nabla c(x,t), u(x,t)) &= 0, \end{aligned}$$

where $x \in \Omega \subset \mathbb{R}^n$ and $t \geq 0$, u denotes the displacement vector, c denotes the chemical concentrations and S and K are the nonlinear material functions.
In the variational case we can prove – assuming a growth condition and certain regularities for S and K – an existence theorem for the class of time dependent Young measure solutions.

This existence theorem extends a result of Demoulini [1] for the nonconvex wave equation. Its proof uses time discretization and relaxation methods. An inportant ingredient is the fact that the quasiconvexification of a $\mathcal{C}^1$-function is in $\mathcal{C}^1$ as recently shown by Ball, Kirchheim and Kristensen [2].
Further properties of the Young measure solutions were studied.

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2.9 Nonlinear Functional Analysis

Organizers : Hansjörg Kielhöfer, Jean L. Mawhin

Key note lecture

Applications of critical point theory to variational problems on $\mathbb{R}^n$

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In this talk we will discuss some recent advances in critical point theory and their applications to elliptic problems on $\mathbb{R}^n$.

The abstract set up deals with the existence of critical points of C^2 functionals of the type

$$f_\epsilon(u) = f_0(u) + \epsilon G(u), \qquad u \in E \tag{1}$$

where E is a Hilbert space and f_0 possesses a finite dimensional, possibly non compact manifold Z of critical points. Suppose that Z is non degenerate (in the sense that $KerD^2 f_0(z) = T_z Z$ for all $z \in Z$) and let Γ denote the functional G constrained on Z. Then near any *topologically stable* critical point of Γ there exists a critical point of f_ϵ, provided ϵ is small enough. See [1] for the case that Z is compact and [2, 3] for the general case.

The preceding abstract setting provides a unified frame for a broad variety of problems, variational in nature. A typical example is the existence of homoclinics and chaos in dynamical systems, see [2, 4]. Other applications include: (i) the existence of semiclassical states for nonlinear Schrödinger equations with potential, see [5]; and (ii) the problem of finding a conformal metric with a prescribed scalar curvature, see [6, 7].

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Invited lectures

New Existence Theorems in Nonlinear Elastostatics

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Nonlinear elasticity deals with the deformation of solid bodies under the influence of external loadings. As such it forms one of the pillars of classical continuum physics - dating back to Cauchy. In spite of its age, properly formulated elasticity theory produces formidable nonlinear problems, the resolution of which is in many cases beyond the reach of present-day mathematical analysis. In particular, there are no general existence results in nonlinear elastostatics. In this talk we discuss the mathematical difficulties, which are a direct consequence of restrictions dictated by the physical phenomenon. Next we present some recent results of the speaker yielding global branches of classical solutions based upon a novel Leray-Schauder approach. Lastly we compare/contrast our results with the famous minimum-energy results of Ball.

Pattern Formation of the Stationary Cahn-Hilliard Model

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We investigate critical points of the free energy $E_\varepsilon(u)$ of the Cahn-Hilliard model over the unit square under the constraint of a mean value m. We show that for any fixed value m in the so-called spinodal region and to any mode of $D_uE_\varepsilon(m)$ there are critical points of $E_\varepsilon(u)$ having the characteristic symmetries and monotonicities of that mode provided $\varepsilon > 0$ is small enough. As ε tends to zero these critical points have singular limits where the symmetries and monotonicities are preserved. Therefore they form characteristic patterns for each mode.

It is remarkable that all singular limits are global minimizers of $E_0(u)$ which is not obvious since minimizing properties of the critical points of $E_\varepsilon(u)$ are not known.

Our method consists of a global bifurcation analysis of critical points of the energy $E_\varepsilon(u)$ where the bifurcation parameter is the mean value m. This new analytic approach suggests a path following device to obtain the critical points (near global minimizers) numerically.

Smooth Curves of Variational Solutions

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We consider positive solutions of the homogeneous Laplace equation $-\Delta u + \lambda u = u^q$ with Neumann boundary conditions and q subcritical. There are several families of symmetric multi-peaked solutions known, whose existence was shown by minimizing an energy functional subject to several side conditions (cf. e.g. [1]; the symmetries were discussed in [2]). In this talk we derive that these solutions in fact lie on smooth curves. We have to combine symmetry arguments, variational technics, and bifurcation theory.

Coauthor: Zhi-Qiang Wang

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Some results on the forced pendulum equation

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I present two results on the periodic problem for the forced pendulum equation [2], [3]. Both are inspired by an interesting example first analyzed in [1].

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Contributed talks

Multipoint Boundary Value Problems

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Consider the following multipoint boundary value problem for a second order ordinary differential equations

$$\begin{cases} y''(x) = f(x, y(x), y'(x)) + e(x) \qquad x \in]0,1[\\ ax(0) + bx'(0) = 0 \\ cx(1) + dx'(1) = \sum_{i=1}^{m} \alpha_i x(\xi_i) + \sum_{j=1}^{n} \beta_j x'(\tau_j) \end{cases} \tag{1}$$

where $f : [0,1] \times R^2 \to R$ is a Caratheodory function, $e \in L^1([0,1])$. Given $\xi_i, \tau_j \in (0,1)$, $\alpha_i, \beta_j \in R$, all of the α_i ,respectively β_j, have the same sign for $i = 1, 2, ..., m$ and $j = 1, ..., n$ with $0 < \xi_1 < ... < \xi_m < 1$, $0 < \tau_1 < ... < \tau_n < 1, a, b, c, d,$ are real and $ad - bc \neq 0$.

Our objective is to prove the existence of solutions of problem (1) under suitable conditions on the nonlinearity f.

Multiple solutions of a quasilinear elliptic boundary value problems

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In this talk, we give a description of the set of solutions for the following problem:

$$\begin{cases} -\left(|u'|^{p-2} u'\right)' = |u|^{\alpha-1} u + \lambda |u|^{\beta-1} u \qquad x \in]0,1[\\ u(0) = u(1) = 0 \end{cases}$$

where $p > 1, 0 < \beta < 1 < \alpha,$ and λ is a real parameter.

Theoretical and numerical study of multi-phase flows through order parameter formulation

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In this work, we study a mathematical model for multiphase flows. It was proposed for example in [2] in order to study the evolution of an alloy of two incompressible fluids in a channel under shear. It deals with a coupling of the Cahn-Hilliard equations (1)-(2) and the incompressible Navier-Stokes equations (3)-(4)

$$\frac{\partial \varphi}{\partial t} + v.\nabla\varphi - \operatorname{div}(B(\varphi)\nabla\mu) = 0, \tag{1}$$

$$\mu = -\alpha\Delta\varphi + F'(\varphi), \tag{2}$$

$$\frac{\partial v}{\partial t} - 2\ \mathrm{div}(\eta(\varphi)D(v)) + (v.\nabla)v + \nabla p = \mu \nabla \varphi, \tag{3}$$
$$\mathrm{div}(v) = 0, \tag{4}$$

where $B(\varphi)$ is a mobility coefficient, $\eta(\varphi)$ the viscosity of the alloy, and μ stands for a chemical potential in which F is the bulk free energy of the alloy.

This system is provided with Neumann boundary conditions for φ and μ, and by Dirichlet and/or periodicity conditions for the velocity. The Gibbs free energy F is caracterised by its double-well structure reprensenting the two phases of the alloy [3].

In a first part, we deal with theoretical properties of this system in the case of a channel under shear [1]. Under quite general assumptions on F, we prove the existence of global weak solutions even if the mobility is allowed to degenerate. Then, if the mobility does not degenerate, we show the existence and uniqueness of strong solutions which are global in 2D and local in 3D.

Then we prove a result of asymptotic stability in both dimensions 2 or 3 for the uniform alloy of composition $\varphi = \omega$, if ω lies in a metastable region of F [3]. More precisely we establish that if the initial condition is close enough from the equilibrium $\varphi = \omega$, and if the initial velocity is small enough, then there exists a unique global strong solution which converges to the stationary solution when t goes to infinity.

In a second part, we investigate the numerical approximation by finite differences of this model in 2D. In particular we show that even if the convective term is predominant in (1), the presence of the Cahn-Hilliard term limits the effects of the numerical diffusion of classical upwind schemes.

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On problems with nonlinear functional boundary conditions

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Boundary value problems for ordinary differential equations and functional differential equations are considered. The boundary conditions are described by nonlinear functionals. Sufficient criteria for the solvability are obtained. These criteria are applicable to essentially nonlinear systems. The case of boundary conditions given by nonlinear monotone mappings is studied. The topological structure (including arcwise connectedness) of the solution set is investigated. Boundary value problems with nonlinear boundary conditions arise, in particular, in the theory of control and differential games.

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Lower and upper solutions in the theory of ϕ – Laplacian problems

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In [2], the existence of solutions of the problem

$$-(\phi(u'(t)))' = f(t, u(t), u'(t)),\ u(a) = u(b),\ u'(a) = u'(b) \tag{1}$$

in the sector $[\alpha, \beta]$ is proved. Here α and β are respectively a lower and an upper solution of problem (1), ϕ is an increasing homeomorphism from $\mathbb{R}$ onto $\mathbb{R}$ and f is a Carathéodory function which satisfies some Nagumo type conditions.

It is well known that, even if ϕ is the identity, existence of lower and upper solutions in the reversed order, i.e. $\alpha \geq \beta$, in general does not imply the existence of solutions of problem (1). If $\phi = id$ optimal conditions on $f(t, u, u') \equiv f(t, u)$, which guarantee the existence of solutions of problem (1) in $[\beta, \alpha]$, are known [1].

In this work, we study problem (1) with $f(t, u, u') \equiv f(t, u)$ and ϕ^{-1} a locally Lipschitz function. Existence of extremal solutions in $[\beta, \alpha]$ via the monotone method is obtained. Such results are based on anti-maximum comparison principles for the operator $-(\phi(u'))' - Mu$ $(M > 0)$, which are stronger than the classical anti-maximum principle since we need to compare two solutions of the problem

$$-(\phi(u'))'(t) - M\,u(t) = \sigma(t), \quad u(a) = u(b),\ u'(a) = u'(b).$$

These comparison results are derived from those obtained in [3] for the Neumann problem. Our results are optimal in the sense that if ϕ is the identity we obtain the best possible estimate on M given in [1]. Furthermore, we indicate functions ϕ^{-1} which are not locally Lipschitzian and f so that there exist a lower solution α, an upper solution $\beta \leq \alpha$ and there exists no solution lying between β and α.

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On the ensemble of resonance of a fourth order equation.

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In this talk we will try to give an answer to a question that appears in the book of Fučík [1]. The problem is to give a description of the set A_1 of all pairs (a, b) such that the system

$$\begin{cases} u^{iv} = au^+ - bu^- \\ u(0) = u(\pi) = u''(0) = u''(\pi) = 0, \end{cases}$$

has a non-trivial solution. Here $u^+ = \max\{u, -u\}$ and $u = u^+ - u^-$. This problem appears when one looks for a priori bounds in a problem

$$\begin{cases} u^{iv} = f(u) \\ u(0) = u(\pi) = u''(0) = u''(\pi) = 0, \end{cases}$$

where f grows linearly.

In this talk we describe the set A_1 as a countable set of analytic curves. We shall also analyze their intersections and the asymptotic behavior. This situation resembles the case of second order in which analytic curves go through the eigenvalues of the operator.

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Remarks on a periodic boundary value problem for second order impulsive integro-differential equations

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Consider the following second order integro-differential equation with impulses at fixed moments

$$\begin{array}{ll} -u''(t) = f(t, u(t), [Ku](t)), & t \in J \setminus \{t_1, t_2, \dots, t_p\}, \\ \Delta u|_{t=t_k} = I_k(u'(t_k)), & k = 1, 2, \dots, p, \\ \Delta u'|_{t=t_k} = \bar{I}_k(u(t_k)), & k = 1, 2, \dots, p, \end{array} \tag{1}$$

where $J = [0, 2\pi]$, $0 = t_0 < t_1 < t_2 < \cdots < t_p < t_{p+1} = 2\pi$, $f \colon J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $I_k, \bar{I}_k \colon \mathbb{R} \to \mathbb{R}$, $\Delta u|_{t=t_k} = u(t_k^+) - u(t_k^-)$, $\Delta u'|_{t=t_k} = u'(t_k^+) - u'(t_k^-)$ and K is the integral operator defined by $[Ku](t) = \int_0^T a(t, s)u(s)ds, \quad a \in C[J \times J; \mathbb{R}]$.

Note that this equation has the peculiarity that the impulses are in some sense crossed, since $\Delta u|_{t=t_k}$ depends on $u'(t_k)$ and $\Delta u'|_{t=t_k}$ depends on $u(t_k)$. Equation (1) has been considered in several papers with different boundary conditions. Here we shall study the periodic problem, i.e.,

$$u(0) = u(T), \quad u'(0) = u'(T). \tag{2}$$

In [2] problem (1)-(2) is studied without impulses in u and with linear impulses in u' (i.e. $I_k = 0$, $\bar{I}_k(x) = c_k x$). In [1] it is considered with linear impulses at u and u' (i.e. $I_k = c_k x$, $\bar{I}_k(x) = \bar{c}_k x$). Both papers are presented in the more general scheme of Banach spaces.

By establishing certain existence and comparison results, and using the monotone iterative technique, we shall obtain a criteria on the existence of minimal and maximal solutions for (1)-(2).

In the scalar case we improve and simplify results in [1] and [2]. Besides, it is easy to generalize our results for integro-differential equations in Banach spaces.

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The sweeping processes without convexity

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The sweeping process introduced by Moreau in the Seventies describes the behaviour of a moving point that is constrained both to belong in each time moment t to some closed set $K(t)$, and to have a velocity "normal" to the boundary of this set. Due to mechanical applications (such as friction and resistence) this moving point may be treated as the velocity of a particle as well. The case of convex set $K(t)$ was detaily investigated by J.J.Moreau, C.Castaing, M.Valadier, M.D.P.Monteiro Marques and other researchers. Some results were also obtained if $K(t)$ is the complement of an open convex set by a suitable way depending on t.

We propose a new approach to study the sweeping processes associated with not necessarily convex sets. Passing to the mathematical formulation, we have to find a solution of the problem

$$\begin{cases} -\dot{x}(t) & \in & \mathbf{N}_{K(t)}(x(t)) \\ x(0) & = & x_0 \in K(0) \\ x(t) & \in & K(t) \qquad \forall t \end{cases} \tag{1}$$

where $\mathbf{N}_K(x)$ means the Clarke normal cone to K at the point $x \in K$, and the mapping $t \mapsto K(t)$ is supposed to be Lipschitzean. Notice that in the nonconvex case statement of the problem depends essentially on the meaning of normal vectors. Moreover, as shown by an example, the problem (1) in general can have no solutions if we consider the Bouligand normal cone in the place of the Clarke one. We present two existence results for solutions of (1) in a Hilbert space. One of them requires local compactness of the graph of $K(t)$, while other involves the assumption permitting to control the lack of convexity of $K(t)$ through some variational inequality. It seems that the last property known in the literature as φ-convexity (see, e.g., [1]) is strictly related with existence and uniqueness of the projection onto the set. Also we study the variational properties which naturally generalize φ-convexity and their relations with regularity of the closed sets. Finally, we prove uniqueness and regularity theorems for solutions of (1) under assumptions that the moving closed set $K(t)$ satisfies some of these properties.

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Polynomial solutions on matrix differential equations of Ricati type

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By introducing and using the properties of polynomial part of the expansion of a root of polynomial in scalar case, up to now polynomial solutions of Riccati differential equations and some wider classes of algebraic differential equations are described - issue studied in many papers from many various authors.

Matrix equations of Riccati type with λ-matrixes as coefficients is solved in this thesis. λ-matrix with the basic properties of previously mentioned polynomial is defined, its existing is proved in a terms of eigenvalues of the oldest matrix and than the solutions of the matrix differential equation are described and also the conditions for their existence are given.

For commutative matrices the results get the same form as in the scalar case.

Boundedness of Volterra Difference Equations

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Volterra difference equations mainly arise in the modeling process of some real phenomena or by applying a numerical method to a Volterra integral equation. Actually much of their general quantitative and specially their qualitative theory remains to be developed.

We consider the following system of Volterra difference equations

$$\Delta x(n) = A(n)x(n) + \sum_{j=n_0}^{n-1} K(n,j)x(j) + F(n),\ x(n_0) = x_0, \tag{1}$$

where $\Delta x(n) = x(n+1) - x(n)$; $A(n)$, $K(n,j)$ are $m \times m$ matrices for each $n, j \in N$, and $F : N \to R^m$ is a function.

The goal of this article is to give sufficient conditions under which the solutions of the Volterra difference Equation (1) are bounded. For this purpose, we will establish that the following linear difference Equation (2) is equivalent to the Equation (1):

$$\Delta y(n) = B(n)y(n) + L(n,n_0)x_0 + H(n),\ y(n_0) = x_0, \tag{2}$$

where

$$B(n) = A(n) - L(n,n),\ H(n) = F(n) + \sum_{j=n_0}^{n-1} L(n,j+1)F(j) \tag{3}$$

and L is an $m \times m$ matrix.

Moreover, weighted norms are used to find sufficient conditions under which Equation (1) has bounded solutions.

A multiplicity result for a floating beam equation

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We consider a strongly indefinite functional whose critical points are the weak solutions of a beam equation of the type

$$\begin{aligned} u_{tt}(x,t) + u_{xxxx}(x,t) + b[u(x,t)]^+ &= c && \text{in } [0,a] \times \mathbf{R} \\ 0 = u_{xx}(0,t) = u_{xx}(a,t) = u_{xxx}(0,t) &= u_{xxx}(a,t) && \text{for all } t \in \mathbf{R} \\ u(x,t) &= u(x,t+2\pi) && \text{for all } x \in [0,a],\ t \in \mathbf{R} \\ u(x,t) &= u(x,-t) && \text{for all } x \in [0,a],\ t \in \mathbf{R} \\ u(x,t) &= u(a-x,t) && \text{for all } x \in [0,a],\ t \in \mathbf{R} \end{aligned}$$

where b and c are positive constants and $[u]^+ = \max(0,u)$. This kind of equation, modelling a floating beam, has been studied by Lazer and Mc Kenna by means of a finite dimensional reduction. They prove the existence of two nontrivial solutions when b belongs to a certain interval.
We get the existence of two nontrivial solutions for $b > -\Lambda_1^-$, and the existence of four nontrivial solutions for $b > -\Lambda_k$ (with $k \geq 2$) and b close to Λ_k. Here we denote by Λ_i^- the sequence of the negative eigenvalues of the operator $Lu = u_{tt} + u_{xxxx}$.
In the proof we describe the geometry of a suitable associated functional and use a limit category argument.
This is a joint work with Claudio Saccon.

Lower and Upper Solutions Method for Third Order Nonlinear Periodic Problem

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We study in this work an existence result for the nonlinear third order periodic boundary value problem $x''' = f(t, x, x', x'')$ using topological degree and the method of lower and upper solutions for continuous functions. With our technique we can bound the solution and its derivative.

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Quasilinear Elliptic Equations in the whole space

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We look for positive (weak) solutions of the following problems:

$$-\Delta_p u(x) + \alpha u^{p-1}(x) = \lambda h(x) u^{q-1}(x) + g(x) u^{r-1}(x), \quad x \in \mathbb{R}^N, \qquad (P_\lambda)$$

with $1 < p < \infty$, $1 < q < p < r \le p^*$, where $p^* = \frac{Np}{N-p}$ if $1 < p < N$, $p^* = \infty$, if $p \ge N$, λ is a real parameter, $\alpha \ge 0$, h and g verify some integrability conditions. When $p = 2$ the problem corresponds to the classical *Laplacian* and even in this case the results are new. We will study three different cases:

We will emphasize the differences of behaviour with the case of the Dirichlet problem in a bounded domain.

We would like to point out that for $p = 2$ and the critical case the problem is related to some scalar curvature problems in $\mathbb{R}^N$.

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Setting of the dual problems in mathematical physics

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A solution of boundary value problem of mathematical physics is used at calculation of physical fields. These problems are a differential corollary of conservation laws of some physical magnitudes (u — basic variable). For magnitude u in domain $V \subset R^3$ the functional equation is given:

$$L(u) = f \tag{1}$$

Let the "natural" boundary conditions are given on a surface bounding the domain

$$pn = \phi \tag{2}$$

where "fluxes" p of magnitude u are introduced with the help of operator M

$$p = M(u) \tag{3}$$

Thus

$$Div\ M = L \tag{4}$$

There is a dual setting of problem (1), (2). It consists of solution of the equations

$$Div\ p = f \tag{5}$$

being a corollary of relations (1), (4). Besides the "equations of a compatibility" are introduced:

$$P(p) = 0 \tag{6}$$

being the solvability conditions of the equation (3).

Variational principles are connected, as a rule, with each of the dual problems (1), (2) and (5), (6), (2). The simultaneous application of these variational principles allows energetically estimating a solution of the problem from above and from below.

We offer new setting of the dual problem which in most cases essentially simplifies realization of applied calculations of physical fields. It consists of solution of the functional equations

$$P(p) + A\{Div\ p - f\} = 0$$

with boundary conditions

$$P_n = \phi; \quad Div\ p - f = 0$$

The new variational principle is offered. The effective methods of a numerical realization of the problem in new setting are indicated. Special examples are included. The possible generalizations are considered.

Upper and lower solutions for first order discontinuous BVPs

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Existence of Carathéodory – type solutions for the problem

$$(P) \qquad x'(t) = f(t, x(t)) \text{ for a.e. } t \in [0,T], \quad B(x(0), x) = 0,$$

where f and B are not necessarily continuous functions, is studied in the frame of upper and lower solutions.

Condition $B(x(0), x) = 0$ is defined as an extension of the periodic boundary conditions ($x(0) = x(T)$) which involves the behavior of the solution along the whole interval $[0, T]$. For instance, under the previous formulation we will be able to consider functional conditions such as

$$x(0) = \int_0^T x(s)ds \quad \text{or} \quad x(0) = \max_{t\in[0,T]} x(t).$$

Existence of solutions for problem (P) is studied in [1], with absolutely continuous upper and lower solutions. We shall see that the regularity of these functions can be relaxed to obtain finer results in practical situations: we shall define the concepts of upper and lower solutions in the space of functions of bounded variation, covering the useful case of piecewise absolutely continuous upper and lower solutions (see [2, 3]).

In [1] the following assumptions on f are required: $f(t,x) = q(x)g(t,x)$ where $q : \mathbb{R} \to (0, \infty)$ needs not be a continuous function, and g is a Carathéodory function (in particular, continuous with respect to its second variable). We shall see how to prove an existence result for (P) assuming no continuity hypothesis over g: essentially, this is done replacing the Carathéodory conditions by the ones introduced in [4].

Finally, we consider the situation in which the upper and the lower solution are given with no ordering. To prove a proper existence and localization result in this case we have to couple the basic ideas presented in [5] with adequate new arguments (suitable to deal with upper and lower solutions of bounded variation).

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Equivalence of two definitions of index of periodic solutions of an evolution equation of second order

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We consider the evolution equation

$$\ddot{u} + c\dot{u} + \ell u = f(t, u)$$

where ℓ is a coercive self-adjoint operator with dense domain in a Hilbert space H and f is periodic in time.

If u is a periodic solution in t of such equation, we can define the index of u in two ways: either as the fixed point of the Poincaré map or in terms of a standard functional setting for periodic problems.

We prove the equivalence of both definitions. In the proof we follow the ideas of Krasnoselskii and Zabreiko.

This is joint work with R. Ortega.

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The Fucik spectrum of general Sturm-Liouville problems

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We consider the general Sturm-Liouville boundary value problem

$$-(pu')' + qu = \alpha u^+ - \beta u^-, \quad \text{in } (0, \pi),$$
$$a_0 u(0) + b_0 u'(0) = 0, \quad a_1 u(\pi) + b_1 u'(\pi) = 0,$$

where $u^\pm = \max\{\pm u, 0\}$. The set of points $(\alpha, \beta) \in \mathbb{R}^2$ for which this problem has a non-trivial solution is called the Fucik spectrum. When $p \equiv 1$, $q \equiv 0$, and either Dirichlet or periodic boundary conditions are imposed, the Fucik spectrum is known explicitly, and consists of a countable collection of curves, with certain geometric properties. In this talk we show that similar properties hold for the general problem above, and also for a further generalization of the Fucik spectrum. We also discuss some spectral type properties of a positively homogeneous, 'half-linear' problem, and use these results to consider the solvability of a nonlinear problem with jumping nonlinearities.

Bounded solutions for nonlinear elliptic equations in unbounded domain

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We prove the existence of a bounded, radial solution in unbounded domain of the nonlinear elliptic problem

$$\begin{array}{l} \Delta u = f(\|x\|, u) \ \text{ for } \ \|x\| > 1, \ x \in \mathbb{R}^n, \ n \geq 3 \\ u(x) = 0 \ \text{ for } \ \|x\| = 1 \end{array} \tag{1}$$

under some asymptotic and sign condition on f.

Under stronger assumptions it is proved that this solution must be of constant sign.

The problem is at resonance, since the function $u(x) = 1 - \|x\|^{2-n}$ is a solution for coresponding homogeneous BVP (with zero right-hand side of (1)). The theory of coincidence degree developed in [1], [2] does not apply here, because the differential operator (with the boundary condition taken into account) is not Fredholm.

To prove the existence of solution of (1) we use perturbation method together with some fixed point theorem in the space of bounded and continuous functions.

A good survey of known results on the existence of positive solutions of (1) is in [3]. However, only solutions convergent to zero at infinity are considered therein.

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Adjoint Operators to Nonlinear Operators

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If on a nonvoid set the nonnegative identical symmetric function is given then this set is called weakly metric space. An operator which maps one weakly metric space to another weakly metric space is considered. In this situation the concepts of adjoint spaces and adjoint operator are introduced. Different properties of adjoint operators are establshed. The role of adjoint operator concept for nonlinear equation solvability is explained. More explicitly the Lipshitz operators and its adjoint operators in Banach spaces are investigated.

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2.10 Variational Methods

Organizer : Mariano Giaquinta

Key note lecture

Minimal geodesics on groups of volume-preserving maps and generalized solutions of the Euler equations

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The three-dimensional motion of an incompressible inviscid fluid is classically described by the Euler equations, but can also be seen, following Arnold, as a geodesic on a group of volume-preserving maps. Local existence and uniqueness of minimal geodesics have been established by Ebin and Marsden. In the large, for a large class of data, the existence of minimal geodesics may fail, as shown by Shnirelman. For such data, we show that the limits of approximate solutions are solutions of a suitable extension of the Euler equations or, equivalently, as sharp measure-valued solutions to the Euler equations in the sense of DiPerna and Majda.

Invited lectures

Existence of equivariant harmonic maps

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One way to obtain harmonic maps between Euclidean spheres in nontrivial homotopy classes is to reduce the effective dimension of the problem by imposing certain symmetry conditions. Several situations where this reduction leads to ODE problems have been discussed (cf. [1] for an overview), beginning with Smith's paper [3] about harmonic joins. In this talk, examples are given where the reduced problem is of higher dimension.

We minimize energy in Sobolev spaces of mappings between spheres which are equivariant with respect to certain torus actions. Doing so, we get weakly harmonic minimizers. It is now a matter of regularity theory [2] to deduce conditions under which these minimizers are actually smooth harmonic maps. These conditions imply existence of harmonic representatives of the nontrivial elements of the following homotopy groups: $\pi_{n+1}(S^n) \cong Z_2$ $(n \geq 3)$, $\pi_{n+2}(S^n) \cong Z_2$ $(n \geq 5$ odd). Moreover, each of these classes contains infinitely many harmonic maps of arbitrarily high energy.

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Critical points of the area functional by gluing technics

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We prove a 'gluing theorem', which states roughly that if two (appropriate) constant mean curvature surfaces (resp. minimal surfaces) are juxtaposed, so that their tangent planes are parallel with opposite orientation, and very close to one another, then there is a new constant mean curvature surface (resp. minimal surface) near this configuration, which is a topological connected sum of the two surfaces. Even though, gluing constructions for geometric objects are by now well understood and somewhat commonplace (e.g. in the context to CMC and minimal surfaces, the pioneering work was that of N. Kapouleas and, probably more relevant to the results here, is the recent work of S.D. Yang, in which geometric connected sums of complete minimal surfaces of finite total curvature are constructed), our procedure is both easier to understand, gives more information. What is more, the technics can be easily adapted to other problems (e.g. existence (and uniqueness) of Ginzburg-Landau vortices, existence of CMC or minimal hypersurfaces, existence of singular or regular solutions of semilinear elliptic equations, ...).

A simple context for our result is when we are given two orientable compact nondegenerate CMC surfaces, Σ_1 and Σ_2, with nonempty boundaries. Let us asume that $0 \in \Sigma_1 \cap \Sigma_2$, that $T_0\Sigma_1 = T_0\Sigma_2$ are equal to the $x\,y$-plane and have opposite orientation at this point. Then we prove that there is S_ϵ a one-parameter family of smooth, immersed CMC surfaces with boundary, for $\epsilon \in (0, \epsilon_0]$, such that, S_ϵ converges in the $\mathcal{C}^\infty$ topology to $\Sigma_1 \cup \Sigma_2$ away from the origin, and the dilated surface $\epsilon^{-1} S_\epsilon$ converges in the $\mathcal{C}^\infty$ topology to a catenoid with vertical axis. Moreover, the boundary of S_ϵ is the union of the boundaries of the Σ_i, each possibly transformed by a small rigid motion.

We also give sufficient conditions for S_ϵ to be embedded and finally we prove that these surfaces S_ϵ are "generically" nondegenerate (This question of nondegeneracy is quite important in the moduli space theory).

The above result can be used to establish a general connected sum theorem for complete, noncompact, embedded CMC surfaces (resp. minimal surfaces). For example, using this theorem and starting from two Delaunay surfaces, we are able to produce new embedded four-ended CMC surfaces which are quite different from the previously known examples.

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Contributed talks

Periodic solutions and rigid rotation of Ginzburg-Landau vortices

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Ginzburg-Landau vortices are finite action critical points of the functional

$$\mathcal{E}(A,\Phi) = \frac{1}{2}\int_{\mathcal{D}} |dA|^2 + |D_A\Phi|^2 + \frac{\lambda}{4}(|\Phi|^2-1)^2 dx \qquad GL$$

where $-iA$ is an S^1 connection, Φ is a section of the associated complex line bundle and $D_A\Phi = d - iA\Phi$ is the covariant derivative. The action is invariant under an infinite dimensional group of gauge transformations $\rho(\chi) : (A,\Phi) \mapsto (A+\nabla\chi, \Phi e^{i\chi})$ for smooth functions χ. The domain $\mathcal{D}$ is taken to be either S^2 or a disc in the plane. Time periodic solutions of systems describing the dynamics of Ginzburg-Landau vortices are obtained by a direct variational method. The existence of such solutions was conjectured in [1]. The systems studied include the Abelian Higgs model, which is a system of semi-linear wave equations, as well as a system of non-linear Schrödinger type, introduced in [2]. The solutions describe vortices moving in rigid rotation about one another. A previous result of this type was proved in [3] by perturbative techniques.

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The loaded differential Equations and their Applications

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The equations prescribed in $\Omega \subset R^n$ are called loaded, if they contain some operations of the traces of desired solution on manifolds (of dimension which is strongly less than n) from Closure $\overline{\Omega}$. The loaded equations are examined in Works [1], [2].

Assume we are given a bounded and regular connected open Set $\Omega \subset R^N$ and positive Number T(T is a final instant of time). Let $Q = \Omega\times(0,T), \Sigma = \partial\Omega\times(0,T), \overline{\Omega} = \Omega\cup\partial\Omega$.

Problem. Find a Solution $u(x,t)$ of the Boundary Value Problem:

$$\frac{\partial u}{\partial t} = \Delta u + \sum\nolimits_{k=1}^{m} M_k(x,t)u + f,\ \{x,t\} \in Q,$$
$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial \vec{n}} + \sum\nolimits_{k=1}^{n-1} c_k(\sigma,t)\frac{\partial u}{\partial \vec{\tau}_k} = \sum\nolimits_{k=m+1}^{m+l} M_k(\sigma,t)u + g,\ \{\sigma,t\} \in \Sigma,$$
$$u(x,0) = u_0,\ x \in \Omega,\quad u(\sigma,0) = u_1,\ \sigma \in \partial\Omega,$$

where $M_k(\theta,t)u = \int_{\Gamma_k(t)} \mathcal{M}(\theta,\xi,t) D_\xi^{\alpha_k(t)} u(\xi,t)d\xi,$

$\Gamma_k(t) - (n-1)$ – dimensional Manifolds in $\overline{\Omega}$ for $\forall t \in [0,T]$,

$D_\xi^\gamma = (1-\Delta_{|\partial\Omega})^{\gamma/2}$ – fractional integral-differential Operator of γ-order,

$\Delta_{|\partial\Omega}$ – the Laplace-Beltrami Operator,

$\vec{n}, \vec{\tau}$ – the unique outer normal and tangent vectors to $\partial\Omega$.

Theorem 1. Let $f \in L_2(0,T;\Xi^{-1}(\Omega))$, $g \in L_2(0,T;H^{-1/2}(\partial\Omega))$, $u_0 \in L_2(\Omega)$, $u_1 \in L_2(\partial\Omega)$. Then the **Problem** has unique Solution $u(x,t)$ which satisfies

$$u \in L_2(0,T;H^1(\Omega)) \cap L_\infty(0,T;L_2(\Omega) \cap L_2(\partial\Omega))$$

and depends continuously from f, g, u_0, u_1.

Theorem 2. Let $f \in L_2(Q)$, $g \in L_2(\Sigma)$, $u_0 \in H^1(\Omega)(u_1 = u_{0|\partial\Omega)})$. Then the **Problem** has unique Solution which satisfies $u \in L_\infty(0,T;H^1(\Omega)) \cap H^1(0,T;L_2(\Omega)$ $\cap L_2(\partial\Omega))$ and depends continuously from f, g, u_0.

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On the Heat Equation with Integral Mean-Values

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Eequations with integral mean-values arise in the search of approximative solutions of partial differential equations. For example, this approach was used for the calculation of the physical and heat engineering properties of the oiling in the bearings [1].

Let $Q = \{x,t|0 < x < 1, 0 < t < T\}$. We examine the following boundary value problem:

$$\begin{cases} (\pm)v'(t) = \dfrac{\partial^2 u(x,t)}{\partial x^2} + f \text{ in } Q, \\ u(0,t) = u(1,t) = 0 \text{ in } (0,T), \\ v(0) = 0, \text{ where } v(t) = \int_0^1 u(\xi,t)d\xi, \end{cases} \tag{1}$$

$$W_1 = \{u|u \in L_2(0,T;H^2(0,1) \cap H_0^1(0,1)),\ v'(t) \in L_2(0,T)\},$$

$$W_2 = \{u|u \in L_2(0,T;H_0^1(0,1)),\ v'(t) \in L_2(0,T)\}.$$

We examine problem (1) both with the sign plus and the sign minus.

Theorem 1. For every $f \in L_2(Q)$, there is a unique $u \in W_1$ such that (1) holds.

Theorem 2. For every $f \in L_2(0,T;H^{-1}(0,1))$, there is a unique $u \in W_2$ such that (1) holds. We examine the following generalization of problem (1):

$$\frac{\partial v(x,t)}{\partial t} = \Delta u(x,t) + f(x,t) \text{ at } \Omega \times (0,T),$$

$$u(x,t) = 0 \text{ at } \partial\Omega \times (0,T), \quad v(x,0) = \varphi(x) \text{ at } \Omega,$$
$$v(x,t) \equiv \sum\nolimits_{j=1}^{m} \alpha_j(x) u(x_j,t).$$

Here $\Omega \subset R^n-$ bounded Region, $\Delta-$ Laplace's Operator.

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Integral inequalities associated with Inequalities of Opial type

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We derive some new integral inequalities of the form (∗):

$$\int_I u|h|^p|h'|dt \le (p+1)^{p/(p+1)} \left(\int_I r|h'|^{p+1}dt\right)^{1/(p+1)} \left(\int_I s|h|^p|h'|dt\right)^{p/(p+1)}, \ h \in \ H,$$

where $I = (\alpha,\beta), -\infty \le \alpha < \beta \le \infty$, $p > 0$, H is a wide, determined class of functions h, absolutely continuous on I, satisfying the limit conditions $h(\alpha) = 0$ or $h(\beta) = 0$, the functions r, s and u are any set of functions related by the appropriate weight functions. The inequalities we examine are analogues of integral inequalities of Weyl type which are connected with integral inequalities of Hardy type [1]. To obtain the inequalities (∗) we use at first the uniform method of obtaining integral inequalities of Opial type of the form (∗∗):

$$\int_I s|h|^p|h'|dt \le \int_I r|h'|^{p+1}dt, \qquad h \in, H,$$

where r is an arbitrary weight function, s depends on a parameter and is chosen appropriately for r in such a way that the inequality (∗∗) holds in a wide class of functions absolutely continuous on I [2]. Next by the optimisation with respect to parameter we obtain the inequality (∗).

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Multiple existence of solutions for semilinear elliptic problems

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In this talk, we consider the multiple existence of semilinear elliptic problems of the form

$$
\begin{aligned}
-\Delta u + u &= g(x,u), \quad x \in \Omega \\
u &= 0 \qquad\quad \text{on } \partial\Omega
\end{aligned}
$$

where Ω is an unbounded domain in $\mathbb{R}^N$ and $g : \Omega \times \mathbb{R} \to \mathbb{R}$ is a continuous function with a superlinear growth. The existence of solutions of the problem above has been studied by many authors. In case that $g(x,t) = | t |^{p-1} t$ with $1 < p < (N+2)/(N-2)$ and $\Omega = \mathbb{R}^N$, it is well known that the problem has a unique positive solution. It is also known in this case that the problem has infinity many sign changing solutions. In this talk, we consider the existence of positive solutions and sign changing solutions for the case that g depends on x and the domain Ω is not necessarily the whole space.

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Recent results about existence of solutions to dynamic contact problems with Coulomb friction

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Despite its practical application, the theory of dynamic contact problems remains underdeveloped. The Signorini contact condition to the hyperbolic system is up–to–now solvable only for half–spaces or for one space dimension. This led to introduction of some (possibly small) physically based viscosity into the elastic problems. Moreover, the Coulomb friction law creates the problem to be nonsmooth, the friction term is nonmonotone and seems to be of the same order as the volume forces.

We shall present our recent results about the existence of solutions for the approximate Signorini condition formulated in velocities. They are based on the penalization of the contact condition and on the proof of a partial regularity of solutions to such auxiliary problem (which allows the limit procedure to the original problem). The proof, employing a certain localization technique, is based on exact trace and inverse trace theorems which allow the bounds for the admissible magnitude of the coefficient of friction to be as high as possible. The bounds depend exclusively on the viscous behaviour of the material and are remarkably better for homogeneous isotropic materials. The coefficient of friction can be solution–dependent.

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Bifurcation of homoclinic solutions for Hamiltonian systems

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We consider the following first order Hamiltonian system

$$Ju'(x) + Mu(x) - \nabla_s F(x, u(x)) = \lambda u(x)$$

where J is a $2N \times 2N$ real matrix such that

$$J = -J^T = -J^{-1} ,$$

M is a $2N \times 2N$ real symmetric matrix such that

$$\sigma(JM) \cap i\mathbb{R} = \emptyset$$

and $F : \mathbb{R} \times \mathbb{R}^{2N} \to \mathbb{R}$ is such that

$$\lim_{|s|\to 0} \frac{F(x,s)}{|s|^2} = 0 .$$

We are interested in existence and bifurcation results of homoclinic solutions for a subset of such systems. By homoclinic solution, we mean a solution $u(x)$ such that $\lim_{x\to\pm\infty} u(x) = 0$.

With global and local hypotheses on F, we prove the existence of non trivial homoclinic solutions when the parameter λ is in a small neighbourhood.

Under certain local conditions on F, we find a bifurcation point for the system.

The proofs are based on variational arguments developed in the reference below.

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The shape of functions yielding best constants in Sobolev-type inequalities

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If for $p, q \in [1, \infty]$ one looks for minimizers u of

$$J_1(v) := \frac{||\nabla v||_{L^p(\Omega)}}{||v||_{L^q(\Omega)}} \qquad \text{on} \quad W^{1,p}(\Omega) \ \cap \int_\Omega v \ dx = 0 \tag{1}$$

or

$$J_2(v) := \frac{||v||_{W^{1,p}(\Omega)}}{||v||_{L^q(\Omega)}} \qquad \text{on} \quad W^{1,p}(\Omega), \tag{2}$$

then already in one space dimension, for $\Omega = [-1, 1]$ the shape of minimizers varies considerably in q, for fixed p.

As shown in [1], the minimizer u_1 of (1) changes of course sign and is odd and monotone for $q \leq 2p + 1$ and not odd but still monotone for $q > 4p - 1$. This improves results in [2] and [3].

The minimizer u_2 of (2) is always of one sign. In fact it is constant for $q \leq p$, but nonconstant, positive, strictly monotone and has exactly one inflection point for $p < q < \infty$, which moves to the boundary as $q \to \infty$. The case $q = \infty$ was treated in [4].

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Gradient flow for Willmore surfaces

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Here we report on joint work with Reiner Schätzle (Freiburg). An immersion $f : \Sigma \to \mathbb{R}^3$ of a closed, oriented surface Σ is called a Willmore surface if it is critical for the Willmore integral

$$W(f) = \int_\Sigma H^2 \, d\mu, \tag{1}$$

where $H = \frac{\kappa_1+\kappa_2}{2}$ is the mean curvature and $d\mu$ is the induced area element. L. Simon used the direct method to prove the existence of a minimizing torus, see also [2]. Here we propose the study of the corresponding L^2 gradient flow

$$\partial_t f = (\Delta H + 2H(H^2 - K))\nu, \tag{2}$$

where ν is the unit normal and $K = \kappa_1\kappa_2$ is the Gauß curvature. It is not clear whether (2) develops singularities in finite time and whether it converges as $t \to \infty$.
We prove a lower bound for the maximal lifespan $T > 0$ of the solution to (2) depending on how much the curvature is concentrated in L^2 for the initial surface. More precisely for $0 < \varepsilon_0 < 8\pi$ we introduce the quantity

$$r_0(f) = \inf\{r > 0 : \exists\, x \in \mathbb{R}^3 \text{ with } \int_{f^{-1}(B(x,r))} |A|^2 \, d\mu \geq \varepsilon_0\} \tag{3}$$

and prove the following result:
Theorem Let $f_0 : \Sigma \to \mathbb{R}^3$ satisfy $\int_\Sigma |A_0|^2 \, d\mu_0 \leq \Lambda < \infty$. There are constants $\varepsilon_0 > 0$, $c_0 < \infty$ depending only on Λ such that $T \geq c_0 \, r_{\varepsilon_0}(f_0)^4 =: T_0$. On $[0, T_0]$ the solution is estimated in terms of the initial data.
If the flow blows up at $T < \infty$, the result implies $r_{\varepsilon_0}(f) \leq \left(\frac{T-t}{c_0}\right)^{1/4} \to 0$ and thus the curvature concentrates.

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Regularity for minimizers of some variational integrals

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We consider integral functionals of the calculus of variations

$$\mathcal{F}(u,\Omega) = \int_\Omega f(Du)\; dx, \tag{1}$$

where Ω is an open subset of $\mathbb{R}^n$, $n \geq 2$, $u : \Omega \to \mathbb{R}^N$, $N \geq 1$ and the integrand f satisfies the following growth assumption

$$\mid z \mid^p \leq f(z) \leq L(1+ \mid z \mid^q), \tag{2}$$

$q \geq p \geq 2$, $z \in \mathbb{R}^{nN}$. A wide literature has been devoted to the study of regularity properties of local minimizers of (1) when $q = p$. More recently, the case $q > p$ has been considered. In particular, in the scalar case ($N = 1$) regularity of local minimizers has been proved, provided q and p are not too far apart. Counterexamples show that restrictions on the exponents p, q have to be assumed in order to have regular minimizers, [1, 2, 3]. Now we consider the vectorial case $N \geq 1$ and we deal with higher integrability. In our setting a local minimizer of $\mathcal{F}$ is a function $u \in W^{1,1}_{loc}(\Omega)$ such that

$$f(Du) \in L^1_{loc}(\Omega) \tag{3}$$

and

$$\int_{supp\varphi} f(Du)\; dx \leq \int_{supp\varphi} f(Du + D\varphi)\; dx, \tag{4}$$

for every $\varphi \in W^{1,1}(\Omega)$ with $supp\varphi \subset\subset \Omega$. By (2) and (3) we only have that $u \in W^{1,p}_{loc}(\Omega)$: we prove (see [4]) that the minimality of u boosts its integrability from $W^{1,p}_{loc}(\Omega)$ to $W^{1,q}_{loc}(\Omega)$ provided

$$q < p + 2\min\{1, \frac{p}{n}\}. \tag{5}$$

Once we filled the gap between p and q, standard technique allows us to get higher differentiability and further higher integrability.

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Closed Surfaces Minimizing the Bending Energy under Prescribed Area and Volume

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Let $\Sigma \subset \mathbb{R}^3$ be a closed surface, and h be its mean curvature whose sign is positive for convex surfaces. We consider a variational problem for the bending energy

$$W(\Sigma) = \int_\Sigma (h - c_0)^2 dS$$

under the prescribed area $A(\Sigma)$ and volume $V(\Sigma)$. Minimizers of this problem are interpreted as models of the shape of red blood cells [1,2]. Here, the constant c_0 denotes the spontaneous curvature for the red blood cell membrane. If Σ is a critical point, then it satisfies

$$\delta W(\Sigma) + \lambda_1 \delta A(\Sigma) + \lambda_2 \delta V(\Sigma) = 0,$$

where δ means the first variation, and λ_i's are the Lagrange multipliers. This is a second order elliptic partial differential equation for h.

Many results on the existence of critical points as well as their stability have been obtained numerically or by formal computations. For instance, Jenkins [3] obtained numerically bifurcating families of critical points near the sphere which are surfaces of revolution (in the case $c_0 = 0$). Subsequently Peterson [5] and Ou-Yang and Helfrich [4] carried out formal analysis of the stability/instability of the lowest-mode solution.

The purpose of this work is (i) to point out some subtleties overlooked in those formal analyses, (ii) to give a solid mathematical basis for the bifurcation analysis, and (iii) to prove, among other things, that there do exist local minimizers near the sphere unless c_0 coincides with an exceptional value.

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Multiple solutions for an elliptic equation with non–homogeneous boundary conditions

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Let us consider the nonlinear elliptic problem

$$\begin{cases} \Delta u \ + \ |u|^{p-2}\, u = 0 & \text{in } \Omega \\ u = g & \text{on } \partial\Omega \end{cases} \tag{1}$$

where Ω is an open smooth bounded subset of $\mathbb{R}^N$, $N \geq 2$, $g \in C(\partial\Omega) \cap H^{1/2}(\partial\Omega)$ and $p > 2$ is fixed.

If $g \equiv 0$, it is well known that (1) has infinitely many distinct solutions for $2 < p < \frac{2N}{N-2}$ if $N \geq 3$ or $p > 2$ if $N = 2$. In this case the energy functional is even in a Banach space, then such results have been proved by using variational methods and the properties of the genus for symmetric sets.

On the contrary, if $g \not\equiv 0$ the problem (1) loses its symmetry and the energy functional is not invariant under any group of symmetries.

However, we prove that it is possible to apply a perturbative method introduced by Rabinowitz and developed by Bahri – Berestycki and Struwe. Hence, for $p > 2$ but not too larger, the existence of infinitely many solutions of (1) with higher and higher energy can be stated (see [2]).

Further generalizations of this result are contained in [1] and [3].

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Convexity estimates for mean curvature flow of mean convex surfaces

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We consider a family $\mathcal{M}_t$ ($t \geq 0$) of n–dimensional immersed surfaces in $\mathbb{R}^{n+1}$ evolving by mean curvature flow under the assumption that the initial surface $\mathcal{M}_0$ is smooth, closed and mean convex (i.e. with nonnegative mean curvature). Then the flow is defined up to some critical time T at which the curvature becomes unbounded. Our aim is to describe the singular behaviour as $t \to T$ using rescaling techniques.

The paper [2] considered the case of surfaces satisfying a bound of the type $H \leq C/\sqrt{T-t}$ for the mean curvature H. It was shown that any limit of rescalings of such surfaces is a self-similar homothetically shrinking solution of the flow.

Here we develop a different approach, based on a–priori estimates, which can be applied without restrictions on the growth of H. We denote by $\lambda_1, \ldots, \lambda_n$ the principal curvatures of the evolving surfaces $\mathcal{M}_t$ and let S_k $(k = 1, \ldots, n)$ be the elementary symmetric polynomial of degree k evaluated in $\lambda_1, \ldots, \lambda_n$. In particular, $S_1 = H$. Then our main result is (see [3,4])

Theorem 1 *Let $\mathcal{M}_t$ be a family of surfaces evolving by mean curvature flow with $\mathcal{M}_0$ closed and mean convex. Then, for any $\eta > 0$ there exists C_η such that the estimate*

$$S_k \geq -\eta H^k - C_\eta$$

holds on $\mathcal{M}_t$ for any $t \in [0, T[$ and for all $k = 2, \ldots, n$.

The arbitrariness of η breaks the scaling invariance in the above inequality and implies that near a singularity, where H becomes unbounded, each S_k becomes nonnegative after rescaling:

Corollary 2 *Let $\mathcal{M}_t$ be a solution of mean curvature flow as in Theorem 1. Then any smooth rescaling of the singularity for $t \to T$ is convex.*

Once we know that the rescaling is convex, we can use the results of [1] about convex eternal solution of the flow to describe the profile of the surfaces which do not satisfy the growth restriction mentioned above. We show that in this case the rescaling procedure yields a translating solution of the flow.

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Some general quadratic integral inequalities of the second order

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Integral inequalities of the second order of the form

$$(1) \quad \int_I sh^2 dt \leq \int_I rh''^2 dt, \; h \in H,$$

where $I = (\alpha, \beta), -\infty \leq \alpha < \beta \leq \infty$, r and s are real functions of the variable t, H is a class of functions h absolutely continuous on I, have been considered by many authors. They have been also derived by Florkiewicz and Wojteczek (see [1]) using the uniform method of obtaining integral inequalities involving a function and its derivatives.

Now we shall derive integral inequalities of the form

$$(2)\ \int_I (rh''^2 + sh'^2 + uh^2)dt \geq 0,\ h \in \hat{H},$$

using a generalization of this uniform method.

This generalization of the uniform method lets us for a given function r and auxiliary functions φ_1, φ_2 to determine directly the weight functions s and u and use them to build a wide class $\hat{H}$ of functions h for which the inequality (2) holds.

Then, using the inequality (2), we shall obtain some integral inequalities with constant weight functions, e.g. classical Hardy, Littlewood and Polya inequality of the form

$$(3)\ \int_I (h''^2 - h'^2 + h^2)dt \geq 0,$$

(see [2]) and examine the classes of functions h for which these inequallities hold.

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Bifurcation Results for a Class of Quasilinear Elliptic Systems on R^n

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In this paper we prove the existence of a branch of solutions for the system

$$\begin{aligned} -\Delta_p u &= \lambda a(x)|u|^{p-2}u + F_1(x,\lambda,u,v),\ x \in R^n \\ -\Delta_q v &= \lambda d(x)|v|^{q-2}v + F_2(x,\lambda,u,v),\ x \in R^n \end{aligned}$$

under certain conditions for p,q,F_1, F_2. This fact is achieved by applying degree theory. It was possible to carry out these methods by working between certain equivalent weighted and homogeneous Sobolev spaces.

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Posters

Direct gradient method for semilinear elliptic systems

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The subject of this paper is the numerical solution of semilinear elliptic systems

$$\begin{cases} T_i(u_1,\dots,u_r) \equiv -\mathrm{div}\,(a_i(x)\nabla u_i) + f_i(x,u_1,\dots,u_r) \;=\; g_i(x) \quad \text{in } \Omega \\ \qquad\qquad\qquad Qu_i \equiv \alpha(x)u_i + \beta(x)\partial_\nu u_{i|\partial\Omega} = 0 \end{cases} \tag{1}$$

$(i=1,\dots,r)$, where $a_i(x) \geq m > 0$ and the eigenvalues λ of the matrices $\left\{\frac{\partial f_j(x,\xi)}{\partial \xi_i}\right\}$ fulfil $0 \leq \kappa \leq \lambda \leq \kappa' + \gamma \sum\limits_{j=1}^{r} |\xi_j|^{p-2}$. We investigate the application of the Hilbert space version of the gradient method (GM) to system (1). Whereas the usual application of the GM is done after discretization , our approach is, in contrast, to execute the iteration directly in the corresponding Sobolev space, which demands the solution of linear auxiliary problems.

It is proved that for any $u^0 = (u_1^0,\dots,u_r^0) \in H^2(\Omega)^r$ with $Qu_i = 0$ $(i=1,\dots,r)$ in trace sense we can suitably define $M > 0$ such that the sequence

$$u_i^{k+1} = u_i^k - \frac{2}{M+m}(-\Delta)^{-1}(T_i(u^k) - g_i) \quad (k \in \mathbf{N};\ i=1,\dots,r)$$

converges to the weak solution $u^* = (u_1^*,\dots,u_r^*)$ of (1) according to the estimate

$$\|u^k - u^*\|_{H_0^1(\Omega)^r} \;\leq\; \text{const.}\cdot \left(\frac{M-m}{M+m}\right)^k .$$

The solution of the auxiliary equations can be achieved by discretization, thus linear algebraic systems have to be solved instead of nonlinear ones obtained from discretizing (1). (A suitable FEM or FDM might be used here, similarly to the case of the Dirichlet problem.)

We examine some special domains in the case of polynomial nonlinearity when the auxiliary equations can be sol ved directly (without discretization). This includes domains that can be suitably transformed to a ball, cube or an annulus. Numerical examples of the solution of elliptic RDE equations are given.

Bifurcation of some semilinear elliptic equations

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We investigate the exact number of positive solutions of the semilinear boundary value problem

$$\left.\begin{array}{r} \Delta u + f(u) \;=\; 0 \\ u_{|\partial B_R} = 0 \end{array}\right\} \tag{BVP}$$

where $B_R \subset \mathbf{R}^n$ is the ball centered at the origin with radius R and f is a C^2 convex function on $[0,\infty)$. Our aim is to determine the bifurcation diagram of positive solutions versus the radius R and to classify every strictly convex C^2 function according to the shape of the bifurcation diagram.

As is well-known, every positive solution of (BVP) is radially symmetric, hence it satisfies an ODE boundary value problem on the interval $[0, R]$. Then positive solutions of (BVP) can be obtained by the shooting method. We investigate the number of positive solutions via the time-map. Our aim is the describe the shape of the time-map, which is achieved through the following three characteristic properties:

- the domain of the time-map
- the limit of the time-map at the boundary points of its domain
- the monotonicity of the time-map on the maximal subintervals of its domain.

Using this the exact number of positive solutions of (BVP) can be determined.

We present different results on the above properties for $n \geq 1$ and establish the bifurcation diagram for certain convex functions. In the one-dimensional case a full characterization is given for strictly convex C^2 functions.

2.11 Viscosity Solutions

Organizer : Neil Trudinger

Key note lecture

Homogenization of Hamilton–Jacobi PDE and Hamiltonian dynamics

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I will discuss some recent work (done jointly with D. Gomes) concerning the homogenization of Hamilton–Jacobi PDE in the spirit of Lions–Papanicolaou–Varadhan. The principal new ideas concern the interpretations and implications of the "effective" Hamiltonian for the long time asymptotics of certain solutions of the related Hamiltonian ODE.

Invited lectures

Singularities of Concave Solutions to Hamilton-Jacobi Equations

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After a general introduction to the properties of the singular set (points of non-differentiability) of viscosity solutions to nonlinear first order partial differential equations, the authors investigate the structure of such a set. In particular, a new technique introduced in [1] for studying propagation of singularities will be presented and compared with earlier methods.

Finally, the special case of a Hamilton-Jacobi equation of the form

$$u_t + H(Du) = 0 \quad \text{a.e. in} \quad \mathbb{R}_+ \times \mathbb{R}^n$$

with initial condition

$$u(x,0) = u_0(x)$$

will be considered. It is well known that the solution of the above problem is concave with respect to x whenever the initial datum u_0 is concave and the Hamiltonian H is convex (see e.g. [2]). Actually, using Hopf's formula, one can prove that u is concave in $\mathbb{R}_+ \times \mathbb{R}^n$. For such a solution the authors show that singularities propagate, from any initial singular point (t_0, x_0), along a Lipschitz arc $(t, x(t))$ for all times $t \geq t_0$ up to $+\infty$. Moreover, the singular arc $x(t)$ satisfies the generalized equation of characteristics

$$\dot{x}(t) \in \nabla H(D^+u(t,x(t))) \qquad t \geq t_0 \text{ a.e.}$$

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On the rate of convergence in the homogenization of HJ equations

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The homogenization of partial differential equations can be regarded as an asymptotic process in which one looks at the limiting behaviour of solutions of a PDE with a rapidly oscillating structure as the frequency of oscillations tends to infinity.

We shall consider here fully nonlinear first order scalar partial differential equations of Hamilton-Jacobi type such as

$$u^\epsilon(x) + H(x, \frac{x}{\epsilon}, Du^\epsilon(x)) = 0 \quad \text{ in } \mathbb{R}^N \tag{1}$$

where ϵ is a small positive parameter and the Hamiltonian H satisfies

$$\xi \mapsto H(x,\xi,p) \quad \text{is} \quad Z^N - \text{periodic} \tag{2}$$

for each $(x,p) \in \mathbb{R}^{2N}$. Equations of type (1) arise for example, via dynamic programming, in optimization problems for non linear control systems with rapidly oscillating dynamics. Although the importance of homogenization of Hamilton-Jacobi equations the first step on this subject was taken in 1985 by P. L. Lions, G. Papanicolau and S.R.S.Varadhan who established that, as ϵ tends to 0, the limit problem of equations (1) is given by

$$u(x) + \overline{H}(x, Du(x)) = 0 \quad , \quad x \text{ in } \mathbb{R}^N \tag{3}$$

where the effective Hamiltonian $\overline{H}$ is obtained by solving the so-called cell problem (which, in the control theoretic interpretation, can be seen as an ergodic control problem) . The next major contributions are due to L.C.Evans (1989) who developed the perturbed test functions methods for studying the homogenization problem in the framework of the theory of viscosity solutions. Further reaserch in this direction has been performed by M.C. Concordel, K. Horie and H. Ishii, O. Alvarez.

The question of estimating, in terms of ϵ , the rate of the uniform convergence of solutions of equations (1) to the solution of equation (3) has not been tackled up to now, at least in our knowledge. The purpose of this talk is to present some recent results in this direction obtained in collaboration with H. Ishii. Our main result is as follows:

Theorem Assume that H satisfies assumption (2),

$$\nu|p| - M \leq \; H(x,\xi,p) \leq \nu|p| + M \quad (x,\xi,p \in \mathbb{R}^N),$$

and $\|D\ H\|_\infty \leq M$, for some constant $M > 0$ and let u^ϵ,$u \in BUC(\mathbb{R}^N)$ be respectively the viscosity solutions of equations (1) and (3).
Then there is a constant $C > 0$ independent of $\epsilon \in (0,1)$ such that

$$|u^\epsilon(x) - u(x)| \leq C\epsilon^{1/3} \quad (x \in \mathbb{R}^N).$$

Almost Periodic Homogenization of Hamilton-Jacobi Equations

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I shall discuss the almost periodic homogenization of the Hamilton-Jacobi equation

$$\lambda u^\varepsilon(x) + H(x, x/\varepsilon, u^\varepsilon(x), Du^\varepsilon(x)) = 0 \quad \text{ in } \mathbb{R}^N.$$

Here $\lambda > 0$ is a given constant, H is a given continuous real function on $\mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R} \times \mathbb{R}^N$, which is often called the Hamiltonian, u^ε represents the unknown function, and $\varepsilon > 0$ is a small parameter to be sent to zero. The question addressed here is the characterization of the limit function of u^ε as $\varepsilon \to 0$. Main assumptions on $H(x,y,r,p)$ are its coercitiveness in the p variable and its almost periodicity in the y variable. The results I shall explain describe the limit partial differential equation which the limit function should solve and show the uniform convergence of u^ε to the solution of the limit equation under the assumptions above together with some other mild assumptions.

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Hessian Measures

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Hessian measures are defined through the action of certain second order, partial differential operators on associated classes of subharmonic functions. Special cases are the distributional Laplacian acting on classical subharmonic functions and the Monge-Ampere measure of Aleksandrov, as determined by the action of the Monge-Ampere operator on the class of convex functions. In the complex case, a measure arises through the action of the complex Monge-Ampere operator on certain plurisubharmonic functions.For these and more general operators,the associated classes of subharmonic functions are the viscosity subsolutions, and one is led to the general question of how the operator acts on its viscosity subsolutions. In this talk,we present a recent result,with Xu-jia Wang,on the weak continuity of operators given as elementary symmetric functions of the eigenvalues of the Hessian matrix of second derivatives.We also explore the extension of our techniques to other classes of operators,including quasilinear operators.

Contributed talks

Propagation of Maxima and Strong Maximum Principle for Degenerate Elliptic equations

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We consider $2nd$ order partial differential equations of the form

$$F(x, u, Du, D^2u) = 0, \quad \text{in } \Omega, \tag{1}$$

where $\Omega \subseteq \mathbb{R}^N$ is an open set, $F: \overline{\Omega} \times \mathbb{R} \times \mathbb{R}^N \times S(N) \to \mathbb{R}$, Du corresponds to the gradient of u, D^2u corresponds to the matrix of second derivatives of u and $S(N)$ is the set of symmetric $N \times N$ matrices. We require that F satisfies a very weak ellipticity condition.

We extend a Boundary Hopf Lemma and the Strong Maximum Principle to semi-continuous viscosity subsolutions of (1). We test our assumptions on several examples involving for instance the p-Laplacian and the minimal surfaces operator. We can cover equations that are singular at $p = 0$ and very degenerate operators such as the ∞-Laplacian and some first order Hamilton-Jacobi equations. Moreover we characterize the set of propagation of maxima of the subsolutions of $2nd$ order Hamilton-Jacobi-Bellman equations. More precisely we consider $2nd$ order operators which can be represented as the supremum or the infimum of a parametrized family of linear operators. We show that the set where a maximum propagates contains the reachable set of a suitable deterministic controlled system as well the reachable set of a controlled stochastic differential equation. We also derive a Strong Maximum Principle for fully nonlinear equations satisfying a suitable version of Hörmander hypoellipticity condition.

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On the Liouville property for viscosity solutions of fully nonlinear elliptic problems

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The classical Liouville theorem states that every bounded either from above or from below harmonic function in $\mathbb{R}^N$ is constant. This property still holds for nonnegative, superharmonic functions in $\mathbb{R}^N$, provided that $N \leq 2$.

In recent years, the Liouville property has been generalized to semilinear elliptic equations stating the nonexistence of positive solutions in unbounded domains and it has been the object of a keen interest in the literature, also for its connection with the problem of the a priori bounds and the existence of positive solutions of superlinear boundary value problems in bounded domains, see e.g. [1,2,4.5] and the references therein.

I will present some results, obtained in a joint work with Fabiana Leoni (see [3]), concerning the Liouville property for nonnegative viscosity supersolutions of a class of fully nonlinear uniformly elliptic equations in $\mathbb{R}^N$. The first one regards the problem:

$$u \geq 0\,, \quad -\mathcal{M}^+_{\lambda,\Lambda}(D^2u) \geq 0 \qquad \text{in } \mathbb{R}^N\,,$$

where $\mathcal{M}^+_{\lambda,\Lambda}$, for fixed $0 < \lambda \leq \Lambda$, is the Pucci extremal operator

$$\mathcal{M}^+_{\lambda,\Lambda}(D^2u) = \sup_{A \in \mathcal{A}_{\lambda,\Lambda}} \operatorname{tr}(AD^2u)$$

with $\mathcal{A}_{\lambda,\Lambda} = \{A \in \mathcal{S}_N \;:\; \lambda|\xi|^2 \leq A\xi \cdot \xi \leq \Lambda|\xi|^2,\ \forall \xi \in \mathbb{R}^N\}$. I will show that u is constant provided that $N \leq 1 + \frac{\Lambda}{\lambda}$.

The second one is relative to the problem:

$$u \geq 0\,, \quad F(x, D^2u) - h(x)u^p \geq 0 \qquad \text{in } \mathbb{R}^N,$$

where F is a uniformly elliptic operator with ellipticity constants $0 < \lambda \leq \Lambda$. Under some optimal restrictions on $h \geq 0$ and p, the only solution is $u \equiv 0$.

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Lyapunov functions for infinite-dimensional systems

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We study Lyapunov functions for infinite dimensional dynamical systems governed by general maximal monotone operators. We obtain a characterization of Lyapunov pairs by means of the associated Hamilton-Jacobi partial differential equations, whose solutions are meant in viscosity sense, as evolved in works of Tataru [1] and Crandall-Lions. Our approach leads to a new sufficient condition for Lyapunov pairs, generalizing a classical result of Pazy [2].

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Gradient estimates, interior gradient blow up and comparison principle of nonlinear pde's

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A comparison principle for semicontinuous viscosity sub- and supersolutions of general fully nonlinear second order elliptic equations $F(x, u, Du, D^2u) = 0$ is proved. The case of uniformly elliptic equations as well as degenerate elliptic ones is considered. As an application, existence and uniqueness of the continuous viscosity solutions is obtained. Under the precise conditions for the validity of the comparison principle Lipschitz regularity of the continuous viscosity solutions is given. If these conditions fail, existence of continuous viscosity solutions which are not Lipschitz in the interior of the domain is proved. As a consequence of the interior gradient blow ups the viscosity solutions form interior jump discontinuities with an infinite gradient at the points of discontinuity in the strictly elliptic case. The concept of jump discontinuous viscosity solutions for nonlinear problems with interior gradient blow-up phenomena is discussed.

The minimum time function for control systems with controls in L^p, $p \geq 1$

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We study the properties of the minimum time function to reach the origin and its relative reachable set for the solutions $y_x(\cdot, u)$ of a control system of the form

$$\dot{y}(t) = Ay(t) + Bu(t) \; t > 0, \qquad y(0) = x \in \mathbb{R}^n$$

where the admissible controls belong to the set $U_{p,k} = \{L^p([0, +\infty[, \mathbb{R}^m) : \; \|u\|_p \leq k\}$. Moreover we want to characterize such a function, denoted by $T_p(x, k)$, and the reachable set $R_p = \cup_{k>0}\{x : \exists t \geq 0, \; \exists u \in U_{p,k} : y_x(t, u) = 0\} \times \{k\}$; by means of an opportune boundary value problem associated to the (HJB) equation.

In case $p = +\infty$ everything is well known both for regularity and characterization of T_∞ and R_∞ (see e.g. [1]). If $p > 1$, under controllability conditions there are only regularity results for the k–sections $T_p(\cdot, k)$ (see [2]), and there is no characterization of T_p, R_p through the (HJB) equation. If $p = 1$, (to our knowledge) nothing is known.

As new results we prove that, under the Kalman condition, if $p > 1$ then $(x,k) \mapsto T_p(x,k)$ is continuous on R_p; while if $p = 1$ then T_1 is continuous on the interior of R_1 (R_1 fails in general to be an open set). Moreover for $p \geq 1$ we characterize the function T_p and the reachable set R_p by means of the unique bounded bilateral viscosity solution W_p to the boundary value problem given by

$$\max_{w_0 \geq 0,\ |(w_0,u)|=1} \{-\frac{\partial W}{\partial x}(x,k)(Axw_0 + Bu) + \frac{\partial W}{\partial k}(x,k)|u|^p - e^{-k}w_0^p + W(w_0^p + |u|^p)\} = 0$$

on $(\mathbb{R}^n \setminus \{0\}) \times]0, +\infty[$, and the boundary conditions

$$W(0,k) = 0 \quad \forall k \geq 0, \quad W(x,0) = 1 \quad \forall x \neq 0.$$

More precisely, one obtains that for any fixed $k > 0$ one has

$$T_p(x,k) = -\log(1 - e^k W_p(x,k)); \quad R_p = \{(x,k) : W_p(x,k) < e^{-k}\}$$

if $p > 1$, while for $p = 1$ $\mathrm{Int}\, R_p = \mathrm{Int}\{(x,k) : W_p(x,k) < e^{-k}\}$.

We remark that these results are proven also for a wide class of nonlinear systems with controls bounded in L^p- norm ($p \geq 1$).

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Existence, uniqueness and regularity of the solution of a Dirichlet problem

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The aim of this lecture is to continue the study begun in [4] and to extend the local regularity properties of the solutions of the elliptic second order divergence form operators to global ones obtaining, for the solution of a Dirichlet problem, the global Hölder regularity.

Let us consider a bounded open set Ω in $\mathbb{R}^n$, $n \geq 3$, and $H_0^{1,p,\lambda}(\Omega)$ the subset of functions $u \in H_0^{1,p}(\Omega)$ such that its first derivatives lie in the Morrey space $L^{p,\lambda}(\Omega)$ and the following Dirichlet problem

$$\begin{cases} \mathcal{L}u \equiv -\sum_{i,j=1}^n (a_{ij}(x)u_{x_i})_{x_j} = div(f) & \text{a. e. in } \Omega, \\ u \in H_0^{1,p,\lambda}(\Omega), & 1 < p < \infty,\ 0 < \lambda < n. \end{cases}$$

The coefficients a_{ij} are discontinuous, precisely they are assumed to be in the subset $V.M.O.$ of the John-Nirenberg space whose elements have norm on the balls vanishing as the radius of the balls approaches zero. The known term $f = (f_1, \ldots, f_n)$ is supposed such that f_i belongs to the Morrey space $\in L^{p,\lambda}(\Omega)$, $\forall i = 1, \ldots, n$.

Under these assumptions there exists the unique solution $u \in H_0^{1,p,\lambda}(\Omega)$ of the above Dirichlet problem.

Moreover, there exists a constant k independent on f such that

$$\|u\|_{C^{0,\alpha}(\bar{\Omega})} \leq k\|f\|_{L^{p,\lambda}(\Omega)}, \text{ where } \alpha = 1 - \frac{n}{p} + \frac{\lambda}{p},\ p > n - \lambda.$$

The technique used is similar to that used for the nondivergence form elliptic equation in [3]. In this paper the authors derive a $C^{1,\alpha}_{loc}$−regularity result, related to that of Caffarelli contained in [1] and [2] about the $W^{2,p}$ viscosity solutions, using an integral representation formula for the second derivatives of the solutions of the above equation.

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Some results on first order Hamilton-Jacobi equations with discontinuous coefficients

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I will discuss Hamilton-Jacobi equations of the form

$$H(x, u, Du) = g(x),$$

where the function g can be discontinuous. It can be shown that viscosity super and subsolutions can be characterized by optimality principles. These principles allow to obtain explicit representation formulas for minimal and maximal solutions, and to prove uniqueness of the viscosity solution under suitable conditions. Examples where the solution is not unique can also be produced.

C. Global Attractors and Stability

3.1 Global Attractors and Limits

Organizers : Jack K. Hale, Geneviève Raugel

Key note lecture

Compact attractors

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The first part of the talk will discuss the necessity of the classical hypotheses for existence of compact global attractors. The second part will concentrate on diffusively balanced conservation laws.

Invited lectures

Attractors of Parabolic Problems and Perturbations of the Diffusion

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We are interested in studying the continuity properties of the asymptotic dynamics for second order parabolic problems when the diffusion coefficient varies in a singular fashion.

For $\epsilon \in (0, \epsilon_0)$, for certain $\epsilon_0 > 0$, the family of problems we consider are of the type

$$\begin{cases} u_t^\epsilon - \mathrm{div}(a_\epsilon(x)\nabla u^\epsilon) + u^\epsilon = f(u^\epsilon), & \text{on } \Omega \\ a_\epsilon \dfrac{\partial u^\epsilon}{\partial n} = g(u^\epsilon), & \text{on } \partial\Omega \end{cases}$$

where $f, g : \mathbb{R} \to \mathbb{R}$ are locally Lipschitz functions, $\Omega \subset \mathbb{R}^n$ is a bounded and smooth domain and a_ϵ, the diffusion coefficient, satisfies that there exist positive constants m_0, M_ϵ such that $m_0 \leq a_\epsilon(x) \leq M_\epsilon$.

Under several growth restrictions on f and g we will pose this problem in an L^q or $W^{1,q}$ setting and by imposing some dissipativity assumptions we will prove the existence of a global attractor $\mathcal{A}_\epsilon$. We will also obtain uniform bounds on $\mathcal{A}_\epsilon$ in the $H^1(\Omega)$ and $C^0(\bar{\Omega})$-norm.

We are interested in the following perturbation of the diffusion

$$a_\epsilon(x) \xrightarrow{\epsilon \to 0} \begin{cases} a_0(x) & \text{uniformly on } \Omega_1 \\ \infty & \text{uniformly on compact subsets of } \Omega_0 \end{cases}$$

where Ω_1 is a subdomain of Ω. This kind of perturbation may appear, for instance, in composite material, where the diffusivity behavior of the material may differ significantly from one part to another.

The limiting problem that we obtain is given by

$$\begin{cases} u_t - \operatorname{div}(a_0(x)\nabla u) + u = f(u) & \text{on } \Omega_1 \\ a_0 \dfrac{\partial u}{\partial \vec{n}_0} = g(u) \qquad \text{on } \partial\Omega \\ u_{|\Omega_0} =: u_{\Omega_0}, \quad \text{(constant)} & \text{on } \Omega_0, \\ \dot{u}_{\Omega_0} + \dfrac{1}{|\Omega_0|}\displaystyle\int_{\Gamma_0} a_0 \frac{\partial u}{\partial \vec{n}} + u_{\Omega_0} = f(u_{\Omega_0}) \end{cases}$$

and this problem has an attractor $\mathcal{A}_0$ which lies in a bounded subset of $C^0(\bar{\Omega})$.

We will show that the family of attractors $\{\mathcal{A}_\epsilon\}_{\epsilon\geq 0}$ is upper semicontinuous at $\epsilon = 0$ in the $H^1(\Omega)$ and $C^0(\bar{\Omega})$-norms, that is

$$\sup_{u_\epsilon \in \mathcal{A}_\epsilon} \operatorname{dist}(u_\epsilon, \mathcal{A}_0) \xrightarrow{\epsilon\to 0} 0.$$

where dist is the distance either in $H^1(\Omega)$ or in $C^0(\bar{\Omega})$. Notice that to obtain the upper semicontinuity in the uniform topology a delicate analysis of the behavior of the solutions near $\partial\Omega_0$ is needed.

On the time-periodic solutions to the Navier-Stokes equations of compressible flow

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We prove the existence of the time-periodic solutions with given positive mass to the Navier-Stokes system of compressible isentropic flow:

$$\partial_t \varrho + \operatorname{div}(\varrho u) = 0,$$

$$\partial_t(\varrho u^i) + \partial_{x_j}(\varrho u^i u^j) - \mu\Delta u^i - (\lambda+\mu)\partial_{x_i} \operatorname{div} u + a\partial_{x_i}\varrho^\gamma = \varrho f^i(t,x),\ i = 1,2,3.$$

with a given time-periodic driving force f. The unknown density $\varrho(t,x)$ and the velocity $u(t,x) = [u^1, u^2, u^3]$ are functions of the time t and the spatial variable $x \in \Omega$, where Ω is a bounded regular domain. The homogeneous Dirichlet boundary conditions are imposed on u.

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Attractors for dissipative problems on unbounded domains

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We consider parabolic problems of reaction–diffusion type on large or unbounded spatial domains $\Omega \subset \mathbb{R}^d$. Under suitable conditions on the nonlinearity, global existence and the existence of an absorbing ball can be established by using weighted energy estimates with a weight decaying at infinity, [1,2,6,7]. The method of weighted norms can even improve existing bounds for the absorbing ball in the case of bounded domains, [7]. Compactness properties of the semiflow are obtained in the weaker toplogy induced by the weighted norm. In this way it is possible to construct a global attractor $\mathcal{A}^\Omega$ which is closed and bounded in $L^\infty(\Omega)$ and compact and attracting with respect to the weighted norm $\|\cdot\|_\rho = \sup\{\, \rho(x)|u(x)| : x \in \Omega\}$, [4,6,8].

Using examples we illuminate the essential differences between the dynamics on bounded and on unbounded domains. For instance, systems which are gradient flows when considered on bounded intervals will display nontrivial recurrent behavior when considered on the real line with initial data in $L^\infty(\mathbb{R})$. Another instance is convergence to a whole continuum of steady states like in diffusive mixing, [4].

The attractor $\mathcal{A}^\Omega$ can be compared with $\mathcal{A}^{\mathbb{R}^d}$ by extending all functions on Ω by 0 outside of Ω. Upper semicontinuity for Ω approaching $\mathbb{R}^d$, that is

$$\mathrm{dist}_{\|\cdot\|_\rho}(\mathcal{A}^{\Omega_n}, \mathcal{A}^{\mathbb{R}^d}) \to 0 \quad \text{for } \Omega_n \supset \{|x| > r_n\} \text{ with } r_n \to \infty,$$

is obtained easily. Lower semicontinuity is false in general. However, considering a joint limit in the sense of [5], where the time and the domains go to infinity simultaneously, gives a limit attractor $\mathcal{A}^*$ which coincides with $\mathcal{A}^{\mathbb{R}^d}$.

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The global attractor of the viscous Cahn-Hilliard homotopy

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For the viscous Cahn-Hilliard homotopy it is possible to ascertain detailed information on the equilibria which allows the relative energies and Morse indices to be calculated, and a Morse decomposition based on energy can be constructed. Additionally, near the nonlocal Allen-Cahn end of the homotopy, a weak lap number principle is proven. This information permits a partial unraveling of many of the connections on the global attractor and demonstrates that the structure of the global attractor in the metastable region can be distinctly different from earlier predictions for the Cahn-Hilliard equation in zero mean mass (spinodal) case. Implications for a model for shape memory materials known as Ericksen's bar will be discussed if time permits.

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Contributed talks

Global Attractor in Overdamped Dynamics of tilted Frenkel-Kontorova Models

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We study a model for nonlinear conduction in charge density wave materials:

$$m\ddot{u}_n + \dot{u}_n = -h_1(u_n, u_{n+1}) - h_2(u_{n-1}, u_n) + F,$$

where subscripts on h denote partial derivatives, $m \geq 0$, $F \geq 0$, $h_{12} \leq -c < 0$, and $h(x+1, y+1) = h(x, y)$.

For the gradient dynamics case $m = 0$, we proved in [1] that there is a globally attracting time-periodic orbit (in the sense that there is a $T > 0$ such that $u_n(t+T) = u_n(t)+1$ for all t and n), in each of the following classes of configuration if F is sufficiently large that there is no equilibrium in that class:

1. finite chain with free ends;

2. spatially periodic configurations: $u_{n+q} = u_n + p$ for all n, for some integers p, q;

3. weakly rotationally-ordered configurations of irrational mean spacing.

The key tool is the monotonicity of the flow with respect to a partial order on configurations (u_n).

In the overdamped case $0 \le m \le (4\mu)^{-1}$, where $\mu = \sup h_{11} + h_{22}$, the flow is again monotone, this time with respect to a certain partial order on sequences $((u_n, \dot{u}_n))$. I shall discuss the extent to which the above results generalise to this case.

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On equivalence of exponential and asymptotic stability under changes of variables

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Lyapunov's notion of (global) asymptotic stability of an equilibrium is a key concept in the qualitative theory of ordinary differential equations and nonlinear control. In general, a far stronger property is that of *exponential* stability, which requires decay estimates of the type "$\|x(t)\| \le ce^{-\lambda t}\|x(0)\|$." In this talk, we show that, for differential equations evolving in finite-dimensional Euclidean spaces $\mathbb{R}^n$ (at least in dimensions $\neq 4, 5$) the two notions are one and the same under coordinate changes. The same statement holds if perturbed differential equations are considered

Of course, one must define "coordinate change" with care, since under diffeomorphisms the character of the linearization at the equilibrium is invariant. However, if, in the spirit of both structural stability and the classical Hartman-Grobman Theorem, we relax the requirement that the change of variables be *smooth at the origin*, then all obstructions disappear. Thus, we ask that transformations be continuously differentiable, and they have inverses that are continuously differentiable except possibly at the origin, and globally continuous.

Closely related to our work is the fact that all asymptotically stable *linear* systems are equivalent (in the sense just discussed) to $\dot{x} = -x$; see e.g. [1]. The basic idea of the proof is based upon projections on the level sets of Lyapunov functions, which in the linear case of course be taken to be quadratic. It is natural to use these ideas also in the general nonlinear case, and Wilson's paper [2], remarked that level sets of Lyapunov functions are always homotopically equivalent to spheres. Indeed, it is possible to obtain, in great generality, a change of coordinates rendering the system in normal form $\dot{x} = -x$ (and hence exponentially stable), and several partial versions of this fact have appeared in the literature.

It is perhaps surprising that, at least for unperturbed systems, the full result seems not to have been observed before, as the proof is a fairly easy application of results from differential topology. Those results are nontrivial, and are related to the generalized Poincaré conjecture and cobordism theory; in fact, the reason that we only make an assertion for $\neq 4, 5$ is closely related to the fact that the original Poincaré conjecture is still open.

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Limit sets for reaction transport equations

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Our aim is to propose a hyperbolic generalisation of the well-known reaction diffusion equation and to study the structure of its ω–limit sets.

We consider moving and interacting particles, characterised by their position $x \in \Omega \subset \mathbb{R}^n$ in space and by their direction of movement $v \in V = S^{n-1}$. The evolution of the density $u(t, x, v)$ is governed by the following isotropic reaction transport equation

$$\left(\frac{\partial}{\partial t} + v \cdot \nabla + \mu L\right) u = \frac{1}{|V|} f(\bar{u}),$$

combined with the initial condition $u(0, x, v) = u_0(x, v)$ and appropriate boundary conditions on $\partial\Omega$. Here μ denotes the scattering parameter, $\bar{u}(t, x) = \int_V u(t, x, v)dv$ is the total density, $|V|$ the measure of V, and $Lu = u - \frac{1}{|V|}\bar{u}$ is the local deviation from the mean. The linear part on the left hand side models the spreading of particles due to a transport process, the nonlinearity f contains the reaction laws. The name isotropic refers to the usage of the total density $\bar{u}$ both in the scattering and in the reaction part. Equations of this type were proposed in [3,4] to model interacting bacteria when their motion consists of straight runs interrupted by re-orientations.

The linear existence theory of transport equations is well-established, cf. for instance [1] and the references therein.

On suitably defined Hilbert spaces we prove global in time existence of solutions by an argument based on a Lyapunov function approach. Here we assume a dissipativity condition on f, namely $\limsup_{|z|\to\infty} f(z)/z < 0$.

The main step in proving that orbits are relatively compact uses the far-reaching regularity estimates in [2] for velocity averages in transport equations. By classical results on the ω–limit sets of orbits, cf. [5], we can conclude that $\omega(u_0)$ is non-empty, compact, and connected. Furthermore, the Lyapunov function ensures that any ω–limit set is contained in the set of equilibrium solutions of the above reaction transport equation.

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Asymptotic Behavior of Some Nonlinear Dissipative Hyperbolic Equations on $\mathbb{R}^N$

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We consider the semilinear hyperbolic problem

$$u_{tt} + \delta u_t - \phi(x)\Delta u + \lambda f(u) = \eta(x),\ x \in \mathbb{R}^N,\ t > 0,$$

with the initial conditions $u(.,0) = u_0$ and $u_t(.,0) = u_1$ in the case where $\delta > 0$, $N \geq 3$ and $(\phi(x))^{-1} := g(x)$ lies in $L^{N/2}(\mathbb{R}^N)$. The energy space $\mathcal{X}_0 = \mathcal{D}^{1,2}(\mathbb{R}^N) \times L^2_g(\mathbb{R}^N)$ is introduced, to overcome the difficulties related with the noncompactness of operators which arise in unbounded domains. We derive various estimates to show local existence of solutions and existence of a global attractor in $\mathcal{X}_0$. We show that the global attractor has finite Hausdorff dimension and we prove the existence of an exponential attractor of finite fractal dimension that attracts all solutions at exponential rate. The compactness of the embedding $\mathcal{D}^{1,2}(\mathbb{R}^N) \subset L^2_g(\mathbb{R}^N)$ is widely applied.

Global attractors of reaction-diffusion equations without uniqueness

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We study the existence of global attractors for reaction diffusion equations in which the nonlinearity is a difference of two maximal monotone maps.

Even though for these equations it is possible to prove the existence of at least one solution corresponding to each initial condition, this solution can be non-unique. So, this equation does not generate a semigroup of operators. Taking the union of all possible solutions of a certain class we define a multivalued semiflow. By applying the theory of attractors for multivalued semiflows we obtain under certain conditions the existence of a global compact attractor.

This result is applied to several physical processes such as a model of combustion in porous media and some models of transmission of electrical impulses in nervous axons.

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Geometry of heteroclinic cascades in scalar semilinear parabolic PDE

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We study the long-time behaviour of one-dimensional reaction-diffusion equations

$$u_t = u_{xx} + f(u, u_x, x), \quad x \in [0,1],\ t \in \mathbf{R}^+$$

with Neumann boundary conditions. Particular properties of the corresponding semiflow on the space of x-profiles, such as the gradient-structure or the linearization being of Sturm-Liouville-Type, allow for a detailed geometrical description of the global attractor.

The main tool for such investigations, beside the general geometric theory for semilinear parabolic equations, introduced by D.Henry [1], are the nodal properties of the stationary solutions. G.Fusco and C.Rocha recognized that one can describe these nodal properties in terms of a permutation of the equilibrium solutions [2]. This permutation which is determined already by the stationary equation, an ODE, can provide crucial information about the long-time behavior of the full PDE.

The attractor consists of the unstable manifolds of all equilibria. These are partitioned into sets of heteroclinic orbits, connecting to the same equilibrium. The main problem is to understand the structure of these sets and how they are assembled in the attractor. A first important step in this direction was a result of C.Rocha and B.Fiedler [3], using Conley-Index and giving a criterion for these sets to be empty or not. But as an inspection of several examples shows, their result is not sufficient to provide a satisfactory geometrical description of the attractor.

Mainly we present a result, giving more insight in the structure of the sets of connecting orbits by taking into account the decomposition of stable and unstable manifolds according to different exponential rates. We give a criterion in terms of the permutation, deciding exactly in which strong-stable and strong-unstable manifold the heteroclinic connections between given equilibria take place. This information seems also to be crucial for the understanding of the attractor as a whole. The main ingredient of the proof will be a detailed study of the permutation of equilibria. Finally, we suggest a new concept for the equivalence of attractors.

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Posters

Optimal Stabilization of Almost Periodic Systems

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Consider a system of differential equations of perturbed motion

$$\dot{x} = X(t, x; u) \tag{1}$$

where $x = (x_1, ..., x_n)$, $X = (X_1, ..., X_n)$, $u = (u_1, ..., u_r)$ -is a control function, and $X(t, x; u)$-is an almost periodic function in t.

Suppose that function $X(t, x; u)$ is defined, continuous and satisfying Lipschitz condition in x in the domain

$$t \in R, \|x\| < H \ (H = const). \tag{2}$$

Suppose that criterion of motion $x(t)$ is given as

$$I = \int_{t_0}^{\infty} \omega(t, x_1[t], ..., x_n[t]; u_1[t], ..., u_r[t])\, dt, \tag{3}$$

where $\omega(t, x, u)$ is nonnegative function.

Theorem. If there exist almost periodic in t, positive definite,

continuously differentiable function $V^0(t, x)$ and functions $u_j^0(t, x)$, which are satisfying in the domain (2) the following conditions:

1) function $w(t, x) = \omega(t, x; u^0(t, x))$ is nonnegative, and $w(t, x)$ may equal zero only in the points of set which does not include any samitrajectory of the system (1) $x(x_0, t_0, t), (t_0 < t < +\infty)$ entirely (except the trivial solution);

2) an equality

$$B[V^0; t, x; u^0(t, x)] = 0 \tag{4}$$

holds, where

$$B[V; t, x; u] = \frac{\partial V}{\partial t} + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} X_i(t, x; u) + \omega(t, x; u) = \frac{dV}{dt} + \omega(t, x; u); \tag{5}$$

3) an equality

$$B[V^0; t, x; u] \geq 0, \tag{6}$$

holds for each control functions u_j, then functions $u_j^0(t,x)$ solve the problem of optimal stabilization and an equality

$$\int_{t_0}^{\infty} \omega(t, x^0[t]; u^0[t])\, dt = \min \int_{t_0}^{\infty} \omega(t, x[t]; u[t])\, dt = V^0(t_0, x(t_0)) \tag{7}$$

holds.

3.2 Nonautonomous Attractors

Organizer : Mark Vishik

Key note lecture

Attractors of Nonautonomous Evolution Equations

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1. Nonautonomous evolution equations and the corresponding processes. Time-symbols of such equations and their hulls. Uniform attractors for a family of processes with translation compact symbols. Uniform attractor for nonautonomous 2D Navier-Stokes system, reaction-diffusion systems, dissipative nonlinear hyperbolic equations and other systems.

2. Upper and lower bounds for the fractal dimension of the attractors for the equations with quasiperiodic symbols. Upper bounds for Kolmogorov ε-entropy of the uniform attractor for the above equations with translation compact forcing terms and interaction functions.

3. Trajectory attractor for evolution equations without uniqueness of solution to the corresponding Cauchy problem or for equations for which the uniqueness is not proved yet. Construction of trajectory attractor for 3D-Navier-Stokes system, for dissipative hyperbolic equation with arbitrary polynomial interaction term and for other equations. Convergence of the trajectory attractors of finite dimensional Galerkin approximations to the trajectory attractor of corresponding system.

4. Trajectory attractor for an evolution equation that singularly depends on a parameter. Trajectory attractor for an evolution equations with rapidly oscillating coefficients and their limit – the trajectory attractor of the averaged equations. Examples.

The exposition follows the papers of V.Chepyzhov and M.Vishik.

Invited lectures

Kolmogorov Epsilon-entropy of Attractors of Non-autonomous Evolution Equations

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It is well known that global attractors of many autonomous dissipative equations and systems of mathematical physics have finite fractal dimension. Among such equations are: the 2D Navier-Stokes system, the reaction-diffusion systems, the Ginzburg-Landau equation, the nonlinear hyperbolic equation with damping and others.

In this report we consider uniform attractors of the corresponding non-autonomous equations and systems whose coefficients, external forces or interacting functions depend on time explicitly. All time-dependent terms of a particular equation are called the time symbol of this equation. The symbol is a quasi-periodic function, or an almost periodic function, or a general translation compact function with values in an appropriate Banach function space. It is proved that the uniform attractors of the mentioned above non-autonomous equations have finite fractal dimension if the time symbols are quasi-periodic. Moreover the upper estimate of the fractal dimension of the uniform attractor has two summands. The first summand coincides with the upper estimate of the global attractor of the corresponding autonomous equations, while the second summand is the number of rationally frequencies of the quasi-periodic symbol of the equation. At the same time, examples show that the fractal dimension of uniform attractors of non-autonomous equations can be infinite, when the symbol is an almost periodic function of time. This observation leads to the study of the Kolmogorov ε-entropy of uniform attractor since the ε-entropy describes the complexity of infinite dimensional compact sets in functional spaces.

We prove the following theorem. If the time symbol of the non-autonomous equation is a general translation compact function then the ε-entropy of the uniform attractor does not exceed the sum of two terms: the first term is upper estimate of the global attractor of the corresponding autonomous equation multiplied with $\log_2\left(\frac{1}{\varepsilon}\right)$ and the second term is the ε-entropy of the hull of the initial time symbol of the equation measured on the finite time interval having the length of the order $\log_2\left(\frac{1}{\varepsilon}\right)$. We note that in general the second term is the main term of the estimate as $\varepsilon \to 0$. (In the quasi-periodic case this term has the form $k \cdot \log_2\left(\frac{1}{\varepsilon}\right)$, where k is the number of rationally independent frequencies of the time symbol). In particular, the functional dimension of the uniform attractor does not exceed the functional dimension of the hull of the symbol of the non-autonomous equation under consideration. These results are applicable to various equations and systems arising in mathematical physics.

This research is the joint work with Professor Mark Vishik.

Long time behavior in phase-field models with memory

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We present two thermodynamically consistent phase-field models with memory. One is based on the linearized version of the Gurtin-Pipkin heat conduction law (cf. [1]), the other is characterized by a linear heat conduction law of Coleman-Gurtin type (cf. [2]). The formulation of Cauchy-Neumann initial and boundary value problems for these evolution systems is framed in a history space setting. Namely, the summed past history of the temperature is regarded itself as a variable along with the temperature and the phase-field. Well-posedness results are discussed as well as long term behavior of solutions. In particular, regarding the first model, suitable conditions are stated in order to ensure the existence of a uniform absorbing set. In the second case, taking advantage of a higher dissipation in the equation for the temperature, we can prove the existence of a uniform attractor.

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Long time behavior of models of Cahn-Hilliard equations in deformable media

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We are interested in the study of the long time behavior of models of Cahn-Hilliard equations in elastic deformable continua. More precisely, we consider models of generalized Cahn-Hilliard equations that take into account the effects of internal microforces introduced by M. Gurting, coupled with the stationary Navier equation of linear elasticity (we assume that the displacement gradient is small and that the deformation is infinitesimal).

We prove that the variational formulation associated with the problem can be uncoupled, defining a type of system that we call weakly coupled (nonautonomous) system.

We define and study the notions of (uniform) attractor and of (uniform) exponential attractor for a weakly coupled system and then obtain the existence of finite dimensional uniform attractors for the weakly coupled Cahn-Hilliard equations.

Existence, uniqueness, and asymptotic behaviour of the solutions of nonlinear hyperbolic equations

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Two hyperbolic equations of Mathematical Physics are considered, the first related to traveling waves in a strip, while the second refers to nonlinear models of the vibrating string and membrane, obtained without making any assumptions on the "smallness" of the motion and assuming a not necessarily linear stress-strain relationship. In the first case, it is proved that the corresponding semigroup possesses a trajectory attractor, constituted by the union of all bounded solutions of an associated set of equations. Regarding the vibrating string and membrane models, it is possible to prove the existence of a global solution and of an absorbing set.

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Attractors for nonautonomous dynamical systems

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We introduce a general nonautonomous dynamical system on a fiber bundle. Let $P \neq \emptyset$ be the base of this fiber bundle and $X(p)$ denote the fibers which are assumed to be Polish spaces. This dynamical system is given by a *cocycle*

$$\phi(t,p,\cdot) : X(p) \to X(\theta_t p)$$
$$\phi(t+\tau,p,\cdot) = \phi(t,\theta_\tau p,\phi(\tau,p,\cdot)), \quad \phi(0,p,\cdot) = \mathrm{id}_{X(p)}$$

for $p \in P$, t, $\tau \in \mathbb{R}^+$ where θ is a flow on P:

$$\theta_{t+\tau} = \theta_t \circ \theta_\tau, \qquad \theta_0 = \mathrm{id}_P$$

for t, $\tau \in \mathbb{R}$. Note we do not assume that P carries any structure. In particular, we do not suppose that P is compact. An attractor of the nonautonomous dynamical system is a multifunction $p \mapsto A(p)$ with compact images such that we have a *generalized invariance*:

$$\phi(t,p,A(p)) = A(\theta_t p).$$

In addition, we have *pull back convergence*:

$$\lim_{t\to\infty} \mathrm{dist}(\phi(t,\theta_{-t}p, D(\theta_{-t}p)), A(p)) = 0$$

where D is a multifunction contained in the *basin of attraction*. If P carries a measurable structure then A is called a *random attractor*.

Conditions for the existence and uniqueness of such an attractor will be formulated. We will apply this general result to show the existence of attractors for the nonautonomous and the stochastic Sine Gordon equation and the 3D stochastic Navier Stokes equation (see [1]). In another application we will prove the existence of attractors for numerical schemes with nonequidistant steps (see [2]). In addition, we will study Lyapunov functions for nonautonomous dynamical systems introduced above (see [3]).

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Contributed talks

Dynamics of the slightly compressible Navier-Stokes equations

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The subject of this talk is to compare the long time behavior of the solutions of a slightly compressible Navier-Stokes equations and the ones of the incompressible Navier-Stokes equations (INSE), in a two dimensional bounded set Ω. We conclude by comparing the bounds of the fractal dimensions of the global attractors related to the two considered systems.

The considered slightly compressible model of Navier-Stokes equations reads,

$$\begin{array}{l} \partial_t u^\varepsilon - \nu \triangle u^\varepsilon - u^\varepsilon \wedge \text{rot } u^\varepsilon + \nabla \pi^\varepsilon = f \\ \varepsilon^2 \partial_t \pi^\varepsilon + \text{div } u^\varepsilon = 0 \\ u^\varepsilon(0) = u_0^\varepsilon \quad\; \pi^\varepsilon(0) = \pi_0^\varepsilon, \end{array} \tag{1}$$

where $u \wedge \text{rot } u = \text{rot } u \ (u_2, -u_1)^t = (\partial_1 u_2 - \partial_2 u_1) \ (u_2, -u_1)^t$.

We consider Dirichlet boundary conditions on $\partial\Omega$. This is an approximation of (INSE) which corresponds formally to $\varepsilon = 0$ without forced initial data on the pressure.

The dynamic of the problem (1) was firstly studied in [3] but the size and the fractal dimension of the global attractor was not estimated uniformly with respect to ε. Here, we introduce new results [1] [2]. We denote by P the orthogonal Leray projection on the subspace of $L^2(\Omega)^2$ of free divergence functions and $u_{||}^\varepsilon = (I - P)u^\varepsilon$,

Theorem 1 *For smooth enough initial data $(u_0^\varepsilon, \varepsilon\pi_0^\varepsilon)$ and for $\varepsilon \leq \varepsilon_0$ (ε_0 fixed), there exist δ and K, two nonnegative constants independent of ε, such that*

$$|u_{||}^\varepsilon(t)|_2^2 + |p^\varepsilon(t)|_2^2 \leq 4(|u_{0||}^\varepsilon|_2^2 + |p^\varepsilon(0)|_2^2)e^{-\delta t} + \varepsilon^2 K,$$

with $\nabla p^\varepsilon = \varepsilon \nabla \pi^\varepsilon + (I - P)\,(Pu^\varepsilon \wedge \mathrm{rot}\,(Pu^\varepsilon) + \nu \bigtriangleup Pu^\varepsilon)$.
For other boundary conditions, an asymptotic expansion (w. r. to ε) can be given,

$$u^\varepsilon_{\|} = \varepsilon^2 u^1_{\|} + ..., \quad Pu^\varepsilon = u + \varepsilon^2 u^1_{\perp} + ..., \quad \pi^\varepsilon = \pi + \varepsilon^3 p^1 + ...$$

where (u, π) are solution of (INSE), $u^1_{\|}$, p^1 are explicitly given in function of (u, π).

Theorem 2 *For an adequate norm and for* $\varepsilon \le \varepsilon_1$, ε_1 *small enough, the fractal dimension of the attractor* $\mathcal{A}^\varepsilon$ *in the variable* $(u^\varepsilon, \varepsilon\pi^\varepsilon)$ *is bounded by* $G^2 = \frac{|f|_2^2}{\nu^4 \lambda_1^2}$.
The fractal dimension of the global attractor related to (INSE) is bounded by G. This difference is only due to the fact that the system (1) is not fully dissipative.

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Hausdorff dimension estimates by use of adapted Carathéodory outer measures and applications to invariant sets with an equivariant tangent bundle splitting

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The contribution is concerned with upper bounds for the Hausdorff dimension of a flow invariant compact set $K \subset M$ which arises from an autonomous C^2-vector field f on M. Special Carathéodory outer measures are constructed which majorize the Hausdorff measures (see [1]). They are defined via covering elements which are tubular neighborhoods of arcs of smooth curves to approximate the fiber structure of the sets. New coordinates are introduced in order to separate the components of the vector field which act normally to the flow lines. The Hausdorff dimension bounds are formulated in terms of the eigenvalues of the symmetric part $\frac{1}{2}\pi(p)(\nabla f(p) + \nabla f(p)^*)\pi(p)$ of the operator which generates the associated system in normal variations with respect to the direction of the vector field (see [2]). Here $\pi(p)$ is the orthogonal projector onto the linear subspace orthogonal to the vector $f(p)$ and $\nabla f(p)$ is the covariant derivative of f in p. Assuming special properties of the stable and unstable manifolds of equilibrium points the results are derived for vector fields having a finite number of such equilibrium points in the considered invariant set.

Suppose that the compact and flow-invariant set $K \subset M$ without equilibrium points possesses an equivariant tangent bundle splitting $T_K M = E^0 \oplus E^1 \oplus E^2$ with respect to the flow. Then an entropy term of the flow can be indroduced into the dimension estimates. Considering the long-time behavior of the flow dimension bounds are derived including uniform Lyapunov exponents with respect to the splitting of the tangent bundle.

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Posters

On the Bounded Solutions of Autonomous Ordinary Differential Equations

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Consider an autonomous system of ordinary differential equations

$$\frac{dx}{dt} = X(x), \tag{1}$$

where $x, X \in R^n$, t is time, X is continuous function, satisfying a Lipschitz condition on any compact. Denote

$$x(t) = x(x_0, t) \tag{2}$$

the solution of the system (1) with initial data $x(0) = x_0$. Suppose that the semitrajectory (2) is bounded under $t \geq 0$, and all its ω-limit points are initial points of stable semitrajectories. Then $x(t) = y(t) + z(t)$, where $y(t)$ is almost periodic function, and $z(t)$ is vanishing function, i.e. $\lim\limits_{t\to\infty} z(t) = 0$.

On the Periodic Solutions of Ordinary Differential Equations

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Consider a system of ordinary differential equations

$$\frac{dx}{dt} = X(t, x), \tag{1}$$

where $x, X \in R^n$; t is time, X is continuous function, satisfying a Lipschitz condition on any compact. Besides we assume the function $X(t, x)$ to be periodic in t with the period ω_1:

$$X(t + \omega_1, x) \equiv X(t, x).$$

Theorem. Let $x(t)$ be any periodic solution of the system (1) with the period ω_2. If ω_1/ω_2 is an irrational number, then for every t_0 the function $X(t, x(t_0))$ is constant.

3.3 Order-Preserving Systems

Organizer : Hiroshi Matano

Key note lecture

Order Preserving Systems in the Presence of Symmetry

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Given an equation with certain symmetry such as symmetry with respect to rotation or translation, an important questions is whether or not the symmetry of the equation is inherited by its solutions.

In this lecture, which is based on a joint work with Toshiko Ogiwara (Proc. Royal Soc. Edinburgh 129 (1999), Discrete and Continuous Dynamical Systems 5 (1999)), we will first discuss the above question in a general framework of order-preserving dynamical systems under a group action and establish a theory concerning symmetry or monotonicity properties of stable equilibrium points.

In short, our result states that if G is the group of symmetry and if u is a stable equilibrium point (or a fixed point) of the dynamical system in an ordered metric space, then either $Gu = \{u\}$ or Gu is a totally ordered set homeomorphic to $\boldsymbol{R}$.

The results we present here are extensions of those of Mierczyński - Poláčik and Takač in the late 1980's, in which they required that the dynamical system be strongly order-preserving and the group of symmetry be compact and connected. We do not require the compactness of the group, which makes the theory applicable to the study of travelling waves. We have also relaxed the assumption that the dynamical system be strongly order-preserving, which allows our theory to deal with some degenerate equations and equations on unbounded domains. It should be noted that our idea is closely related to the so-called sliding method introduced by Berestycki and Nirenberg. In other words, it is, in some sense, an abstract formulation of sliding method.

As applications of our theory, we will show, among other things, the following:

(i) rotational symmetry of solutions of nonlinear elliptic equations;

(ii) instability of periodic solutions for equations of surface motion;

(iii) monotonicity of travelling (or "pseudo-travelling") waves.

We will also show, under additional assumptions, that stability implies asymptotic stability (or stability with asymptotic phase). Using this general result, we can show uniqueness and asymptotic stability of travelling or pseudo-travelling waves for a class of nonlinear diffusion equations and systems, including degenerate ones.

These results for the travelling waves put the earlier results by Volpert-Volpert-Volpert, Vega, Chen and other people in a more unified perspective.

Invited lectures

Order-preserving random dynamical systems

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We present the general concept of an order-preserving (or monotone) random dynamical system and we apply well-developed deterministic ideas to study the long-time behaviour of the trajectories and the properties of random attractors of these systems. As a main example we consider the random dynamical system generated by a coupled system of reaction–diffusion equations of the form

$$\partial_t u_j = \Delta u_j + f_j(u_1, \ldots, u_m) + \sum_{k=1}^{m} h_{kj}(u_j) \cdot \sigma_k(t, \omega), \quad j = 1, \ldots, m,$$

in a smooth bounded domain $D \subset \mathbb{R}^d$ with the Neumann boundary conditions. Here Δ is the Laplace operator, $f = (f_1 \ldots, f_m) : \mathbb{R}^m \mapsto \mathbb{R}^m$ is a C^1-mapping satisfying the so-called cooperativity condition, $h_{kj} : \mathbb{R} \mapsto \mathbb{R}$ are smooth functions and $\{\sigma_k(t, \omega)\}$ is a family of (generalized) random stationary processes (white noise case is not excluded). We give the conditions that guarantee the existence of random equilibria and we study their stability properties under various additional assumptions concerning the functions $f_j(u_1, \ldots, u_m)$ and $h_{kj}(u_j)$. We note that in contrast with the deterministic autonomous or periodic case random order-preserving systems can possess nontrivial completely ordered omega-limit sets. Our consideration relies crucially on the concept of sub- and super- equilibria which for the deterministic case is well-known, and for the random case was introduced in [1]. We also use random and stochastic comparison principles.

The results presented are based on ideas and methods which were developed in collaboration with L. Arnold [1,2] and with P. Vuillermot [3,4].

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Principal spectrum for nonautonomous linear parabolic PDEs

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We generalize the concept of principal eigenvalue to the case of a nonautonomous linear parabolic partial differential equation

$$u_t = \Delta u + a(t,x)u, \quad t \in \mathbb{R},\ x \in \Omega,$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with sufficiently smooth boundary and $a\colon \mathbb{R} \times \bar{\Omega} \to \mathbb{R}$ is a bounded continuous function uniformly Hölder in both variables. The equation is complemented with Dirichlet boundary conditions.

The idea of the generalization is the following. Consider a linear skew-product semiflow generated by the family of equations

$$u_t = \Delta u + b(t,x)u, \quad t \in \mathbb{R},\ x \in \Omega,$$

where b belongs to the closure in the supremum norm of the set of all time-translates of a. For that semiflow there is a canonically defined one-dimensional invariant subbundle S ([2]), which can be characterized as the set of all global solutions that are of the same sign for all $t \in \mathbb{R}$ ([1]).

The *principal spectrum* of the equation is defined as the dynamical (Sacker–Sell) spectrum (see [3]) of the skew-product semiflow restricted to S.

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Stable subharmonic solutions of reaction-diffusion equations

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We consider the Neumann problem for time-periodic reaction-diffusion equations. The asymptotic behavior of most bounded solutions of such equations is governed by stable periodic solutions. We address the question whether a stable periodic solution can be subharmonic, that is, whether its minimal period can be larger than the period of the equation.

Special attention will be given to nonlinearities that are spatially homogeneous, that is, they do not depend on the spatial variable explicitly. We show that, unlike in the nonhomogeneous case, the answer depends on the shape of the domain. Stable subharmonic solutions cannot occur on a convex domain, but they do occur on some nonconvex domains.

Posters

Star Product for Bimodal Maps

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This communication will discuss a symbolic star product of the dynamics of iterated cubic maps from the real line to itself. We also characterize the subsets in the parameters space that are self-similar.

3.4 Qualitative Theory of Parabolic Equations

Organizer : Peter Poláčik

Invited lectures

On the gradient structure of the FitzHugh–Nagumo system

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We consider the system of partial differential equations

$$\begin{cases} u_t = d_1 \Delta u + f(u) - v & \\ v_t = d_2 \Delta v + \delta u - \gamma v & x \in \Omega. \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x) & \end{cases} \tag{1}$$

with Neumann or Dirichlet boundary conditions on $\Omega \in \mathbb{R}^n$ and sufficient conditions on f to ensure the existence of a global attractor for the semigroup generated by (1). A typical nonlinearity f is the cubic function $f(u) = \mu(u - u^3)$.

Due to the abundance of results on gradient-like systems, the existence of a Lyapunov functional for (1) has been considered in the literature for particular parameter regimes.

For example, if $d_2 = 0$ the system generated by (1) is gradient-like in the region $\delta \le \gamma^2$ and the same result holds for a shadow system obtained from (1) by a limiting process as $d_2 \to +\infty$.

We present a Lyapunov functional for (1) holding in the general case ($0 < d_1, d_2 < +\infty$) and draw some immediate consequences for the structure of the attractor.

This is a collaboration work with Pedro Freitas.

Reaction-diffusion equations on thin domains

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We consider reaction-diffusion equations on thin domains obtained by squeezing a given domain in some space direction by a factor $\epsilon > 0$. We show that as $\epsilon \to 0$ the corresponding dynamical systems tend to a limit system, generated by a parabolic equation on a graph. We also obtain the upper-semicontinuity of attractors as $\epsilon \to 0$, thus extending some earlier results of Hale and Raugel proved for domains lying under the graph of a function. (Joint work with Martino Prizzi)

Reaction-Diffusion Systems with Skew-Gradient Structure

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We consider the reaction-diffusion system

$$\begin{cases} Tu_t = D\Delta u + f(u) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega. \end{cases} \tag{RD}$$

Here Ω is a bounded domain in $\mathbb{R}^N$, $u = u(x,t) \in \mathbb{R}^n$, $T = diag(\tau_1, \cdots, \tau_n)$ with $\tau_i > 0$, $D = diag(d_1, \cdots, d_n)$ with $d_i > 0$, and $f : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a smooth function of u. We say that the above system has *skew-gradient* structure if the nonlinear term f is written as

$$f(u) = Q\nabla_u H(u)$$

with some function $H(u) : \mathbb{R}^n \mapsto \mathbb{R}$ and $n \times n$ matrix $Q = diag(1, \cdots, 1, -1, \cdots, -1)$. In short, the skew-gradient system is a sort of activator-inhibitor system which resembles a gradient system but has nonlinearities with different sign.

In this talk, we discuss some fundamental properties of reaction-diffusion systems with skew-gradient structure. Any stationary solution of the skew-gradient system (RD) corresponds to a critical point of the functional

$$J[v] = \int_\Omega \left\{ \frac{1}{2} \langle D\nabla_x v, Q\nabla_x v \rangle - H(v) \right\} dx.$$

We study its stability as a steady state of (RD) and min-maximizing property as a critical point of $J[v]$.

Contributed talks

Logarithmic Sobolev inequalities and the rate of convergence to equilibrium for parabolic equations of Fokker-Planck type

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In this talk we are concerned with the large-time behavior of the Cauchy problem for linear Fokker–Planck type equations (advection-diffusion equations). These problems appear as homogeneous versions of kinetic Fokker–Planck models, e.g. In particular, we use the entropy method to analyze the rate of convergence to the equilibrium. Specifically, we consider the IVP for the *Fokker-Planck type equation*

$$\rho_t = \operatorname{div}(\nabla\rho + \rho\nabla A), \quad x \in \mathbb{R},\, t > 0; \quad \rho(t=0) = \rho_I \in L^1_+(\mathbb{R}^n)$$

with a given potential $A = A(x)$ such that the steady state $\rho_\infty = e^{-A} \in L^1(\mathbb{R})$.

For a wide class of problems the solution converges (in a weighted L^2-space) exponentially to the unique steady state, which is due to the spectral gap of the Hamiltonian that governs the evolution of the symmetrized problem. To extend this result to more general initial data we analyze the time decay of the *relative entropy* $e(t) = e_\psi(\rho(t)|\rho_\infty) = \int_{\mathbb{R}} \psi(\frac{\rho(t)}{\rho_\infty})\, \rho_\infty(dx)$, generated by the convex function $\psi(\sigma)$, $\sigma > 0$ with $\psi(1) = \psi'(1) = 0$. The idea of our analysis is to derive a differential inequality for the entropy production $I(t) = I_\psi(\rho(t)|\rho_\infty) = \frac{d}{dt} e_\psi(\rho(t)|\rho_\infty) \le 0$ and the entropy production rate $I'(t)$. For uniformly convex $A(x)$, i.e. $\left(\frac{\partial^2 A(x)}{\partial x^2}\right) \ge \lambda \mathbf{I}$, $x \in \mathbb{R}$, we get

$$\frac{d}{dt} I(t) = -2\lambda I(t) + r_\psi(\rho(t)), \tag{1}$$

with a non-negative remainder $r_\psi(\rho(t))$. This shows the exponential decay of $I(t)$. An integration in t then gives

$$I(t) = e'(t) \le -2\lambda e(t), \tag{2}$$

and the relative entropy converges to 0 exponentially with rate -2λ.

(2) is the entropy version of a convex Sobolev inequality. For the physical entropy ($\psi(\sigma) = \sigma \ln \sigma - \sigma + 1$) and Gaussian steady states $\rho_\infty = M_a$ (with standard deviation a) (2) can be transformed to the well-known *Gross logarithmic Sobolev inequality* (L. Gross, *Lecture Notes in Mathematics*, **1563**, 1993):

$$\int_{\mathbb{R}} f^2 \ln\left(\frac{f^2}{\|f\|^2_{L^2(dM_a)}}\right) M_a(dx) \le 2a \int_{\mathbb{R}} |\nabla f|^2 M_a(dx), \quad \forall f \in L^2(dM_a).$$

From the precise form of the remainder r_ψ in (1) one can deduce sharpness conditions on ρ, ρ_∞, and ψ such that (2) becomes an equality.

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The stability of solutions of the differential equations

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In [1] the stability criteria for solutions of systems of differential equations in the critical case of one zero root, based on analysis of spectrum of the Jacobi matrix for the right-hand side of the equation in a neighborhood of the solution is received. This method was generalized for investigation of stability of solutions of differential and difference equations in a Banach spaces and in all possible critical cases [2]. In the paper [3] this generalization was used for investigation of a domain of stability of systems of differential equations. In the paper [4] the Aizerman's problem was decided for a self-adjoint matrix. In the papers [5,6] the problems of stability of differential equations with lateness, of stability of the differential equations with a small parameter attached to derivative are investigated. In this paper we diffuse these results for receiving of some sufficient conditions of stability of differential equations of parabolic and hyperbolic types. In particular we study systems of partial differential equations of the following type

$$\left\{\begin{array}{l} \dfrac{\partial u_1(t,x_1,x_2)}{\partial t} = a_{11}(t)\dfrac{\partial^2 u_1(t,x_1,x_2)}{\partial x_1^2} + a_{12}(t)\dfrac{\partial^2 u_1(t,x_1,x_2)}{\partial x_2^2} + \\ \quad + a_{13}(t)\dfrac{\partial^2 u_2(t,x_1,x_2)}{\partial x_1^2} + a_{14}(t)\dfrac{\partial^2 u_2(t,x_1,x_2)}{\partial x_2^2} + g_1(t,x,u), \\ \dfrac{\partial u_2(t,x_1,x_2)}{\partial t} = a_{21}(t)\dfrac{\partial^2 u_1(t,x_1,x_2)}{\partial x_1^2} + a_{22}(t)\dfrac{\partial^2 u_1(t,x_1,x_2)}{\partial x_2^2} + \\ \quad + a_{23}(t)\dfrac{\partial^2 u_2(t,x_1,x_2)}{\partial x_1^2} + a_{24}(t)\dfrac{\partial^2 u_2(t,x_1,x_2)}{\partial x_2^2} + g_2(t,x,u) \end{array}\right. \tag{1}$$

with starting conditions

$$u(t_0;x_1,x_2) = u_0(x_1,x_2). \tag{2}$$

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Periodic Orbits and Attractors for Autonomous Reaction-Diffusion Systems

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Let $n \in \mathbb{N}$ and $\Omega \subset \mathbb{R}^n$ be a bounded domain with smooth boundary. It is a well known fact that bounded solutions of an autonomous reaction-diffusion equation of the form $u_t = \lambda u_{xx} + q(u)$, $t > 0$, $x \in \Omega$ with Dirichlet-boundary conditions and $\lambda > 0$ tend to stationary solutions. Far less is known about solutions of systems of autonomous reaction-diffusion equations such as

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \lambda\Delta \begin{pmatrix} u \\ v \end{pmatrix} + g\begin{pmatrix} u \\ v \end{pmatrix}, \quad t > 0, \quad x \in \Omega \tag{1}$$

with Dirichlet boundary conditions and $\lambda > 0$. In order to understand the dynamics of reaction-diffusion systems, the description of the dynamics of the model system

$$\frac{d}{dt}\begin{pmatrix} u \\ v \end{pmatrix} = \lambda\Delta \begin{pmatrix} u \\ v \end{pmatrix} + f\begin{pmatrix} u \\ v \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} -v \\ u \end{pmatrix}, \quad t > 0, \quad x \in \Omega \tag{2}$$

can be looked at as a first step. We will see that the dynamics of (2) seriously differs from the dynamics of one equation; in particular, periodic motion can occur.
If $\dim \Omega = 1$, i.e. Ω is an interval, then we can prove a Poincaré-Bendixson result for (2) which means that all solutions tend either to zero or to a periodic orbit. We examine these periodic orbits and prove qualitative statements on the dynamics of all solutions of (2) using torsion number results. This is a concept for the examination of symmetric reaction-diffusion systems which is related to oscillation numbers for one equation.
If $\dim \Omega > 1$, then the existence of a stable periodic orbit can be proved under appropriate smoothness and monotonicity conditions on f. Furthermore, we can describe a part of the global attractor including the stable periodic orbit, the zero solution and all connections between them.

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Recents Results in Micromagnetism

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In this work, joint with Gilles Carbou, we study a model of ferromagnetic material governed by a nonlinear Laudau-Lifschitz equation coupled with Maxwell equations (1).

$$\begin{gathered} \frac{\partial u}{\partial t} + u \wedge \frac{\partial u}{\partial t} = 2u \wedge (\alpha\Delta u + H) \ \text{ in } \mathbb{R}^+ \times \Omega, \\ \mu\frac{\partial(H+\bar{u})}{\partial t} + \text{curl } E = 0, \ \ \varepsilon\frac{\partial E}{\partial t} - \text{curl } H = 0 \text{ in } \mathbb{R}^+ \times \mathbb{R}^3, \end{gathered} \tag{1}$$

with appropriated boundary conditions and initial data. Let us assume that

$$u_0 \in I\!H^1(\Omega)\ ,\ H_0 \in I\!L^2(\mathrm{I\!R}^3)\ ,\ E_0 \in I\!L^2(\mathrm{I\!R}^3),\ \ |u_0| = 1\ a.e.\ ,\ \mathrm{div}\ (H_0 + \bar{u}_0) = 0. \qquad (\mathcal{H})$$

This system describes electromagnetic waves propagation in a ferromagnetic medium confined to the domain Ω. In the ferromagnetic model the magnetic moment denoted by u links the magnetic field H with the magnetic induction B through the relationship $B = \mu(H + \bar{u})$. Moreover u is a vector field which takes its values on the unit sphere of $\mathrm{I\!R}^3$. Finally ε is the dielectric permittivity and μ is the magnetic permeability. This model is described in detail in [2]. The quasistatic model corresponds to $\mu = 0,\ \varepsilon = 0$. After proving that under the assumption $(\mathcal{H})$, equation(1) has at least one weak solution, we establish :

Theorem 1 *Under assumption $(\mathcal{H})$, if u is a weak solution of (1), each point v of the ω-limit set of the trajectory u is a weak solution of the steady state system.*

Theorem 2 *Consider a sequence $(\varepsilon(n),\ \mu(n))_n$ which tends to $(0,0)$ as $n \to +\infty$ with $\mu(n)/\varepsilon(n)$ bounded. Then under the assumption $(\mathcal{H})$ if u^n denotes a weak solution of (1) with $\varepsilon = \varepsilon(n)$ and $\mu = \mu(n)$, there exists a subsequence $(u^{n_k})_k$ converging to u, solution of the quasistatic model.*

Theorem 3 *Let us assume $u_0 \in H^2(\Omega),\ \frac{\partial u_0}{\partial \nu}\Big|_{\partial\Omega} = 0,\ |u_0| \equiv 1$. Then there exists a time $T^\star_\alpha > 0$ depending only on the size of the data, and a unique u such that for any $T < T^\star_\alpha$, $u \in \mathcal{C}^0\left((0,T); H^2(\Omega)\right) \cap L^2\left((0,T); H^3(\Omega)\right)$.*
Moreover $\lim\limits_{\alpha\to 0} T^\star_\alpha = +\infty$ and for any $T > 0$, $u_\alpha \to u$ in $L^2\left((0,T); H^2(\Omega)\right)$ with

$$\frac{\partial u}{\partial t} - u \wedge H(u) + u \wedge (u \wedge H(u)) = 0 \text{ in } \mathrm{I\!R}_t \times \Omega \qquad (2)$$

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Connecting equilibria by blow-up solutions

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Radial solutions of the equation

$$u_t = \Delta u + \lambda e^u$$

on an N-dimensional ball are studied for $3 \leq N \leq 9$. We discuss blow-up-connections between equilibria. By a blow-up-connection from an equilibrium ϕ^- to an equilibrium ϕ^+ we mean a function $u(\cdot, t)$ which is a classical solution on the interval $(-\infty, T)$ for some $T \in \mathbb{R}$ and blows up at T but continues to exist in an appropriate weak sense for $t \in [T, \infty)$ and satisfies

$$u(\cdot, t) \to \phi^{\pm} \quad \text{as} \quad t \to \pm\infty$$

in a suitable sense.

On a Class of Selfsimilar Solutions of a Third-Order Model for the Shearless Turbulence Mixing Layer

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We study similarity solutions in the problem on the interaction and mixing between two semi-infinite homogeneous turbulent flow fields of different scales. The modelling study of this turbulence mixing layer is based on using a third–order model. This model includes a transport equation for the vertical component $< w^2 >$ of the turbulent kinetic energy and two equations of diffusion type for the triple correlation $< w^3 >$ and for the spectral flux ϵ of the turbulent kinetic energy. We analyze this model in the case of planar geometry. A self-similar solution of this system is a solution of the form

$$< w^2 >= t^{-2\mu} f(\xi), \quad < w^3 >= t^{-3\mu} g(\xi), \quad \epsilon = t^{-3\mu-\nu} h(\xi),$$

$$\xi = t^{\mu-1}(z - z_0), \quad z_0 = \delta_0 t^{1-\mu} + \delta_1, \quad \delta_k \equiv const, \quad k = 1, 2.$$

The free similarity exponent μ has to be determined from a solution of the obtained nonlinear eigenvalue problem. This is a typical situation appearing in problems of nonlinear diffusion where a conservation law does not exist. The boundary conditions are determined by the physical model, the functions $f(\xi)$, $g(\xi)$, $h(\xi)$ tend to given, generally speaking, positive limits at $\xi \to \pm\infty$. To find a solution to this nonlinear eigenvalue problem, we note that self-similar solutions can be found as steady-state solutions of the original system of partial differential equations after rewriting it in the new independent variables $x = t^{\mu}(z - z_0)$, $\theta = \log t$.

In the present article, we study the steady-states solutions and prove that there exists a positive steady state. The existence of a steady-state solution of this type implies that on this solution the system of equations will be transformed, returning to the original variables, into a system of two equations for $< w^2 >$ and ϵ in general. For the function

$< w^3 >$, we get a relation which is similar to a diffusion relation in the algebraic triple-correlation model. In other words, we prove that there is a regime of the turbulent flow which can be described in the framework of more simple model of second-order.

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Bifurcation of an equilibrium point in singularly perturbed parabolic system with transformed argument

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We consider the system of singularly perturbed parabolic equations with transformed argument

$$L(\varepsilon)\partial u/\partial t = D(t)\partial^2 u/\partial x^2 + A(t)u + B(t)u_\Delta + f(t,x,u,u_\Delta) \qquad (1)$$

with periodic condition $u(t, x + 2\pi) = u(t, x)$. Here ε is a small positive parameter, $L(\varepsilon) = diag[I_m, \varepsilon I_p], m+p = n$, $u_\Delta = u(t, x-\Delta)$, Δ is a transformation of the argument, matrices $D(t)$, $A(t)$, $B(t)$ and function $f : \mathbb{R}^{2n+2} \to \mathbb{R}^n$ are 2π periodic with respect to t, $f(t, x, u, v) = O(|u|^2 + |v|^2)$ when $|u| + |v| \to 0$, the matrix $L^{-1}(\varepsilon)D(t)$ is positive definite.

We will search the solution of system (1) in the form of Fourier series

$$u(t,x) = \sum_{k=-\infty}^{\infty} y_k(t)\exp(-ikx). \qquad (2)$$

Substituting (2) into (1), we obtain a countable system of differential equations in Fourier coefficients.

Let the following conditions are valid.

1) All roots of the characteristic equations $det(M_{k4}(t) - \lambda I) = 0$ lie on the halfplane $Re\lambda < 0$, where $M_k(t) = -k^2 D(t)+A(t)+B(t)exp(ik\Delta)$, $M_{k1}(t)$, $M_{k2}(t)$, $M_{k3}(t)$, $M_{k4}(t)$ are blocks of matrix $M_k(t)$.

2) The monodromy operator for linear equations $dv_k/dt = P_k(t,\varepsilon)v_k(t)$, $P_k(t,\varepsilon) = M_{k1}(t)+M_{k2}(t)h_0(t)+\varepsilon M_{k2}(t)h_1(t)$, $h_0(t) = -M_{k4}^{-1}(t)M_{k3}(t)$, $h_1(t) = M_{k4}^{-1}(t)\ [dh_0/dt + h_0 M_{k1}(t) + h_0 M_{k2}(t)h_0]$, has a pair of roots $exp[\alpha(\varepsilon) \pm i\beta(\varepsilon)]$, $\alpha(0) = 0$, $\alpha'(0) \neq 0$, $\beta(0) \neq 0$, and the remaining roots satisfy $|\lambda| \neq 1$.

Since the conditions 1, 2 are fulfilled, the center manifold of the countable system of differential equations exist. If $\alpha'(0)$ and the first Liapunov value for the equation on manifold have opposite signs, an invariant torus of system (1) exist. It will be conditionally stable if $\alpha'(0) > 0$. The solutions on the torus may be periodic or quasiperiodic in dependence of the rotation number. These solutions are obtained by normal forms. If the rotation number on the torus satisfies some condition then subfurcation of periodic solutions occurs.

Inertial Manifolds and a Modified Strong Squeezing Property

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We study dissipative nonlinear evolution equations

$$\dot{u} + Au = f(u)$$

in a Banach space X. We are interested in the existence of positively invariant, exponentially attracting, smooth, finite dimensional manifolds M, the so-called *inertial manifolds*, see [1]. Usually inertial manifolds are seeked as the graph of a smooth function ϕ over PX where P is a finite projector.

It appears that independent of the method used a special condition is utilized predominantly of the form

$$\Lambda_2 - \Lambda_1 > CL(\Lambda_1^\nu + \Lambda_2^\nu),$$

usually referred to as a *spectral gap condition.* This spectral gap condition assumes that the spectrum of A is split into a finite set of eigenvalues with real part less or equal to Λ_1 and the remaining set of eigenvalues with real part greater or equal to Λ_2, and that the difference of Λ_2 and Λ_1 is large enough to majorize $CL(\Lambda_1^\nu + \Lambda_2^\nu)$, where L is a Lipschitz constant for f, $\nu \in [0, 1[$ is an additional parameter depending on f, and C is a technical constant depending on the particular proof used.

Since the spectral gap condition is the most restrictive condition with regards to applications, it is of interest to weaken this condition or even to find sharp conditions.

One way to weaken the assumptions is to work out the essential properties of the semiflow which are used for the construction of the inertial manifold. Such properties are the *cone invariance* and the *squeezing property.* The cone invariance property describes the fact that the difference of solutions cannot leave a certain cone. This property is mainly used to prove the existence of the manifold. Robinson [2] has shown that the cone invariance property is sufficient to prove the existence of an invariant Lipschitz manifold. The squeezing property requires that the difference of each pair of solutions decays exponentially as long as the difference is outside of a possibly another cone; it is used to prove the attraction properties. Together both properties are called *strong squeezing property*, see [2].

We shall replace the strong squeezing property by introducing a *modified strong squeezing property* consisting of a cone invariance property and a modification of the squeezing property. To assume this property is a natural assumption and it is also sufficient for the existence of an inertial manifold having the *asymptotic completeness property.* We show that in general the modified strong squeezing property is weaker than the strong squeezing property.

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Banach contraction principle in parabolic problems

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The differential-functional problem

$$D_t u(t,x) \;-\; \sum_{j,l=1}^{n} a_{jl}(t,x)\, D_{x_j x_l} u(t,x) \;=\; f\left(t,x,u_{(t,x)}\right)$$

is transformed to a fixed point equation $u = \mathbb{T}\, u$ which is solved by means of the iterative method $u^{(k+1)} = \mathbb{T}\, u^{(k)}$. The Banach contraction principle (=BCP) provides the error estimates

$$\|u^{(k+2)} \;-\; u^{(k+1)}\| \;\leq\; \theta\, \|u^{(k+1)} \;-\; u^{(k)}\|, \qquad \theta \in (0,1),$$

provided the Lipschitz condition is satisfied. We discuss the question how far the BCP is from nonlinear cases.

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Unfocused blow up in semilinear parabolic equations

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The aim of this talk is to study the blow up behaviour of radially symmetric solutions u of a semilinear parabolic equation, around a blow up point other than its centre of symmetry. We show that u behaves as if a one-dimensional problem was concerned, that is the possible blow up patterns around an unfocused blow up point are the ones corresponding to the case of dimension $N = 1$.

Collision of Layers in a Scalar Reaction-Diffusion Equation of 1-Space Dimension

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We are concerned with the following scalar reaction-diffusion equation:

$$u_t = \epsilon^2 u_{xx} + u(u-a)(1-u), \quad -\infty < x < \infty,$$

where $\epsilon > 0$ and $0 < a < 1/2$ are parameters. One can observe that for small ϵ, transition layers emerge from appropriate initial data and those move with speed $O(\epsilon)$ as long as each layer is apart from the others of distance $O(1)$. This dynamics of the layers was well studied mathematically. It is also known by numerical computations that two facing layers collide and eventually collapse in a finite time. In this lecture we show rigorously the collision and the collapse of any two facing layers starting from a suitable initial function. We also estimate the life time of the layers. A key tool used in the study is a lower solution globally defined for $t > 0$. This is joint work with M. Mimoto [2].

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Formation of stable inner layers in diffusion equations: necessary and sufficient conditions

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In [1], S.B. Angenent, J. Mallet-Paret and L.A. Peletier classified all stable stationary solutions of a one-dimensional space semilinear boundary value problem which appears as a simplified model in population genetics. Their problem naturally generalises to N-space dimension as follows:

$$\frac{\partial v_\varepsilon}{\partial t} = \varepsilon^2 \triangle v_\varepsilon + v(1-v)[v - a(x)]$$

with $\partial v_\varepsilon / \partial \hat{n} = 0$ on $\partial\Omega$ and $\Omega \subset \mathbb{R}^N$. In particular, in [1] every solution develops inner transition layer with interface located at the zeros of $a(x) - (1/2)$. For the ease of presentation we work with the nonlinearity $f(x, v) = (v+1)(1-v)[v - a(x)]$.

We know from [2] that given a smooth hypersurface S in Ω, one of the necessary conditions for a family v_ε of stationary solutions of the above equation to develop an inner transition layer, as $\varepsilon \to 0$, with interface S is that the N-vector equation holds

$$\int_S a(x)\hat{n}(x)dS = 0$$

where $\hat{n}(x)$ stand for the outward normal vector on S.

Given a nodal hypersurface S of $a(x)$ we will give sufficient conditions on a for the existence of a family of solutions as above having S as interface. We suppose that S lies between two arbitrarily narrow N-dimensional annulus on each of which $a(x)$ is radially symmetric and satisfies two technical conditions. Our method of proof relies on techniques based on Γ-convergence and on sub- and super-solutions. More specifically we take two ε-families of conveniently defined functionals whose Γ-limits, as $\varepsilon \to 0$, have each of them, an isolated local minimiser in $BV(\Omega; \{\alpha(r), \beta(r)\})$, the space of functions of bounded variation in Ω which assume only two values: $\alpha(r)$ and $\beta(r)$. Here $\alpha(r) = (1/3)[a(r) - (3a^2(r)+9)]^{1/2}$ and $\beta(r) = (1/3)[a(r) + (3a^2(r)+9)]^{1/2}$. Close to each of these minimisers (in the L^1-topology) there is a local minimiser of the original functional. These, by its turn, will generate a sub- and a super-solution of the original equation and existence of a solution, for ε small, follows.

Sufficient conditions for the existence of such solutions for equations like the above one but with variable diffusivity and the nonlinearity $f(x,v)$ satisfying the generalized equal-area condition $\int_a^b f(x,\xi)d\xi = 0, \ \forall x \in \Omega$, where a and b are zeros of f (not necessarily consecutive), will be provided.

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On the stabilization of bounded solutions to parabolic systems with analytic nonlinearity and Liapunov functionals

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In this communication we consider a boundary value problem to parabolic systems in bounded domains. We assume that the problem has analytic dependence of unknown function and its derivatives. We also assume that the problem has a Liapunov functional. Let $u(x,t)$ denote the solution of problem in the classical sense. We say that the solution $u(x,t)$ stabilizes (stabilizes in backward time) if $u(x,t)$ converges as $t \to +\infty (t \to -\infty)$ to one stationary solution. It is shown that the uniformly bounded for $t \geq 0 (\leq 0)$ solution to the problem stabilizes (stabilizes in backward time). Let us assume that the problem is dissipative. It is shown that the global attrator of this problem consists of stationary solutions and connected orbits and the flow on global attrator is gradient like. In contrast, for one parabolic equation with many space variable (and of course for parabolic system with one space variable) with Liapunov functional the stabilization theorem becomes false. The example of such nonconvergent bounded solution was constructed by P.Polćhik and K.Rybakovskii. This example shows that it is impossible to omit the assumption about the analytic dependence of nonlinearity.

3.5 Stability of Fronts and Pulses

Organizer : Björn Sandstede

Key note lecture

Dynamics of pulses and fronts

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Travelling waves form an important class of patterns that arise in a variety of applied problems. The issue considered in this talk is the interaction between several well-separated stable travelling waves. Suppose that a certain dissipative partial differential equation posed on either the real line or an unbounded cylinder exhibits a stable pulse. We begin by describing the ordinary differential equations that govern the interaction of finitely many, well-separated copies of that pulse [1]. Equilibria of these differential equations correspond to multi-hump pulses whose stability properties were studied, for instance, in [5,6,7,8]. If infinitely many pulses are concatenated, the situation is far more complicated. As a first step towards a more complete description, the stability of spatially periodic wave trains with large period is considered. It has been proved by Gardner that their spectrum consists entirely of essential spectrum; here, we locate their spectrum [2]. A different type of interaction occurs for travelling pulses near the onset to instability. Suppose that the homogeneous asymptotic rest state towards which the pulse converges destabilizes. This bifurcation is often referred to as the Turing bifurcation; typically, small spatially-periodic stationary patterns are created near the asymptotic rest state. We show that the interaction between the pulse and these small Turing patterns produces a continuum of time-periodic travelling pulses that resemble a superposition of the pulse and the Turing patterns [3,4]. The main technique used in the proofs of the aforementioned results are exponential dichotomies together with Lyapunov-Schmidt and center-manifold reductions.

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Invited lectures

The stability of large pulse solutions

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First, it is shown that there exist various families of large-amplitude homoclinic (multi-)pulse solutions in singular perturbed reaction-diffusion equations (on the real line) under relatively mild conditions. Second, the stability of these pulses is investigated using the Evans function. It is established that the natural slow/fast decomposition of the Evans function exhibits singularities: the slow component can have poles at (some of) the zeros of the fast component. The eigenvalues of the linearized stability problem, i.e. the zeros of the Evans function, can be determined explicitly by the recently developed 'NLEP method'; this method gives a leading order approximation of the slow component of the Evans function. As a consequence, it can be shown that only the 1-pulse solutions can be stable: all homoclinic multi-pulse solutions must be unstable. The methods will be applied to the Gierer-Meinhardt problem.

Convergence to travelling waves in damped hyperbolic equations

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We discuss the stability of travelling waves for a nonlinear damped hyperbolic equation on the real line. This system depends on a characteristic time $\eta > 0$ as a parameter, and reduces to the parabolic Fisher equation as $\eta \to 0$. Using energy estimates and a hyperbolic version of the maximum principle, we show that the travelling waves are stable against perturbations in a weighted Sobolev space. In the important case of fronts travelling with minimal speed, we prove in addition that the perturbations decay to zero like $t^{-3/2}$ as $t \to +\infty$ and approach a universal self-similar profile, which is essentially independent of η, of the nonlinearity, and of the initial data.

Slowly-modulated two-pulse solutions and pulse-splitting in the 1-D Gray-Scott model

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Two pulse solutions play a central role in the phenomena of self-replicating pulses in 1-D reaction-diffusion systems. In this talk, which is on a joint paper with Arjen Doelman and Wiktor Eckhaus, we focus on the 1-D Gray-Scott model as a prototype. We report on an existence and stability study for solutions consisting of two pulses moving apart from each other with slowly varying velocities. In the various parameter regimes, critical maximum wave speeds are identified, and ODE's are derived for the wave speed and for the separation distance between the pulses. The bifurcations in which these solutions are created and annihilated, and in which they gain stability, are determined. Good agreement is found between these theoretical predictions and the results from numerical simulations. The inhibitor component is far away from its homogeneous value, and slowly varying, between the pulses. Hence, the results presented here apply to the strong pulse interaction problem. The main methods used are analytical and geometric singular perturbation theory for the existence demonstration, and the nonlocal eigenvalue problem (NLEP) method developed in our earlier work on stationary pulses for the stability analysis.

Essential instabilities of travelling waves

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We analyze instabilities of travelling waves in reaction-diffusion systems caused by the essential spectrum crossing the imaginary axis. As a typical example, we examine travelling fronts and pulses whose asymptotic states undergo a Turing instability. Under typical assumptions we prove bifurcation of stable modulated travelling waves asymptotic to small amplitude Turing patterns, if the Turing instability is created ahead of the front. If the instability is created behind the front, we show that modulated travelling waves cannot bifurcate and the primary front remains convectively stable. A pulse is shown to move through a Turing pattern, leaving a recovery zone of Turing patterns, which are growing in amplitude, behind.

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Existence and stability of modulating fronts

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Modulating front solutions look like a front like envelope advancing in the laboratory frame and modulating an underlying spatially periodic pattern. They are transient solutions which connect a spatially periodic equilibrium with the unstable spatially periodic ground state. A classical example is the bifurcating front connecting the Taylor vortices with the trivial Couette flow for the Taylor–Couette problem in infinite cylinders. Here we talk about existence and stability of such objects.

Contributed talks

Multi-pulse solutions in problems with the reflection symmetry in an unbounded domain

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Let us consider an evolutionary autonomous system that is invariant under shifts $z \mapsto z + const$ in the unbounded spatial coordinate z and reflection $z \mapsto -z$. We assume that the basic fully symmetrical steady state loses stability via perturbations with the spatial critical wave number $k_c = 0$. That is the case for a variety of the physical problems, for instance: 3D Poiseuille problem on viscous incompressible fluid flows between parallel plates, convection in inclined layer, film flows, chemical reactions and so on (see [1],[2]). If the time periodicity is presumed together with periodicity in additional unbounded directions then near the instability threshold the local study of the problem can be reduced with the use of the spatial dynamics formulation to the analysis of the phase portrait of $SO(2)$-invariant reversible vector field with nonsemisimple eigenvalue 0 of the linearized problem. Quasihomogeneous truncation corresponding to a positive face of the Newton polyhedron of the ODE problem is equivalent to the steady complex Ginzburg-Landau (cGL) equation ([2,3,5]. For the cGL equation a primary family of homoclinic orbits $T_\alpha H_0$, $\alpha \in S^1$ is known explicitly. Under a generic transversality condition with respect to parameter, the existence of homoclinic n-pulse solutions is demonstrated for a sequence of parameter values. The existence of cascades of $2^l 3^m$-pulse solutions follows by showing their transversality and then using induction. The method relies on the construction of an $SO(2)$-equivariant Poincaré map which, after factorization, is a composition of two involutions: a logarithmic twist map and a smooth global map. Reversible periodic orbits of this map corresponds to small reversible periodic or homoclinic solutions of the problem. Using perturbation arguments we relate reversible n–pulse solutions of this equation to n–pulse solutions of the original problem on a spatial center manifold. Thus, we obtain multi–pulse solutions of the PDE problem for parameters near the criticality. In 3D Poiseuille problem these solutions are localized in spanwise direction, but periodic in streamwise direction.

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Stability of travelling fronts for reaction-diffusion-convection systems

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We are concerned with the role of travelling fronts in the asymptotic behaviour of solutions of systems of the form

$$u_t = Au_{xx} + f(u, u_x),\ x \in \mathbb{R}, t > 0, u(x,t) \in \mathbb{R}^N,$$

$$u(x,0) = u_0(x),\ x \in \mathbb{R},$$

where A is a positive-definite diagonal matrix and the nonlinearity f has two ordered equilibria and satisfies conditions sufficient for a comparison principle for this system to hold.

Suppose that f is "bistable" and that there exists a travelling-front solution w with velocity c connecting the two stable equilibria of f. (We have a set of hypotheses on f under which such a front is known to exist.) If u_0 is bounded, uniformly continuously differentiable and such that $\|w(x) - u_0(x)\|$ is small when $|x|$ is large, we can show that there exists $\xi \in \mathbb{R}$ such that

$$\|u(\cdot, t) - w(\cdot - ct + \xi)\|_{BUC^1} \to 0 \text{ as } t \to \infty,$$

(see [1]). Our approach extends an idea developed by Roquejoffre, Terman and Volpert in the convectionless case, where f is independent of u_x. First local stability is proved using the spectrum of the linearisation about w. Then u_0 is assumed to be increasing in x, and convergence proved via a homotopy argument. The result for arbitrary u_0 is deduced by showing that there is an increasing function in the ω-limit set of u_0.

Suppose now that f is "monostable". In this case, there is some $c_0 \in \mathbb{R}$ such that there exist travelling-front solutions for each velocity $c \geq c_0$. We are investigating the stability of the supercritical waves ($c > c_0$) by formulating the problem in exponentially weighted function spaces, where the velocity-dependent weight is chosen so as to render the "monostable" nonlinearity essentially "bistable", and thus amenable to some of the methods used in the "bistable" case. (Such use of weighted spaces goes back to work of Sattinger, and has been discussed by Volpert, Volpert and Volpert for reaction-diffusion systems where f is independent of u_x.)

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Global structure of traveling waves for 3 competing species model with diffusion

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Reaction-diffusion equations are used extensively as continuous space-time models for interacting and diffusing biological species in population dynamics. In mathematical ecology, there are three types of interactions between individuals with the diffusion terms which model the migration of each species; competition for limited resources, prey-predator interaction, and mutualistic relationships. Though these equations are relatively simple, they can exhibit a variety of interesting spatial and spatio-temporal patterns, including traveling fronts and pulses.

In this talk, we study the following 3-component reaction-diffusion systems for three competing species :

$$(1)\quad \begin{cases} u_{1t} = d_1 u_{1xx} + (r_1 - a_{11}u_1 - a_{12}u_2 - a_{13}u_3)u_1 \\ u_{2t} = d_2 u_{2xx} + (r_2 - a_{21}u_1 - a_{22}u_2 - a_{23}u_3)u_2 \\ u_{3t} = d_3 u_{3xx} + (r_3 - a_{31}u_1 - a_{32}u_2 - a_{33}u_3)u_3 \end{cases} \quad (t,x) \in \mathbb{R}_+ \times \mathbb{R},$$

where $u_i(t,x)$ denote the population densities of three competing species at time t and spatial position x. d_i are the diffusion rates, r_i are the intrinsic growth rates, and a_{ij} are the intraspecific and interspecific competition rates, respectively. All of the coefficients are positive constants. We assume that

(H1) $d_1,\ d_2\ <<\ d_3$ (i.e., $d_1 = \varepsilon^2, d_2 = \varepsilon^2 d, d_3 = 1$ for small positive ε),

(H2) If $u_1 = 0$, this system (1) has a stable equilibrium solution $P_2 = (0, u_{*2}, u_{*3})$ with $u_{*2} > 0$ and $u_{*3} > 0$. On the other hand, if $u_2 = 0$, (1) has a stable equilibrium solution $P_3 = (u_1^*, 0, u_3^*)$ with $u_1^* > 0$ and $u_3^* > 0$.

Let us consider the following situation. If the u_1-species invades on the stable equilibrium state P_2, what happens with the population dynamics of the invader u_1-species ? For suitable parameters, we can check numerically that u_1-species propagates into the state P_2 with constant shape and constant velocity and inhibits u_2-species and then it reaches to the state P_3. This motivates us to study the existence and stability of traveling fronts connecting the states P_2 with P_3. We use the analytical singular perturbation method to show the existence and the SLEP method to show the stability. Finally, we consider the global structure of traveling wave solutions; traveling fronts, standing pulses, traveling pulses, with respect to some parameter.

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Instability of Multiple Pulses in a System of Quadratically Coupled Schrödinger Equations

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Many mathematical models studied in modern optics take the form of two or more Schrödinger equations coupled together in a nonlinear fashion. Of special interest are solitary-wave solutions for which all components of the system are localised in time (representing pulses in optical fibres) or in the space directions transverse to propagation (modelling beams in waveguides). This talk is mainly concerned with a system that describes the phenomena of second-harmonic generation and parametric wave interaction in dispersive quadratic media. In particular, the existence and stability properties of multiple pulses (multibump solitary-waves) are investigated within the framework of homoclinic bifurcation theory, employing a functional-analytic approach with strong geometric underpinnings. All of the multiple pulses generated via the bifurcation occurring at a resonant semi-simple eigenvalue scenario are demonstrated to be unstable; we then discuss the generalisation and implications of this result for other, related, systems.

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D. Computational Aspects

4.1 Computer Algebra Tools

Organizer : Jan Sanders

Key note lecture

Algorithms from representation theory

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First a description of the role of $sl(2, \mathbb{R})$ in the computation and classification of nilpotent normal forms is given. This leads to a similar role for the Heisenberg algebra in the case of the total differentiation operator

$$\mathcal{D}_x = \frac{\partial}{\partial x} + \sum_{i=0}^{\infty} u_{i+1} \frac{\partial}{\partial u_i}.$$

One define derivations

$$\mathcal{F}_x = \sum_{i=1}^{\infty} i u_{i-1} \frac{\partial}{\partial u_i}.$$

and

$$\mathcal{E}_x = \sum_{i=0}^{\infty} u_i \frac{\partial}{\partial u_i}.$$

These form a Heisenberg algebra with the usual operator commutation rules. Using representation theory one can now effectively *integrate*, i.e. solve f from $g = \mathcal{D}_x f$. One can now define transvectants or Hirota operators by

$$\tau_n(f, g) = \sum_{r+s=n} \frac{(-1)^r}{r!s!} \mathcal{D}_x^r \mathcal{E}_x^s(f) \mathcal{D}_x^s \mathcal{E}_x^r(g) = \frac{1}{n!} (\mathcal{E}_x \wedge \mathcal{D}_x)^n f \otimes g.$$

Notice that $\tau_n : \ker \mathcal{F} \otimes \ker \mathcal{F} \to \ker \mathcal{F}$, i.e. the bilinear operators are defined on the space of functionals and τ_0 and τ_1 define a Poisson structure there.

One can then deform the usual multiplication by defining

$$f \star_t g = \sum_{n=0}^{\infty} t^n \tau_n(f, g).$$

This defines an associative product, and thereby a noncommutative deformation of the Poisson structure.

The various connections between classical invariant theory, transvectants, Hirota operators, Rankin-Cohen brackets in the theory of modular functions and pseudo-differential operators and the Poisson algebra of functionals, as they are defined in the theory of integrable evolution equations will be discussed.

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Invited lectures

Algorithmic invariant theory and dynamics

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In recent years there has been a lot of progress in computational algebra, especially in the algorithmic treatment of invariant rings and modules of equivariants. The aim of this talk is to show the benefits of this for situations in the theory of equivariant dynamical systems where a general equivariant vector field is required. I will discuss the consequent algorithmic treatment of the center manifold reduction with symmetry. The method of Noether normalization will be explained and the advantages of using Noether normalization in this context illustrated.

Finally, we will comment on a generalization of the orbit space reduction to group actions on the differential equations which may act on the independent variable and may be nonlinear.

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Using the symbolic method to classify evolution equations

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The main purpose of this talk is to demonstrate the existence of (infinitely many) symmetries and conservation laws for λ—homogeneous evolution equations using the symbolic method.

The symbolic method first introduced by Gel'fand and Dikii. The idea is simply to replace u_i, where i is an index — in our case counting the number of derivatives — by ξ^i, where ξ is now a symbol. The basic operation of differentiation, i.e. replacing u_i by u_{i+1}, is now replaced by multiplication with ξ, as is the case in Fourier transform theory. For higher degree terms with multiple u's, one uses different symbols to denote differentiation.

The key fact is the following symbolic formula for the bracket of a differential polynomial Q of degree **m** with a linear differential polynomial:

$$\widehat{[u_k, Q]} = G_k^{(m)} \, \hat{Q},$$

where

$$G_k^{(m)} = \xi_1^k + \cdots + \xi_m^k - (\xi_1 + \cdots + \xi_m)^k.$$

With this one can readily translate solvability questions into divisibility questions solved by diophantine approximation theory, and we can use generating functions to handle infinitely many orders at once.

The complete list of λ—homogeneous integrable evolution equations contains only 10 all-known equations when $\lambda > 0$. This explains why after the gold rush it was so difficult to find any new integrable systems.

The classification of co-symmetries (conservation laws) is more complicated. One often encounters equations with only a finite number of conservation laws. Here a complete classification for 5-th order KdV-like equations is given using generating function.

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A Method for Computing Center Manifolds and Normal Forms

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Center manifold theory and normal form theory are two basic tools in the study of nonlinear differential equations. The idea of these two theories is to use a sequence of near identity nonlinear transformations to obtain a simple form on the so called center manifold. This reduction greatly simplifies the analysis of the dynamical behavior of the system such as bifurcations and instability. Different methodologies have been developed for computing normal forms, based on matrix theory, algebraic manipulation, representation theory, invariant theory, etc. [1–4]. However, for a given system, finding the explicit formulas of normal forms and the associated nonlinear transformations is not easy. Thus, symbolic computer languages such as Maple, Mathematica and Macsyma have been introduced in such computations.

Normal forms are generally not uniquely defined due to the existence of uncertainty in computing the coefficients of nonlinear transformations. This difficulty may be overcome either by imposing additional conditions so that the coefficients of nonlinear transformations can be uniquely determined; or by imposing certain conditions such that some coefficients in the normal forms may be set zero. Such a simplification raises a question: In general how to find a "simplest" or "minimum" normal form? Some results related to simple singularities have been obtained ([5, 6], more references can be found in [5]). However, it is noted that a *unique* normal form may be not a "simplest" form, and simplest normal forms may be not unique.

We will present an early developed perturbation technique for computing the normal forms of a general n-dimensional system described by a set of differential equations for certain cases of singularities [7]. The method does not need the application of center manifold theory, nor requires for solving large matrix systems. In fact, the approach combines the normal theory and center manifold theory in one computation procedure. Moreover, it has been found that the method may be extended to directly compute a unique, "simplest" normal form for certain cases. Symbolic computation using Maple will be discussed, and examples will be presented to show the computation efficiency of the method.

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Contributed talks

Computer Algebra Tools for the Solution of Boundary-value Problems

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The introduced package provides tools for numerical solution of linear boundary value problems, both regular and eigenvalue ones. It has a syntax of the built-in *Mathematica* function **NDSolve** and strongly enhances its functionality. For the solution of mentioned problems two basic methods are applied. The first one is known as *Chasing* method and was generalized by authors particularly for multipoint problems, and the second one is the well-known *Shooting* method. The newest version of *Mathematica* 3.0 contains the function **NDSolve** which has a very limited ability to cope with regular boundary value problems, but not with eigenvalue ones. Unfortunately, the function uses an in some sense erroneous algorithm which leads sometimes to strange and even wrong results. Moreover, it can solve neither boundary value problem containing more than one equation nor multipoint ones. Below we present an example of a wrong result: **NDSolve** can't cope with the following problem: the command

$$\textbf{NDSolve}[\mathtt{y'''[x] + y[x] == 0, y'[0] == 0, y'[2] == 1, y''[0] == 0, y[x], \{x, 0, 2\}}]$$

gnereates the answer: `NDSolve::"unsol": "Not possible to initiate boundary value problem with the chasing method"` - message from **NDSolve**, while **DSolve** gives analytical solution:

$$\textbf{DSolve}[\mathtt{y'''[x] + y[x] == 0, y'[0] == 0, y'[2] == 1, y''[0] == 0, y[x], x]//Simplify}$$

$$\{\{y(x) \to \frac{\left(-i+\sqrt{3}\right) e^{2+i\sqrt{3}-\frac{\left(3+2\,(-1)^{\frac{2}{3}}\right)x}{2}} \left(e^{\frac{3x}{2}} + e^{\frac{i}{2}\sqrt{3}\,x} + e^{\frac{(3+2\,i\,\sqrt{3})\,x}{2}}\right)}{-2\,i\,e^{3} + i\,e^{i\sqrt{3}} - \sqrt{3}\,e^{i\sqrt{3}} + i\,e^{3+2\,i\sqrt{3}} + \sqrt{3}\,e^{3+2\,i\sqrt{3}}}\}\}$$

In contrast, our implementation of the abovementioned methods produces more accurate results for a wider class of problems. In addition to the solution of a regular boundary value problem our functions can solve eigenvalue problems, which is very important in applications (for instance, for linear stability analysis). We present an example of linear stability analysis in Marangoni convection in the spherical layer, which was successfully performed using our tools. Another advantage of the presented tools is also the possibility to input the problem in its natural form.

A computational method for the stability of a class of mechanical systems

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(Joint work with Prof. **Ângelo Barone-Netto** and Prof. **Mauro de Oliveira Cesar**, of the Institute of Mathematics and Statistics, University of São Paulo, Brazil).

Consider the differential system

$$\begin{aligned} \ddot{x} &= -xf(x) \\ \ddot{y} &= -yg(x), \end{aligned}$$

where $f, g\colon \mathbb{R} \to \mathbb{R}$ are smooth functions and $f(0) > 0$, $g(0) > 0$. Clearly the system has the constant solution $x(t) \equiv y(t) \equiv 0$. The problem here is how to decide the $s$tability of this equilibrium solution, that is, what is the asymptotic behaviour as $t \to +\infty$ of the trajectories starting with initial data close to 0.

We will say that the system associated to a given pair of functions f, g is *k-decidable* for some $k \in \mathbb{N}$ if the stability of the equilibrium can be decided from the values of the derivatives up to the order k of f and g at 0. The system will be finitely decidable if it is k-decidable for some k.

Not all pairs of functions are finitely decidable. However finite decidability is generic in C^∞, and we describe an algorithm for finding explicit sufficient conditions for k-decidability. When implemented on a computer the algorithm runs easily enough for $k = 1$ up to 5 or 6.

With the explicit conditions at our disposal it is easy to construct examples of pairs f, g and of sequences of initial data converging to 0 for which either all $y_n(t)$ are unbounded as $t \to +\infty$, or, more interestingly, where $t \mapsto y_n(t)$ is bounded for all n, but $\sup_{t\geq 0} |y_n(t)|$ diverges as $n \to +\infty$.

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4.2 Control and Optimization

Organizer : Fritz Colonius

Key note lecture

Dynamical Aspects of Control

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We consider families of ordinary differential equations where time dependent coefficients are allowed to take their values in a given set. Thus one obtains a family of nonautonomous differential equations indexed by the coefficient functions which may be considered as (open loop) admissible controls. The long time behavior of this family can be described via an associated skew product flow, the control flow, over the base space of admissible controls. The topologically transitive components and the chain transitive components correspond to the control sets and the chain control sets, respectively. If the closure of a control set is a chain control set, then it depends continuously (in the Hausdorff metric) on parameters. Also some related bifurcation problems will be discussed.

Invited lectures

Characterization and Computation of Reachable Sets via Optimal Control

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For controlled and perturbed systems governed by nonlinear dynamics of the form

$$\dot{x}(t) = f(x(t), u(t))$$

on some manifold M, with $u(\cdot)$ from some function space $\mathcal{U}$, the knowledge of reachability properties plays an important role in the analysis of the system behaviour.

For $u(\cdot)$ being a control function e.g. the *reachable set*

$$R(x) := \{y \in M \mid \exists u(\cdot) \in \mathcal{U} : \phi(t, x, u(\cdot)) = y\}$$

is an important object; if $u(\cdot)$ models a time varying perturbation, and $x^* \in M$ is a locally stable equilibrium for each $u(\cdot) \in \mathcal{U}$ the *robust domain of attraction*

$$RD(x^*) := \{y \in M \mid \phi(t, y, u(\cdot)) \to x^* \text{as}\, t \to \infty\, \forall u(\cdot) \in \mathcal{U}\}$$

is of interest [4].

In this talk we give a survey on recent work [2, 3] about the characterization of these sets via optimal control techniques. Appropriate formulations are e.g. minimum/maximum time and minimum/maximum distance optimal control problems, which have in common that their optimal value functions are characterized by Hamilton-Jacobi-Bellman PDEs. Furthermore we will present a generalization of the classical Zubov's equation [1] which nicely fits into the Hamilton-Jacobi framework.

Based on these formulations we then discuss numerical methods for the computation of these sets and illustrate them by a number of examples. In particular, we will highlight advantages and limitations of this approach compared e.g. with trajectoriwise or cell mapping algorithms.

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Twist maps and sliding-mode control

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In this talk we will study the dynamical stability properties of a suitably defined approximation of equivalent control in sliding manifold control systems. For this we use both the classical theory of singularly perturbed dynamical systems and the theory of twist maps. Examples of control problems which possess dynamical stability properties are presented. The work discussed here continues the investigation of control design, via singular perturbation theory, for control systems posessing a sliding manifold.

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Controllability of Matrix Eigenvalue Algorithms

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Numerical matrix eigenvalue methods such as the QR algorithm or inverse power iterations provide interesting examples of nonlinear discrete dynamical systems defined on Lie groups or homogeneous spaces. A typical approach from numerical linear algebra to improve convergence properties of such algorithms is via suitable shift strategies for the eigenvalues, see [1]. Such eigenvalue shifts can be viewed as control variables and the resulting algorithms can therefore be analyzed using tools from nonlinear control theory. So far the analysis and design of shift strategies in numerical eigenvalue algorithms has been more a kind of an art rather than being guided by systematic design principles. The advance made during the past two decades in nonlinear control theory indicates that the time may now be ripe for a more systematic investigation of control theoretic aspects of numerical linear algebra.

In this talk we investigate the controllability properties of the well known inverse power iteration for finding the dominant eigenvector of a matrix A, defined by

$$x_{k+1} = \frac{A^{-1}x_k}{\|A^{-1}x_k\|}.$$

Shifted versions of the inverse power method defined by

$$x_{k+1} = \frac{(A - u_k I)^{-1}x_k}{\|(A - u_k I)^{-1}x_k\|}$$

lead to nonlinear control systems on projective space, or more generally on Grassmann manifolds. For cyclic matrices A necessary and sufficient conditions for complete controllability on the projective space are given. These are always satisfied if complex shifts are allowed.

In the real case the situation is more involved and we use recent results on the existence of universally regular controls in our analysis, see [2]. In the real case necessary conditions in terms of the location of the eigenvalues are presented. These may be equivalently rephrased by the algebraic statement that every polynomial with real coefficients splits modulo the characteristic polynomial of the matrix A into linear factors over $\mathbb{R}$. We conjecture that these conditions are also sufficient. If this turns out to be true it may be inferred that complete controllability of the inverse power iteration is a generic property.

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Contributed talks

Controllability of generic control systems near k-singular points

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Control system on n-dimensional smooth manifold is defined by smooth family of vector fields parameterized by the points of $(n-1)$-dimensional sphere. A *generic* control system is a system from a certain open everywhere dense subset in the space of such families endowed with the fine Whitney's C^∞–topology.

At a point of the phase space values of the fields forms *velocity indicatrix* at this point, and any velocity from the indicatrix is admissible one at this point. Attainability of one point from another one is defined in standard manner in the class of piecewise continuous controls.

A point of the phase space has *small time local transitivity property* (=SLTP) if for any time $T > 0$ and any neighborhood V of this point there exists a neighborhood of this point such that any two point of the latter neighborhood are attainable from each other in a positive time smaller than T and along a trajectory lying in the neighborhood V. A *k-singular point* is defined as a point of the boundary of set of points having SLTP such that there is a passage through zero on a plane of support to the velocity indicatrix at this point containing exactly k admissible velocities. In the generic case there can appear such singular points with $k \in \{1, 2, \ldots, n\}$.

Theorem *For $n \geq 3$ any k-singular point of a generic control system on n-dimensional manifold has SLTP if $3 \leq k \leq n$ and it has nor SLTP if $k = 1$.*

For 2-singular points for a generic control system some necessary and some sufficient conditions to have SLTP are found. In [1], [2] the complete answer on local controllability of systems on surfaces was obtained for the generic case. In particular, their controllability near 1-, 2-singular points was studied (in [3] analogous results are obtained for a generic two dimensional dynamic inequality).

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The descent constant in the optimal control problems with Lipschitzian differential inclusions

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We consider the optimal control problem with fixed time endpoints: $J(x(S), x(T)) \to \min$, $(x(S), x(T)) \in C$, $\dot{x}(t) \in F(x(t))$, $G(x(t)) \leq 0$, $\forall t \in [S, T]$ where minimum is sought in the class of absolutely continuous arcs $x(\cdot)$, $x(t) \in R^{d_x}$. Here J, G are real-valued Lipchitz functions; $F(\cdot)$ is Lipchitz multifunction with convex compact values belonging to R^{d_x}, C is close subset of $R^{d_x} \times R^{d_x}$.

For this problem the necessary condition of the strong minimum in the maximum principle (MP) form is well-known [1]. In the important subclass with locally-convex J, G where C and $F(x)$ can by represented by a regular system of equations and inequalities, this MP is equivalent [2] to existence of a nontrivial solution of Eulier equation, i.e. there is no reducing directions simultaneously tangentially to local problem's constrains. But in the case of general problem under consideration the connection of conjugate multipliers and geometrical "descent algorithm" is lost completely. This serious defect of modern MP theory is connected with ineffective traditional way of its derivation (penalty method, Ekeland theorem ets.). The purpose of proposed investigation consist in the restoring of the of noted relation. Consider, for brevity, the problem without state constraint $G \leq 0$ and define the auxiliary objects.

$\Lambda(x) := \{\lambda = (\alpha, l, n_C, \psi(t), n(t))\}$ - the totality of conjugate strings of an admissible trajectory $x(\cdot)$. Here $\alpha \geq 0$, $l \in \partial J(x(S), x(T))$, $n_C \in N_C(x(S), x(T)), n(t) = (n_x(t), n_u(t)) \in clgN_{gr\,F(x(t))}(x(t))$, $\psi(t)$, $n(t)$ are Lipchitz and Borelian measurable functions with values in R^{d_x}, $R^{d_x} \times R^{d_x}$ respectively; N_C, $clgN_{grF(x(t))}$ denotes the cones of external normals and the closure "be graph" of the external normals for the respect sets.

$b(x) := \inf\{\|\psi - n_x\|_{L_1} + \|\dot{\psi} - n_u\|_{L_1} + \|(\psi(S), \psi(T)) - n_c\| : \lambda \in \Lambda(x), \alpha = 0, \|(\psi, n, n_C)\| = 1\}$; $a(x) := \inf\{\|\psi - n_x\|_{L_1} + \|\dot{\psi} - n_u\|_{L_1} + \|(\psi(S), \psi(T)) - n_c - \alpha l\| : \lambda \in \Lambda(x), \alpha = 1\}$

Theorem : Let the admissible trajectory $x_0(\cdot)$ satisfy the inequalities $a(x_0) > 0, b(x_0) > 0$. Then $\forall \epsilon > 0\ \exists \delta > 0$ and a family of admissible trajectories $\{x_r\}$, $r \in [0, \delta]$ such that $\forall\, 0 \leq r' \leq r'' \leq \delta$ the following conditions are satisfied: $\|x_{r'} - x_{r''}\| \leq r'' - r'$, $J(x_{r''}(S), x_{r''}(T)) - J(x_{r'}(S), x_{r'}(T)) \leq -(a(x_0) - \epsilon)(r'' - r')$.

That is, the objective function can be decreased over the "Lipchitz" family of admissible trajectories x_r with the speed $a(x_0)$. The proof of theorem is based on constructive Lusternike's type iteration process. Note, that there is no, in general, the strict descent directions.

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About the Optimality Conditions in Control Problems

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The Optimal Control Problems prescribed by Operator Relations in the Banach Spaces are considered in the Work. The Development of the absolute Minimum Theory based on the V.F.Krotov Lemma is given. Let $K = \{k|k = 1, 2, ...\}, S = \{s|s = 1, 2, ...\}$ are index Sets. We assume that the Families of Sets $\{D_k, k \in K\}, v_k \in D_k$ are given. The Functional $J_k : D_k \to R$ on D_k is given.

Lemma 1. Let the Problems $\{<< D_k, J_k >>, k \in K\}$ satisfy to the Conditions: 1^0. for all $k \in K : J_k(v) \geq J_{k+1}(v), v \in D_k$; 2^0. there are the Integers $p, q \in K$ and the Sequence $\{v_s, s \in S\} \subset D_p$ such that $p < q$ and $\lim_{s\to\infty} = i_q = \inf_{D_q} J_q$. Then $\{v_s, s \in S\}$ is minimizing for the Problems $\{<< D_k, J_k >>, k = p, p+1, ..., q\}$ and any minimizing Sequence for $<< D_k, J_k >>$ satisfies to the Condition 2^0 and is also minimizing for Family of Problems $\{<< D_k, J_k >>, k = p, p+1, ..., q\}$.

Let Y_1, Y_2, U_1 are the Banach Spaces, Y_1 is reflexive, continuously and densely embedding in $Y_2 . Y = Y_1^n, U = U_r$ with Elements $y = \{y_1, ..., y_n\}$ and $u = \{u_1, ..., u_r\}$ accordingly, $\{y, u\}$ are the pair State-Control. Let

$$Y_c \subset Y, \quad U_c(y) \subset U, \quad V_c = \{y, u|y \in Y_c, u \in U_c(y)\} \subset V = Y \times U. \tag{1}$$

We suppose that the pair State-Control satisfies to the following functional Constraints:

$$A_i(y, u) = 0_{Y_2}, i = 1, ..., l_1, \quad A_i(y, u) \geq 0_{Y_2}, i = l_1 + 1, ..., l. \qquad (2).$$

We designate a Set of pairs (y, u) which satisfy (1), (2) as D_1 and assume $D_1 \neq \emptyset$. Functional $J_1(y, u)$ is defined in D_1.

Problem. To find a Solution of the Minimization Problem $<< D_1, J_1 >>$.

Let

$$D_2 = \{y, u|(1)\}, \quad J_2(y, u) = J_1(y, u) - \sum_{i=1}^{l} < \lambda_i(y), A_i(y, u) >, \tag{3}$$

where $\lambda_i : Y \to Y_2, i = 1, ..., l_1, \quad \lambda_i : Y \to P^*, i = l_1 + 1, ..., l$, are nonlinear Operators, P is the Cone of positive Elements in Y_2.

Theorem 1. Let there are a). Operators $\lambda_i, i = 1, ..., l$; b). Sequence $\{y_s, u_s, s \in S\} \subset D_1$ such that

$$\lim_{s\to\infty} J_1(y_s, u_s) = i_2 = \inf_{D_2} J_2. \tag{4}$$

Then $\{y_s, u_s, s \in S\} \subset D_1$ is minimizing for $<< D_1, J_1 >>$, and any minimizing Sequence for the Problem $<< D_1, J_1 >>$ satisfies to the Condition (4).

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Bifurcation of control sets

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We consider control-affine systems on $\mathbb{R}^d$ of the following form:

$$\begin{array}{ll} \dot{x} & = f_0(x) + \sum_{i=1}^m u_i(t) f_i(x), \\ u & \in \mathcal{U}^\rho = \{u : \mathbb{R} \to \mathbb{R}^m, u(t) \in U^\rho \forall t \in \mathbb{R}, \text{meas.}\} \end{array} \quad (1)$$

where $U \subset \mathbb{R}^m$, compact and convex containing 0 and $U^\rho = \rho \cdot U$ for $\rho \in [0, \rho^*] \subset \mathbb{R}$. We assume, that the system (1) has a *singular point* $x^* \in \mathbb{R}^d$, i.e. $f_i(x^*) = 0$ for all $i = 0, \ldots, m$.

Control sets are subsets of $\mathbb{R}^d$ where every point can be steered to another at least approximately. Motivated by numerical experiments with the perturbed *Duffing-van der Pol* equation, the question arised under which conditions there are control sets around the singular point x^*. By linearization of the system (1) at the singular point we get the bilinear control system

$$\dot{x} = A_0 x + \sum_{i=1}^m u_i(t) A_i x, \quad u \in \mathcal{U}^\rho \text{ where } A_i := \left. \frac{\partial f_i}{\partial x} \right|_{x=x^*}. \quad (2)$$

Denote the solution by $\phi(t,p,u)$ with $\phi(0,p,u) = p$. For every $(u,p) \in \mathcal{U}^\rho \times \mathbb{R}^d$ define the *Lyapunov exponent* $\lambda(u,p) := \limsup_{t\to\infty} \frac{1}{t} \ln \|\phi(t,p,u)\|$. There is a Whitneysum-decomposition $\mathcal{U}\times\mathbb{R}^d = \mathcal{V}_1^\rho \oplus \ldots \oplus V_l^\rho, 1 \leq l \leq d$ such that $\Sigma_{Ly}^\rho(\mathcal{V}_i) = \{\lambda(u,p) : (u,p) \in \mathcal{V}_i^\rho\}$ are compact intervals (cf. [2]). Under regularity conditions on (1) and (2) and with

$$0 \in \text{int}\Sigma_{Ly}^\rho(\mathcal{V}_1) \text{ and } \Sigma_{Ly}^\rho(\mathcal{V}_i) \subset \mathbb{R}^- \text{ for } 2 \leq i \leq l,$$

we can show, that there exists a control set D with nonvoid interior and $x^* \in \text{cl}D$. Here we use the Hartman-Grobmann Theorem and the theory of stable and unstable integral manifolds for nonautonomous differential equations (cf.[1]) for constructing an appropriate control function. The control set D is characterized via the stable and unstable manifolds of (1). We show that there is a control set of the system (2) which is tangential to D at x^*.

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Generalized Solutions for Singular Linear-Quadratic Optimal Control Problems

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We study general linear-quadratic optimal control problems with time and end points fixed. It is well known that singular problems of this type may fail to have a minimizer in the class of "ordinary", for example, square integrable controls, even in cases when the problem has a finite infimum. In such cases, the minimizing sequences of trajectories for the problem may converge to a discontinuous curve, implying that the respective optimal controls contain impulses.

From the functional-theoretic point of view, the problem is one of finding the minimizer of a quadratic functional on an affine subspace, $\mathcal{U}$, of finite codimension in $L_2^k[0,T]$. The approach we suggest is the extension of the functional by continuity onto a larger class of controls and finding its minimizers in the extension (closure) of $\mathcal{U}$.

We find a set of conditions necessary and sufficient for the existence of an affine subspace of finite codimension in $L_2^k[0,T]$ on which the functional has finite infimum. These conditions are algebraic with respect to the initial data of the problem. We show that when these conditions are satisfied, there is a unique extension of the functional onto a certain subspace, H_{γ_r}, of the Sobolev space $H_{-n}^k[0,T]$, (n=dimension of the state space). If the functional has finite infimum in $\mathcal{U}$, then the extended functional has a minimum in the closure of $\mathcal{U}$ relative to the topology of H_{γ_r}. The problem of finding a minimizer of the extended functional is equivalent to a regular linear-quadratic optimal control problem, with final state confined to an affine subspace of $\mathbb{R}^n$ instead of a single point.

Under suitable conditions (finiteness of the infimum and controllability of the system), there exists a minimizer of the extended functional that is the sum of a real-analytic function on $[0,T]$ with a distribution of order $r \leq n$ concentrated at the initial and final instants. The trajectories corresponding to such controls are sums of real analytic functions on $[0,T]$ with distributions of order $(r-1)$. If the optimal control is unique, any minimizing sequence of "ordinary" trajectories must approximate the generalized optimal trajectory in the topology of $H_{-(r-1)}^n[0,T]$.

Assuming controllability of the system and existence of a solution for an appropriate Riccati differential equation, we provide an algorithm for the computation of generalized optimal controls.

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Exact boundary controllability for a forth order parabolic equation

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We consider exact boundary controllability problem for a forth order parabolic equation. We obtain exact boundary controllability results when the initial data is infinitely differentiable.

On dual approach to optimization problems with elliptic operator

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We study the optimization problem formulated on solutions of some family of boundary value problems. This family is generated by different boundaries (i.e. by smooth closed lines on the plane so that each of these lines bounds some coherent corresponding area. All boundary value problems have the differential equation generated by means of the same elliptic operator (in our example, it is Laplace's operator). The object function of this problem describes the uniformity of the vector potential with respect to the vertical component under restriction on the horizontal component values. Thus, we vary the boundary of the area in order to obtain the most uniform field, i.e. vector potential (in sense of the some measure) in the fixed subarea of the considered area.

The problem formulation describes an important practical problem, namely, creating the most uniform magnet field in MRI-tomography devices. To obtain results of the medical MRI-test which are coincident with the real state of patients, it is necessary to generate uniform magnet fields of high rate with respect to the vertical intensity. In addition, it is necessary to restrict the other component into some prescribed limits. It is well known that constant magnet fields have a scalar potential satisfying the Laplace equation.

We propose an effective numerical recursive algorithm which is based on a two-step procedure in each recursion. This procedure contains the boundary value problem with a modified boundary on each recursion (as the first step) and the special auxiliary linear programming problem (as the second step). The last one is intended for two aims: 1) to find the most convenient parts of the boundary for a variation into the current recursion and 2) to find the points of the subarea which influence most essentially an improvement of the object function. A dual form of the auxiliary problem has been a new approach to solving this multidisciplinary problems. The convergence theorem of the method and several results on the degree of convergence are obtained in this work. This approach may also be used in other boundary value problems with elliptic operators.

Flight Autopilot Design for a Remotely Piloted Vehicle using LQ and feedback controller theories

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In the last years, several control theories have been widely developed. They are generally applied to control task such as trajectory tracking and mission optimization. In this paper, we consider a Remotely Piloted Vehicle (RPV airplane), the linear quadratic (LQ) and feedback methods are used to perform the control law of the autopilot.

The equations of the complete airplane motion are first linearized and the flight parameters decoupled. The total motion is equivalent to a sum of superposed modes.

The LQ design of the autopilot consists in the minimization of a cost function. The result is then obtained by the solution of the Riccati equation. When using the feedback theory, the adjustement of the autopilot gains is based on the pole placement procedure. The response of the complete system (nonlinear RPV airplane with autopilot) is finally simulated. The two methods has proved to be well adapted for this design. The results show that the system is stable with adequate values of rapidity and damping ratio for all the flight modes.

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Stabilization of Overhead Traveling Cranes by Nonlinear Feedback

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Overhead traveling cranes are widely used in factories for the transfer of heavy loads. In our previous papers [1], [2], a set of nonlinear differential equations was derived as an exact mathematical model of the crane, which makes three-dimensional motion. In this paper, we present a solution to the problem of global feedback stabilization of the nonlinear system of the overhead traveling cranes. The crane system has three motors, and torques of the motors can be taken as the control inputs to the system. By using Lyapunov's direct method, we derive a nonlinear feedback control law that ensures the global asymptotic stability of the equilibrium state of the crane system. It will also be proved that the stabilizing control is of switching type.

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Local theory for differential equations of Carathéodory-type

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A nonautonomous differential equation $\dot{x} = f(t,x)$ is of *Carathéodory-type* if $f : D \subset \mathrm{IR} \times \mathrm{IR}^N \to \mathrm{IR}^N$ is *measurable in* t (for fixed x) and C^k *in* x (for fixed t). Two examples: *Random differential equations* (RDE) $\dot{x} = f(\theta_t\omega, x)$ driven by a metric dynamical system θ are pathwise of Carathéodory-type [1]. *Bilinear control systems* $\dot{x} = [A_0 + \sum_{i=1}^N u_i(t)A_i]x$ for fixed controls $u : \mathrm{IR} \to U$, $U \subset \mathrm{IR}^N$ compact and convex, are of Carathéodory-type [4]. In this talk we present three new results:

1. For linear systems $\dot{x} = A(t)x$ a compact *dichotomy spectrum* (related to the Sacker-Sell spectrum [4]) is established under very general conditions.

2. *Gap conditions* on the dichotomy spectrum for the smoothness class of integral manifolds [2] and foliations are given.

3. Poincaré's non-resonance condition [3] on eigenvalues known from the autonomous case is generalized to a non-resonance condition on compact spectral intervals of the dichotomy spectrum.

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Control of Distributed Autonomous Robotic Systems

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Self-organized and error-resistant control of distributed autonomous robotic units is achieved by using a specifically constructed dynamical system. The robotic units have to be assigned to targets in a manufacturing environment with obstacles in a cost effective way. Besides the dynamic control of the robotic units in space, the underlying optimization problems are the (two-index) assignment problem or the $\mathcal{NP}$-hard three-index assignment problem.

The used differential equations are based, first, on the selection of modes which appears in pattern formation of physical, chemical and biological systems (see e.g. [1] or [2]). Coupled selection equations based on these pattern formation principles can be used as dynamical system approach to assignment problems [3], [4]. The obtained solutions always respect the constraints of the considered assignment problem [3]. Second, a model of Behavioural Forces is used, which has been successfully applied to describe self-organized crowd behaviour of pedestrians [5].

This approach includes collision avoidance as well as error resistivity, i.e., in systems where failures are of concern, the system covers up for sudden external changes like breakdowns of some robotic units. Furthermore, the control guarantees always feasible solutions, i.e., no spurious states cause the system to fail which is of great importance in industrial applications. Computer simulations verify these results and demonstrate the capability of this approach.

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Lyapunov functions method in practical stability

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Let us consider a system of differential equations

$$\frac{dx}{dt} = f(x,t) \tag{1}$$

where x is a n-dimensional vector, the vector function $f(x,t)$ satisfies the conditions of the existence and uniqueness theorem, $f(0,t) \equiv 0$, $t \in [t_0, T]$.

We denote $x(t) = x(t, x_0, t_0)$ the trajectory of the system (1) at Cauchy condition $x(t_0) = x_0$ and also assume that $\Phi_t \subset R^n$ are compacts, $0 \in \Phi_t$, $t \in [t_0, T]$, $G_0 \subseteq \Phi_{t_0}$, $0 \in G_0$.

Definition. The unperturbed solution $x(t) \equiv 0$ of the system (1) is said to be $\{G_0, \Phi_t, t_0, T\}$-stable if $x(t, x_0, t_0) \in \Phi_t$, $t \in [t_0, T]$ as soon as $x_0 \in G_0$.

Theorem. For the trivial solution of the system (1) to be $\{G_0, \Phi_t, t_0, T\}$-stable it is necessary and sufficient that there exists a continuous nonincreasing on the system (1) solutions Lyapunov function $V(x,t)$ such that

$$\{x \in R^n : V(x,t) \leq 1\} \subseteq \Phi_t, t \in [t_0, T], \tag{2}$$

$$G_0 \subseteq \{x \in R^n : V(x, t_0) \leq 1\}. \tag{3}$$

Corollary. If the trivial solution of the system (1) is $\{G_0, \Phi_t, t_0, T\}$-stable and G_0 is compact then the function

$$V(x,t) = \begin{cases} 1 + \min\limits_{y \in \partial G_0} \rho(\varphi(t, t_0, x), y), x \in \{x \in R^n : \varphi(t, t_0, x) \in R^n \backslash G_0\}, \\ 1 - \min\limits_{y \in \partial G_0} \rho(\varphi(t, t_0, x), y), x \in \{x \in R^n : \varphi(t, t_0, x) \in G_0\}. \end{cases}$$

satisfies all the theorem conditions. Here ρ is a metric equivalent to Euclidean one, $\varphi(t, t_0, x) = x_0$ on the solution $x(t, x_0, t_0)$.

Further, if the set of initial data is starry compact, then it is possible building Lyapunov function which belongs to differentiable functions class. We also estimate the optimal sets of initial conditions in structural forms for linear system and concrete phase constraints [1,2].

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4.3 Dynamics and Algorithms

Organizer : Eusebius Doedel

Key note lecture

Computing Periodic Orbits of Vector Fields

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Periodic orbits are fundamental structures of flows. New methods for computing periodic orbits and their bifurcations with high accuracy are being developed with the use of *automatic differentiation.* This lecture will present these methods, make comparisons with other methods and discuss outstanding problems. Case studies include stiff systems with multiple time scales and the computation of canards.

Invited lectures

Convergence, parameter-identification and C++-code for bifurcation numerics in real life problems

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This talk presents a resume of parts of the results of our *Marburg group.*

In real life problems, the corresponding equations, often PDEs, are known. Numerical computation of bifurcation scenarios is an important task for applications. Results for bifurcation numerics from the last decade, e.g. [4],[5], only cover a few methods, such as specific finite element methods.

We use reaction-diffusion and Navier-Stokes equations as motivating examples. The classical concepts of stability and consistency, yielding convergence, do not apply to the singular situation in bifurcation. So we formulate a new general theory, e.g. [1],[2],[6], modifying these concepts: Stability for bordered systems and the second property, the consistent differentiability, are proved, e.g. for the above examples and for finite difference, finite element and spectral methods. It is particularly important that the stability for bordered systems is the consequence of available properties, e.g. the linearized operator represents a compact perturbation of a monotone operator and for this monotone operator the standard inf-sup-conditions for discretizations are satisfied. Under these conditions the numerical bifurcation scenarios with all the interesting bifurcating solutions, dynamical properties, transformations of the en-foldings back to the original situation, see [2],.. indeed converge to the the original scenarios.

Hence, the huge number of published bifurcation numerics for operator equations, based on bordered systems indeed present converging results: These papers are either directly covered by the above results or can probably be shown to satisfy these conditions.

Based on these results, a C++ code is being developed, such that parameter dependent solution curves can be continued and the bifurcation and dynamical properties be determined. For a new class of problems the appropriate discretization has to be introduced fitting to the side conditions of the code. Then this software yields the above informations for the new problem.

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Numerical bifurcation analysis of delay equations

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In this talk we discuss numerical bifurcation analysis of retarded functional differential equations (RFDEs) with multiple constant delays. Steady state solutions of such equations are independent of the delays and can be found with any standard package for bifurcation analysis of ordinary differential equations. Their stability, however, is determined by the infinite roots of a transcendent characteristic equation. We describe an algorithm which computes the rightmost, i.e. stability determining, roots of this equation. When a steady state solution looses its stability via a Hopf bifurcation a branch of periodic solutions arises. Computing periodic solutions of RFDEs is an infinite dimensional problem because a (periodic) solution is not determined by one point at a given time, instead a function segment with length the maximal delay has to be determined. We discuss numerical computation of the periodic solution and approximation of the dominant Floquet multipliers using a shooting and a collocation approach. We end with a brief discussion of the extra difficulties arising in the study of neutral functional differential equations.

Numerical Investigation of Periodic solutions of multibody systems

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A simple wheelset rolling with constant velocity on a straight track oscillates laterally in a stable periodic motion if its velocity exceeds a certain critical value. The equations of motions of such mechanical multibody systems are differential algebraic equations of index three. Motivated by this application we present numerical methods for the investigation of periodic solutions of multibody systems depending on parameters. A projected collocation method approximates periodic solutions and provides Floquet multipliers, which characterize the stability of an approximated solution. A branch of periodic solutions can be traced using continuation methods, where the Floquet multipliers indicate bifurcations on the branch. Finally, numerical results obtained with a wheelset model are presented.

Ordinary differential equations, differential-algebraic equations and their use in optimization

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We consider the following general smooth optimization problem:

$$\text{Minimize } f(x),\ x \in D \subset \mathbb{R}^N \text{ open, subject to } g(x) = 0 \text{ and } k(x) \geq 0. \qquad (1)$$

Our approach to solve (1) is to integrate an ordinary differential equation (ODE) appropriate to (1). With the slack variables y as well as $z = (x, y)$, $\bar{f}(x, y) = f(x)$,

$\bar{g}(x,y) = (g(x), k(x) - \text{diag}(y)y)$, the projector $Q(z) = D\bar{g}(z)^T(D\bar{g}(z)D\bar{g}(z)^T)^{-1}D\bar{g}(z)$ such an equation reads

$$\dot{z} \;=\; (I - Q(z))(-\nabla\bar{f}(z)) - D\bar{g}(z)^T(D\bar{g}(z)D\bar{g}(z)^T)^{-1}\bar{g}(z). \tag{2}$$

Using a BDF-method to integrate (2) this gives a reliable optimization code, which is particularly well suited for highly nonlinear optimization problems. But in comparison with a classical SQP-method it is not so efficient. This is because one has to solve a linear system of equations to evaluate the right hand side of (2) once. In addition, one is not able to exploit the structures of g and k, since they are destroyed by Q.
To significantly improve the efficiency of that approach, we rewrite (2) as an index 2 differential-algebraic equation (DAE). In this formulation the evaluation of the right hand side is less costly and allows to take advantage of the structures of the constraints. To solve the DAE we use a BDF-code with the speciality that functional iterations instead of Newtons method is used to solve the nonlinear equations in every time step. This substantially decreases the number of evaluations of the first derivatives of f, g and k.
In addition, we discretize the ODE (2) as well as the corresponding DAE with classical one- and linear multistep methods and present discrete convergence results to underpin the power of that approach theoretically.

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Domain on Bifurcation

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The virtual **World of Bifurcation** (WOB) has been established as a www-domain.

WOB combines a database of bifurcation problems with a tutorial on nonlinear phenomena. The name of the domain is `www.bifurcation.de`.

The approach is example-oriented and experimental. The emphasis is on examples that are application-oriented. Most of the examples are dynamical systems.

The first version of WOB has included a set of 12 examples such as a trigger circuit, a reaction in a catalyst particle, Hodgkin-Huxley's nerve model, and voltage collapse in a power system.

It is planned to extend WOB into several directions. Suggestions are welcome. WOB includes figures in postscript format. The figures and other items may be downloaded. Currently large emphasis is placed on developing computer demos that show dynamical systems. A first set of demos is DOS-oriented, a future set of demos will be based on Java.

Contributed talks

Kinetic of growth of Guinier-Preston zones in alloys

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The Guinier-Preston zones directly affect the mechanical and electrical proprieties of metallic alloys. Indeed, for the case of the Al-rich Zn alloys the maximum in resistivity which occurs at a time of the order of minutes for room temperature ageing has been associated with a critical zone size [1]. On the other hand, many studies on Al-Ag single crystals [2] have shown that the GP zones have a faced spherical form for ageing temperatures ranging from 20 to 250C.

In this work, a new mathematical model for analysing the growth of G- P zones in alloys is presented and solved. The new model combines the reaction diffusion equations given by the vacancy pump model [3] and a surface limited mass-transfer boundary condition, which reflects surface-limited mass-transfer between the G-P zones and the matrix. An important result of the new model is the introduction of a new variable, h_m, which is defined as the surface-limited mass-transfer coefficient.

A new implicit finite difference method has been developed for solving the nonlinear Reaction-Diffusion equation [4]. This new approach removes the stability constraints of the explicit treatments presented in the literature and allows one to use considerably larger time steps. The numerical program developed gives the radius temporal variation R(t) of the G-P zones, the obtained results present a good agreement with experimental resistivity curves obtained in Al-Zn alloys [5], for relatively higher times the radius decreases after a critical value R_0 . The influence of the other parameters on the critical size of G-P zones, for example the aging and quenched temperatures, is also studied. Radial concentration of different species and the total concentration decay could also be provided.

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A numerical method for computing unstable quasi–periodic solutions for the 2–D Poiseuille flow

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It is known that the laminar flow in a plane channel becomes unstable to small disturbances for Reynolds number over the critical one, $Re_{\rm cr} = 5772.22$, for $\alpha = 1.02056$, being α the wavenumber. In this work we compute, for several values of α, the branch of unstable solutions which bifurcate from the laminar one. These solutions are periodic on time. Due to the translational symmetry of the channel, they behave as rotating waves, which are steady in the frame of reference moving with the wave speed c. In this way we can find stable and unstable rotating waves, whose stability is examined through the spectrum of the corresponding linearization in the Navier–Stokes equations.

This analysis allow us to confirm previous results with regard to the existence of two Hopf bifurcations for $\alpha = 1.02056$ in the Reynolds–amplitude curve, namely when $Re \approx 4700$ and $Re \approx 7400$. We also verify that the first bifurcation occurs further from the minimum Reynolds of the curve (≈ 4660). From these bifurcations emanates a family of quasi–periodic solutions (modulated waves) which, again due to symmetry, may be viewed as periodic ones when the observer is moving with an appropriate speed. Using techniques from finite–dimensional dynamical systems, we follow, by means of a continuation method, one branch of quasi–periodic orbits. Newton's method, applied to search for fixed points of a suitable Poincaré map of the flow, allows us to find stable and unstable modulated waves.

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Rigorous discretization of subdivision techniques

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Recently efficient set-oriented methods have been proposed for the numerical investigation of dynamical systems [1,2,3]. A basic question arising in implementing these methods is to compute the "set-wise image" of some set: determine all sets of some collection which intersect the image of the given set. In this talk we describe how this discretization question can be tackled rigourosly and furthermore how the numerical effort can be reduced to a minimum. As an example we compute rigorous coverings of stable and unstable manifolds in the Henon map and show that they intersect transversally. We indicate how these methods carry over to ordinary differential equations.

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Two-dimensional global manifolds of vector fields

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We present an algorithm for computing two-dimensional stable and unstable manifolds of three-dimensional vector fields. The main idea is to grow the manifold in concentric (topological) circles. Each new circle is computed as a set of intersection points of the manifold with a finite number of planes perpendicular to the last circle. Together with a scheme for adding or removing such planes this guarantees the quality of the mesh representing the computed manifold. As examples we show the stable manifold of the origin spiralling into the Lorenz atractor, and an unstable manifold in Arneodo's system converging to a limit cycle.

Strong singularities and the continuous Newton method

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The formulation of continuous-time analogues of iterative methods for root-finding and optimization problems can be traced back to Davidenko in 1953, and found a great development in the decade of 1970, with several papers by Abbott and Brent, Boggs, Branin and Smale, among others (see references in [1,3]). Since then, continuous models have been further developed in the context of homotopy techniques and, more specifically, as *trajectory methods* [1]. In the continuous-time setting, a unique model may lead to different iterative techniques, including damped and accelerated methods, through different integration schemes. This approach shifts the convergence analysis of these iterations to a stability study of the continuous system and the discretization method. Continuous models display a better global behavior and, specifically, the difficulties arising when trajectories of Newton-based methods approach singular points in the search of regular roots can be overcome.

The existence of singular roots introduces additional problems. In the context of the discrete Newton method, the basic theorem, due to Reddien [5], states linear convergence when approaching a singular zero from a cone-shaped region with vertex in the root. This result has been later extended in several directions, mainly by Decker and Kelley, Griewank and Keller (see [2,4] and references therein).

In this talk, the local behavior of the continuous-time analogue of Newton method is studied at singular roots x^* of $f \in C^3(\mathbb{R}^n, \mathbb{R}^n)$ through a taxonomy of these into *weak* and *strong* ones and assuming $\nabla \det f'(x^*) \neq 0$. The weak case, defined by the condition $f(x) \in \mathrm{Im} f'(x)$ for all x in the local singular manifold around x^*, characterizes both discrete and continuous-time situations in which a singular root has a spherical attraction domain including singularities, against a common assumption in this context [2]. A theorem on directional stability in the generic case of strong singular roots is then proven via a Lyapunov-Schmidt approach. Forward Euler discretization yields Reddien's result.

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Posters

Analitycal Formulae for Solutions of Difference Equations

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This paper concerns the analytical formulae for solutions of linear difference equations with non-constant coefficients. We present analitycal formulae which are non-recurrent algorithms for obtaining solutions of equations. We use some kind of sum operators and elements of the graph theory.

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4.4 Exponentially Small Phenomena

Organizer : Carles Simó

Key note lecture

Analytical and numerical detection of exponentially small phenomena

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Exponentially small phenomena occur in a wide variety of problems. Always dealing with problems which are analytical, we find these phenomena both in estimates of the remainders in normal forms of vector fields and diffeomorphisms and in averaging methods and, in general, in most of the problems which cannot be detected by using any finite order approach of the classical perturbation theory. They can be revealed by a suitable use of complex variable, by extending the phase space and time to suitable complex strips. They also appear in phenomena like the delay of the bifurcation for fixed points and periodic solutions, in the case of slowly varying parameters, in adiabatic invariance and in the corresponding bifurcation diagrams of all these problems.

We describe different methods to detect this exponential smallness, dealing both with analytical and numerical aspects. In some problems, the use of Melnikov method can give a key to predict the correct order of magnitude. However, some examples will be displayed where a direct use of Melnikov method fails to give the correct estimate. This is specially true in systems involving several fast frequencies. In that case, the correct answer depends strongly on the arithmetic properties of the frequencies, and some bifurcations are induced by the arithmetics.

Numerically it can be a hard task to detect these phenomena. They can be revealed and estimated by going to the complex. Keeping the computations in the reals requires high precision arithmetics.

Invited lectures

Singular separatrix splitting and Melnikov theory for nearly-integrable systems

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Several results are presented, based on [1], [2], [3], about the applicability of the *Melnikov potential* to the detection of the exponentially small splitting of separatrices that takes place in some families of nearly-integrable exact symplectic maps and Hamiltonian flows.

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Exponentially small splitting of separatrices near bifurcations in area-preserving maps

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When the saddle-center bifurcation occurs in an analytic family of area-preserving maps, first a parabolic fixed point appears at the origin and then this point bifurcates, creating an elliptic and hyperbolic fixed point. Separatrices of the hyperbolic fixed point form a small loop around the elliptic point. In general the separatrices intersect transversely and the splitting is exponentially small with respect to the perturbation parameter. We derive an asymptotic formula, which describes the splitting, and study the properties of the preexponential factor.

On accuracy of adiabatic invariant conservation in one-frequency systems

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Consider a Hamiltonian system with one degree of freedom depending on a slowly varying in time parameter. Let for every frozen value of this parameter the phase portrait of the system contain a domain filled with closed trajectories. Let I be the "action" variable in this domain. Suppose that as time tends to $\pm\infty$ the value of the parameter tends to definite limits in such a manner, that the value of the "action" along a trajectory also tends to definite limits $I_{\pm}$. The difference $\Delta I = I_+ - I_-$ is called an accuracy of adiabatic invariant conservation [1]. If the Hamiltonian of the system is an analytic function of its arguments, and the parameter is an analytic function of the "slow time" εt, $0 < \varepsilon \ll 1$, then the value ΔI is exponentially small: $\Delta I = O(\exp(-c/\varepsilon))$, $c =$ const > 0. In [2] the method for the analytic continuation of solutions of the system under consideration is suggested and an estimate from below for the constant c is found. This estimate can not be improved. In the talk the proof of this estimate on the base of a modification of the method of [2] is given.

References

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A new method for measuring the splitting of invariant manifolds

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We study the so-called Generalized Arnold Model (a weakly hyperbolic near-integrable Hamiltonian system), with $d+1$ degrees of freedom ($d \geq 2$), in the case where the perturbative term does not affect a fixed invariant d-dimensional torus. This torus is thus independent of the two perturbation parameters which are denoted ε ($\varepsilon > 0$) and μ.

We describe its stable and unstable manifolds by solutions of the Hamilton-Jacobi equation for which we obtain a large enough domain of analyticity. The splitting of the manifolds is measured by the partial derivatives of the difference ΔS of the solutions, for which we obtain upper bounds which are exponentially small with respect to ε.

A crucial tool of the method is a *characteristic vector field*, which is defined on a part of the configuration space, which acts by zero on the function ΔS and which has constant coefficients in well-chosen coordinates.

It is in the case where $|\mu|$ is bounded by some positive power of ε that the most precise results are obtained. In a particular case with three degrees of freedom, the method leads also to lower bounds for the splitting.

Averaging in multi-frequency slow-fast systems

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We consider the multi-frequency slow-fast system

$$\dot{x} = \omega + \varepsilon f(x, y, \varepsilon), \quad \dot{y} = \varepsilon g(x, y, \varepsilon).$$

Here $x \in \mathbf{T}^n$, $y \in \mathbf{R}^m$, ε is a small parmeter, the constant vector $\omega \in \mathbf{R}^n$ is Diophantine, and the functions f, g are real-analytic. It is well-known that by a change of the variables $X = x + \varepsilon\xi(x, y, \varepsilon), Y = y + \varepsilon\eta(x, y, \varepsilon)$ the system can be reduced to the form

$$\dot{X} = \omega + \varepsilon F_0(y, \varepsilon) + \varepsilon F_1(x, y, \varepsilon), \quad \dot{Y} = \varepsilon G_0(y, \varepsilon). + \varepsilon G_1(x, y, \varepsilon),$$

with F_1 and G_1 exponentially small with respect to ε. The functions F_1, G_1 depend on the change, but they can not be removed in general.

We are interested in estimates for the "smallest possible" F_1 and G_1.

Contributed talks

Time averaging of parabolic partial differential equations: exponential estimates

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We consider a class of systems of nonautonomous parabolic partial differential equations, where the time dependence is a small rapid forcing

$$\begin{aligned}\frac{\partial U}{\partial t} &= \operatorname{diag}(d_1, \ldots, d_n)\Delta U + F(U) + hG(U, \frac{t}{h}, h) \\ U(0) &= U_0 \in H^s_{per}(\Omega, \mathbb{R}^n)\end{aligned}$$

We assume periodic boundary conditions on $\Omega = [0, l]^d$, enough smoothness on the initial conditions by $s > d/2$ and suitable hypotheses of analyticity on the local nonlinearities F and G. Then we can show in [2] a counterpart for parabolic partial differential equations of the well known result by Neishtadt [3] for ordinary differntial equations. By an analytic time dependent coordinate change we get a new equation:

$$\begin{aligned}\frac{\partial V}{\partial t} &= \operatorname{diag}(d_1, \ldots, d_n)\Delta V + F(V) + \tilde{F}(V) + h\alpha(V, \frac{t}{h}, h) \\ V(0) &= U_0 \in H^s_{per}(\Omega, \mathbb{R}^n),\end{aligned}$$

where the nonautonomous remainder α is exponentially small after any time $t > 0$ with a bound of the form $\exp(-\min(c,t)h^{-\frac{1}{3}})$. The correction terms $\tilde{F}$ and the remainder α are nonlocal in space. The proof is based on Galerkin approximation, an adaption of Neishtadt's methods on finite dimensional approximation space and the use of very high regularity. Following Ferrari and Titi one can show, that the solutions $V(t)$ are in Gevrey classes $G^{s/2}_\sigma = D((-\Delta)^{s/2}\exp(\sigma(-\Delta)^{\frac{1}{2}}))$. For positive times the solutions are analytic in the spatial variable and the Fourier modes decay exponentially fast. Applications can be made to the splitting of the homoclinic orbits.

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4.5 Geometric Integrators

Organizer : Jerrold E. Marsden

Key note lecture

Multisymplectic Integrators

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For first order Lagrangian field theories, the existence of the fundamental geometric structures as well as their preservation along solutions can be obtained directly from the variational principle. Using this observation, we constructed in [1] integrators for Lagrangian field theories and identified the structures they preserved, by direct discretiztion of the variational principle.

My talk will be a review of this work.

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Invited lectures

Variational Structure/Numerics of Contact Problems

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The purpose of this talk is to analyze a variational formulation of problems involving contact and friction. Some numerical examples are done using the Coulomb model of friction to illustrate the method.

Geometric Annealing

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The author discusses dynamical approaches to global minimization and suggests an alternative to simulated annealing based on the combination of a temperature-controlled extended Hamiltonian with efficient symplectic integrators.

The problem of finding the global minimum or nearly-global minima of a smooth nonlinear function f of several variables—possibly with respect to one or more constraints—is widespread and fundamental to scientific and engineering research, including, in particular, problems in atomic and molecular physics, design of materials, protein structure, and sphere packing. The most popular methods share certain obvious features: (1) they are iterative in nature, generally requiring a large number of steps to find the global minimum of a complex function, (2) they rely on successive evaluations of f and—often—its gradient to direct the search process, and (3) in the process of approaching the minimum they allow the value of f to increase as well as decrease, in order to surmount local barriers.

Dynamical approaches to the global minimum have been developed based on a gradient flow or a damped Hamiltonian flow. In either model, the local minima of f become exponentially stable equilibria; for this reason, these flows may converge too rapidly to local minima to form the basis of a global minimization procedure. (Although, in the context of mild damping, it is interesting to note that certain numerical discretization methods appropriately damp the symplectic area form.)

In this talk, an alternative approach to global minimization dynamics is developed based on the combination of Nosé's thermostated Hamiltonian with a cooling schedule (as in 'simulated annealing'). Alternative formulations of the Nosé temperature control are derived, with an emphasis on facilitating efficient symplectic integration. The numerical stability of various approaches with respect to variation of the temperature is investigated, and results of experiments with model problems are presented.

Symmetry, pseudospectral methods, and conservative PDEs

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Preserving the structure, for example the symmetry, of continuum objects under discretization has always been part of the field of numerical PDEs. It is well known, for example, that Fourier pseudospectral differentiation matrices are antisymmetric while Chebyshev ones are not. This is intimately related to the linear and nonlinear stability of methods which used these matrices. In this talk I discuss this situation in detail and show how the Chebyshev matrices *are*, in fact, skew-adjoint with respect to an appropriate inner product, and how they can therefore provide conservative discretizations of some wave equations, with corresponding nonlinear stability and Hamiltonian structure. For more complicated (e.g. Euler) equations, normal pseudospectral methods are not conservative, but with care—discretizing the conserved energy and the equation compatibly, and using antialiasing and fast transforms—they can be made to be so.

Symmetry Reduction of Discrete Lagrangian Mechanics on Lie groups

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We show that when a discrete Lagrangian $L : G \times G \to \mathbb{R}$ is G-invariant, a Poisson structure on (a subset) of one copy of the Lie group G can be defined by means of reduction under the symmetry group G of the canonical discrete Lagrange 2-form ω_L on $G \times G$. Alternatively, for the corresponding reduced discrete mechanical system on a Lie group G determined by the (reduced) Lagrangian ℓ we can define a Poisson structure via the pull-back of the Lie-Poisson structure on the Lie algebra g^* by the corresponding Legendre transform. Our main result shows that these two structures coincide and govern the corresponding discrete reduced dynamics. In particular, the symplectic leaves of this structure become dynamically invariant manifolds which are manifestly preserved under the structure preserving discrete Euler-Poincaré algorithm.

Moreover, starting with a discrete Euler-Poincaré system on G one can readily recover, by means of the Legendre transformation, the corresponding Lie-Poisson Hamilton-Jacobi system on g^* analyzed by Ge and Marsden [1]; the relationship between the discrete Euler-Lagrange and discrete Euler-Poincaré equations and the Lie-Poisson Hamilton-Jacobi equations was examined from a different point of view in [2].

References

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Making Waves: Multi-Symplectic Methods for Hamiltonian PDEs

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Various Hamiltonian PDEs can be reformulated as a multi-symplectic PDEs which implies the existence of a local conservation law of symplecticity. We take this conservation law as the starting point for deriving multi-symplectic methods for PDEs in a similar manner as done in case of symplectic methods for Hamiltonian ODEs. We will discuss multi-symplectic schemes based on Runge-Kutta collocation and finite volume methods. Numerical results will be presented for the sine-Gordon equation and a simplified shallow water system.

Contributed talks

Finite-Difference Approximations and Cosymmetry Conservation in Filtration Convection Problem

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We study different conservative finite-difference discretizations of partial differential equations (PDEs) with respect to preservation of cosymmetry property. The concept of cosymmetry was introduced recently by Yudovich [1], [2], and some interesting phenomena was found for both ordinary differential equations (ODEs) and PDEs. It was shown that cosymmetry may be a reason for the existence of continuous family of regimes of the same type and this family is not connected with a symmetry group. Symmetry implies identical spectrum for all points on the family, while the stability spectrum for cosymmetric systems depends on the location of a point. We consider two-dimensional planar Darcy filtration convection of saturated incompressible viscous fluid in a rectangular container filled with porous medium. We are interested in accurate computation of the family of equilibria. The key point here is discretization of nonlinear terms of the PDE, which are represented by the Jacobian. Several finite difference approximations of Jacobian are compared and it was found that the Arakawa scheme provides the most accurate results due to its conservation properties. The numerical results given demonstrate that conservative finite-difference methods can be effectively used to compute the continuous family of stationary regimes. We give some evidence of family degeneration when unappropriate approximations were used, which do not preserve the skew-symmetry of the Jacobian and nullification of the gyroscopic forces.

References

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Equivalence of Newmark and Variational Algorithms and Extensions to Dissipative Mechanical Systems

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We present two related results. First, for conservative systems, we offer an explanation for the unusually good performance of the classical Newmark family of integrators.

This is done by showing that these algorithms, as well as related schemes, are variational in the sense of the Veselov discrete mechanics. Such variational algorithms are examples of symplectic–momentum preserving integrators and exhibit the excellent global energy behavior typical of geometric methods. Analytical results concerning variational integrators can, through the established equivalence, be applied to explain the observed numerical behavior of the Newmark algorithm.

Second, we extend the variational framework to include forced mechanical systems and, in particular, those with dissipation. This is done both by direct discretization of the Lagrange-d'Alembert principle and by the use of a variational formulation of dissipation. We demonstrate that integrators derived in this manner have good numerical behavior in that they correctly estimate the total change in energy and momenta over the integration run. This extends some of the advantages characteristic of geometric integrators for conservative systems to the dissipative realm.

Posters

Constant Temperature Molecular Dynamics using an Extended Lagrangian Method

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We present a new extended phase space method for constant temperature (canonical ensemble) molecular dynamics. Our starting point is the extended Hamiltonian introduced by Nosé to generate trajectories corresponding to configurations in the canonical ensemble. Using a Poincaré time-transformation, we construct a Hamiltonian system with the correct intrinsic timescale and show that it generates trajectories in the canonical ensemble. Our approach corrects a serious deficiency of the standard change of variables (Nosé-Hoover dynamics), which yields a time-reversible system but simultaneously destroys the Hamiltonian structure. A symplectic discretization method is presented for solving the Nosé-Poincaré equations. The method is explicit and preserves the time-reversal symmetry. Extensions are presented for classical spin systems.

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4.6 Numerical Ergodic Theory

Organizer : Michael Dellnitz

Key note lecture

Set Oriented Numerical Methods for Dynamical Systems

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Frequently dynamical systems exhibit complicated temporal behavior. In this case it can be useful to approximate corresponding characterizing statistical quantities such as Lyapunov exponents or invariant measures. For this purpose there have been proposed reliable numerical techniques, which are based on a set-oriented approach rather than on long term simulations of the underlying dynamical system. Here we give an overview of recent developments in this area.

In particular, we explain how to approximate a natural invariant measure, that is, an *SRB-measure* if such a measure exists. These measures provide the information about the frequency by which a typical solution is observed in different parts of state space. The numerical approximation is obtained by adaptive multilevel subdivision strategies based on box coverings of the corresponding invariant set. (Once the invariant measure is known Lyapunov exponents can efficiently be computed by spatial integration, see [1].)

In addition to the stationary statistical behavior obtained by the invariant measure we show how to approximate *almost invariant sets* (cf. [3]). These are regions in state space where typical solutions stay for a relatively long period of time before leaving again. (The concept of *almost invariance* has turned out to be useful in the approximation of so-called *conformations* of molecules of moderate size, see [4].)

There are several convergence results for the (adaptive) numerical methods which are presented here, e.g. the convergence of approximating measures to an SRB-measure. Additionally we justify the techniques for the computation of almost invariant sets by recent analytical results concerning the spectrum of the *Perron-Frobenius operator* (cf. [2]). We illustrate both analytical results and numerical methods by several examples.

References

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Invited lectures

Numerical Problems for Random Dynamical Systems

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We will review the state of the art of the numerical approximation of invariant objects of random dynamical systems, in particular of

– Lyapunov exponents and rotation numbers,
– random point attractors,
– random set attractors,
– invariant manifolds,
– stationary and invariant measures,
– entropy.

Markov modelling of random dynamical systems

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Techniques for estimating the stationary distribution of deterministic systems based on a discrete Markov approximation of the dynamics are well known and have been successfully used in the past. We now extend these techniques to random dynamical systems by defining a suitably averaged Markov model. We find that these constructions are often superior to iterative orbit based methods and for some classes of maps provide rigorous error bounds for our estimates of the stationary distribution.

References

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Finite Representation of the Conley Decomposition

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A finite precision representation of the Conley decomposition of a continuous map acting on a compact space is presented providing a method to find chain transitive sets and isolating neighborhoods in terms of communication classes of a Markov chain associated with a partition of phase space. When an attracting set supporting an SBR measure exists we can prove the convergence of a set approximation of the attractor and show the relation between the invariant measure and the stationary measure of the chain. When the invariant set is Lyapounov stable and transitive, the work of Buescu and Stewart show that the measure is uniquely ergodic. We use this result to show converence of an approximate measure to the SBR measure.

Contributed talks

Numerical Schemes for Stochastic Delay Differential Equations

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We are concerned here with stochastic delay differential equations or *SDDEs*, which generalise both deterministic delay differential equations (DDEs) and stochastic ordinary differential equations (SODEs). This class of problems is of practical interest, e.g. in population dynamics and robotics.

Let $(\Omega, \mathcal{A}, P)$ be a complete probability space with a filtration $(\mathcal{A}_t)$ satisfying the usual conditions; i.e. the filtration $(\mathcal{A}_t)_{t\geq 0}$ is right-continuous, and each $\mathcal{A}_t$, $t \geq 0$, contains all P-null sets in $\mathcal{A}$. Let $W(t)$ be the 1-dimensional Brownian motion given on this space. With $\tau > 0$ we denote by $C := C([-\tau, 0], \mathbb{R})$ the Banach space of all continuous paths $\eta : [-\tau, 0] \to \mathbb{R}$ given the supremum norm $\|\eta\|_C := \sup_{s\in[-\tau,0]} |\eta(s)|$, $\eta \in C$. Let $0 = t_0 < T < \infty$ and consider the stochastic delay differential equation (SDDE) with one fixed lag τ:

$$\left.\begin{array}{rcl} dX(t) &=& f(t, X(t), X(t-\tau))\ dt\ +\ g(t, X(t))dW(t), \quad t \in [0, T] \\ X(t) &=& \Psi(t), \quad t \in [-\tau, 0] \end{array}\right\} \tag{1}$$

where $\Psi(t)$ is an $\mathcal{A}_{t_0}$-measurable $C([-\tau, 0], \mathbb{R})$ valued random variable such that $\mathcal{E}\|\Psi\|^2 < \infty$. The SDDE (1) is interpreted in the Itô sense. If g does not depend on X, the equation has *additive noise*, otherwise it has *multiplicative noise*. An $\mathbb{R}$-valued stochastic process $X(t) : [-\tau, T] \times \Omega \to \mathbb{R}$ is called a *strong solution* of (1), if it is a measurable, sample-continuous process such that $X|_{[0,T]}$ is $(\mathcal{A}_t)_{0\leq t\leq T}$-adapted, $f : [0, T]\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}$ and $g : [0, T] \times \mathbb{R} \to \mathbb{R}$ are continuous functions and X satisfies (1) almost surely. It is well known, that a unique strong solution to Equation (1) exists, if f and g satisfy Lipschitz conditions and a linear growth bounds.

We address the principal issues involved in progressing to the numerical treatment of Equation (1) and our purpose is to ascertain how familiarity with methods used for DDEs and for SODEs can be exploited to yield methods for approximating strong solutions of SDDEs. We will present numerical results. The work is jointly performed with Christopher Baker, at Manchester.

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Young Measures and Invariance

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Invariant measures are an important tool in discussing the statistics of long-term behaviour of dynamical systems. There are two difficulties associated with this approach to dynamics. One is that for discontinuous systems, which are becoming more and more important in applications, the set of invariant measures may prove to be empty. Another problem is that the set of invariant measures may prove to be too big in a sense, and there arises the problem of picking a "physical" invariant measure from this set. In this presentation we make a case for the use of **Young measures**, introduced initially by L. C. Young in the context of chattering controls. Examples of analytic and numerical computation of Young measures for many well-known dynamical systems will be given.

Calculation of the Morse Spectrum of a Dynamical System

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A discrete dynamical system generated by a homeomorphism **F** defined on a vector bundle **E** and linear on fibers is considered. The investigation is motivated by the inspection of Lyapunov exponents of a differential on the tangent space. The Morse spectrum is a limit set of Lyapunov exponents of periodic ε-trajectories as $\varepsilon \to \mathbf{0}$. Our aim is the calculation of the Morse spectrum without any preliminary information on a dynamical system. The investigation is based on methods of symbolic dynamics and all estimates required can be obtained by standard numerical methods. A common scheme of the study is the following. By using a covering of the associated projective bundle **PE** by cells, a dynamical system is associated with the directed graph **G** called a symbolic image. The symbolic image **G** can be considered as a finite discrete approximation of the associated mapping **PF** on the projective bundle. A lot of the effective information on the global structure of a system on the projective bundle can be obtained by the analysis of the symbolic image. Periodic paths on the symbolic image generate the set of Lyapunov exponents that are estimates of Lyapunov exponents of periodic ε-trajectories. The Morse spectrum of the symbolic image is defined as a limit set of Lyapunov exponents of periodic paths. It turns out that the Morse spectrum of a symbolic image is determined by Lyapunov exponents of simple periodic paths. A periodic path is simple if it passes through different vertices. The set of simple periodic paths is finite. This allows one to calculate the Morse spectrum of a symbolic image. The Morse spectrum of a symbolic image is in turn an estimate of the Morse spectrum of the initial mapping $\mathbf{F}$ depending on diameters of covering cells. In this manner a neighborhood of the Morse spectrum of $\mathbf{F}$ can be obtained. By applying a subdivision on the covering, a sequence of embedded neighborhoods of the Morse spectrum converging to the Morse spectrum of $\mathbf{F}$ is constructed. The method discussed above is realized as a computer software. The localization of the Morse spectrum allows one to verify hyperbolicity by mean of a computer.

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4.7 Numerics of Dynamics

Organizer : Celso Grebogi

Key note lecture

Shadowability of Chaotic Dynamical Systems

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In studying their systems, physical scientists write differential equations derived from fundamental laws. These equations are then used to understand, analyze, predict, and control the system's behavior, provided one is able to determine the solutions. As the role of nonlinearity grows in importance for the study of physics, solutions often cannot be obtained in closed form, and numerical solutions must be relied on. Computers are now an integral part of the physicist's *modus operandi*.

A basic question always present when obtaining numerical solutions is to what extent they are valid. This question is especially meaningful when dealing with chaotic dynamics, since local sensitivity to small errors is the hallmark of a chaotic system. Floating-point calculations commonly used to approximate solutions of differential equations or compute discrete maps produce *pseudo-trajectories*, which differ from true trajectories by new, small errors at each computational step. Despite the sensitive dependence on initial conditions, the methods of shadowing have shown that for chaotic systems that are hyperbolic or nearly hyperbolic, locally sensitive trajectories are often *globally insensitive*, in that there exist true trajectories with adjusted initial conditions, called shadowing trajectories, very close to long computer-generated pseudo-trajectories. A dynamical system is hyperbolic if phase space can be spanned locally by a fixed number of independent stable and unstable directions which are consistent under the operation of the dynamics.

In the absence of hyperbolic structure, much less is known about the validity of long computer simulations. Recently it was shown that trajectories of a chaotic system with a fluctuating number of positive finite-time Lyapunov exponents fail to have long shadowing trajectories. In other words, they are globally sensitive to small errors. Such hyperchaotic system has two positive Lyapunov exponents, although finite-time approximations of the smaller of the two fluctuate about zero, due to visits of the trajectory to regions of the attractor with a varying number of stable and unstable directions. The destruction of hyperbolicity caused by this phenomenon leads to global sensitivity – only relatively short pseudo-trajectories will be approximately matched by true system trajectories.

Our discussion of the global sensitivity of trajectories for these non-hyperbolic systems is limited in this talk to the comparison between physical models and computer simulations, but the same questions arise whenever comparing the time behavior of two systems evolving under similar, but slightly different dynamical rules. For example, a natural system and its theoretical *model* differ by modeling errors. In the presence of fluctuating Lyapunov exponents, global sensitivity may lead to trajectory mismatch, in particular when long times are considered. The result is that no trajectory of the theoretical model matches, even approximately, the true system outcome over long time spans.

Invited lectures

Numerical exploration of homoclinic dynamics

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In many instances chaotic behavior in discrete dynamical systems is related to homoclinic and heteroclinic intersections. In this talk we discuss these phenomena from the viewpoint of numerical bifurcation analysis.

First we report on some recent results of J.-M. Kleinkauf [3],[4] on the numerical computation of homoclinic and heteroclinic tangencies in discrete systems. His analysis starts with the observation that these tangencies can be characterized as turnig points of branches of corresponding orbits in sequence spaces. It is then shown how heteroclinic tangencies can be computed numerically in a robust way by solving appropriate finite boundary value problems. Error estimates for the branches are derived including a superconvergence phenomenon for the critial parameter value at which the tangency occurs.

Using these tools we then analyze bifurcation phenomena created through homoclinic tangencies and - more generally - through closed loops of homoclinic and heteroclinic orbits. In a certain sense a homoclinic tangency may be considered as a bifurcation of infinite codimension. We show how this affects the continuation of multi-humped homoclinics and their tangencies and makes it increasingly more difficult to follow them in a reliable way. The above techniques will also be used to extend the methods for evaluating the symbolic dynamics that is created by homoclinic intersections [1]. Finally, we will shed some new light on the well-known result by Fiedler and Scheurle [2] that the discretization of homoclinic orbits generically creates transversal intersections and that these occur in an exponentially small wedge under analyticity assumptions.

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The Computation of Lyapunov Exponents via Spatial Integration

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We propose a new method for the numerical approximation of the largest Lyapunov exponent. This method is based on the computation of a spatial average with respect to an underlying (natural) invariant measure rather than on a long-term simulation of the dynamical system. Thus, this approach can be interpreted as a generalization of the classical power method for eigenvalue problems. We will show that it is particularly advantageous for the detection of so-called *blowout bifurcations* of a synchronous chaotic state, and we illustrate this fact for a system of two coupled Duffing oscillators.

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Modeling and complexity of coupled chaotic systems

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Coupled oscillators are relevant to a large variety of physical and biological phenomena. Arrays of Josephson junctions and coupled solid state lasers are well known examples in physics and electrical engineering. In biology, vital organs such as hearts, auditory, visual, and central nervous systems are complex networks of many small oscillators such as cells and neurons. Coupled equations can also arise from spatial discretization of nonlinear partial differential equations such as the Navier-Stokes equation in fluid dynamics. The dynamics of the fundamental elements, or the individual oscillators in the network, can be either regular or chaotic. Typically, the collective behavior of all the oscillators in the network can be extremely rich, ranging from steady state or periodic oscillations to chaotic or turbulent motions.

The focus of this talk is on modeling and complexity of coupled chaotic oscillators. In particular, it has been known that chaotic dynamics may impose severe limits to deterministic modeling by dynamical equations of natural systems. The obstruction to deterministic modeling is caused by a type of nonhyperbolic behavior in chaotic systems: unstable dimension variability. Roughly, unstable dimension variability means that periodic orbits embedded in the chaotic invariant set have distinct number of unstable directions. Theoretical argument and numerical computations will be presented to show that unstable dimension variability is common in systems of coupled chaotic oscillators. Evidence will also be given indicating that unstable dimension variability may be responsible for complexity observed in these systems.

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Shadowing time for chaotic trajectories

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We explore the statistical aspects of finite-time shadowing in nonhyperbolic dynamical systems. A diffusion model parametrized by information from the finite-time Lyapunov exponent distributions of the system can be used to calculate expected shadowing times and shadowing distances. This explains the mechanism by which long-term shadowing breaks down in cases of "unstable dimension variability", when finite-time Lyapunov exponents fluctuate about zero.

Contributed talks

Dimension and stability of sets embedded in normally hyperbolic invariant manifolds

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We investigate discrete-time dynamical systems on Riemannian manifolds which are generated by smooth maps. Using properties of associated full variational systems and partial variational systems in direction transversal to the considered invariant submanifolds we obtain conditions for orbital stability and instability of such invariant sets. A known stabiliy criterion of Ruelle for periodic orbits of a discrete-time system with invariant measure is formulated in terms of Lyapunov exponents. But in application to concrete systems in synchronization theory such a criterion is not always effective. These difficulties are handled in the present contribution by deriving stability conditions using singular values of the tangent map. A number of these conditions leads to upper or lower estimates of the largest two singular values of the linearized map. The techniques which are developed here are similar to the constructions for continuous-time systems on Riemannian manifolds used in a joint work with G. A. Leonov and A. Noack [1].

Assuming that the discrete-time system is given in feedback control form (see [2]) on the cylinder the conditions for the stability of invariant sets are split into requirements for the linear part (frequency domain conditions) and the nonlinearity. The contribution is also concerned with upper bounds for the Hausdorff dimension of normally hyperbolic invariant sets. The dimension bounds are formulated in terms of singular values of the restricted tangent maps acting in transversal or longitudinal directions [3].

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Verification of chaos in the planar restricted 3–body problem

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We present a computer–assisted method to verify the existence of chaotic behaviour in discrete dynamical systems. Our approach is closely related to the procedure introduced in Stoffer, Palmer [1]. Yet the scheme of [1] is substantially improved leading to a more efficient procedure with a wider range of potential applications. Moreover, the application to the planar restricted three body problem with two primaries of equal masses is offered. As is well known, this problem is described by a four–dimensional system of differential equations. Restricting the flow to an energy surface and defining an appropriate Poincaré section the problem is reduced to a map in $\mathbb{R}^2$ to which our method is shown to apply.

Strictly speaking we do not rigorously establish chaotic behaviour. This is due to the fact that we replace certain rigorous error bounds by what we call realistic estimated upper bounds.

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Wavelet Approach to Nonlinear Dynamics

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We consider a number of dynamical problems which are described by the systems of ordinary differential equations with polynomial nonlinearities and with or without some constraints. We consider variational wavelet approach for constructing explicit solutions of these problems. We have the solutions as a multiresolution expansion in the base of compactly supported wavelets or wavelet packet bases. In the general case we consider biorthogonal wavelet expansions. These solutions are parameterized by solutions of reduced algebraic problems. We consider applications to optimal control problem and nonlinear accelerator physics problems. Also we present applications to a number of Hamiltonian problems and their perturbations. We consider dynamical problems in invariant variational approach via coadjoint orbit picture, semiproducts and metaplectic structure. We construct symplectic, Poisson and quasicomplex structures using generalized wavelets and non-standard (maximum sparse) representations for operators in different coherent, well localized wavelet bases in functional spaces or scale of spaces. We consider applications of our approach to the theory of homoclinic chaos and quasiclassics. Also we consider applications to a number of wave motion problems (nonlinear wave equations) and turbulence problems (Kuramoto–Sivashinsky equations). We consider variational approach for constructing wavelet–Galerkin representations via reduction from initial complicated problems to a number of standard algebraic problems. As a result we obtained explicit representation for well localized coherent structures.

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Remarks on computational aspects of computer assisted proofs of chaotic behavior for ODE's

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I will discuss the rigorous numerics using the computer assisted proofs of chaotic behavior for Lorenz system and Rössler equations [1–3].

The computer part in the above mentioned proofs can be reduced to the following problem:
Compute fast and rigorously with a-priori prescribed error the image of some relatively big rectangle under the Poincaré map for a given ODE.

It should be stressed that we allow for relatively big errors (for example 10% of the size of the important sets in the problem).

In principle this is an easy problem, we just have to discretize both in time (i.e. we use Runge–Kutta , or Taylor method) and in space and then perform finite computations. In the problems mentioned above it required up to 60 hours of computation time to complete the task.

We discuss how the following issues influence the computation time

- the order of the numerical method
- various norms for computation Lipschitz constant of the flow
- Lohner algorithm for error propagation

We show that different approaches lead to drastically different computation times covering range from just minutes to years for the same problem.

The conclusion from these experiments can be formulated as follows:

The most important factor in such computations is propagation of errors due to space discretization. It is reasonable to apply an complicated algorithm (this means that to perform a single time step takes much computer time) if this leads to a significant reduction of the estimate for the Lipschitz constant of the flow.

For example the use of Lohner algorithm in Rössler system can speed up computation by a factor 1000, when compared to original computations from [2].

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Posters

Heteroclinic behavior in rotating Rayleigh-Bénard convection

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We investigate numerically the appearance of heteroclinic behavior in a three-dimensional, buoyancy-driven fluid layer with stress-free top and bottom boundaries, a square horizontal periodicity with a small aspect ratio, and rotation at low to moderate rates about a vertical axis. The Prandtl number is 6.8. If the rotation is not too slow, the skewed-varicose instability leads from stationary rolls to a stationary mixed-mode solution, which in turn loses stability to a heteroclinic cycle formed by unstable roll states and connections between them. The unstable eigenvectors of these roll states are also of the skewed-varicose or mixed-mode type and in some parameter regions skewed-varicose like shearing oscillations as well as square patterns are involved in the cycle. Always present weak noise leads to irregular horizontal translations of the convection pattern and makes the dynamics chaotic, which is verified by calculating Lyapunov exponents. In the nonrotating case the primary rolls lose, depending on the aspect ratio, stability to traveling waves or a stationary square pattern. We also study the symmetries of the solutions at the intermittent fixed points in the heteroclinic cycle.

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E. Applications

5.1 Chemistry

Organizer : Iannis G. Kevrekidis

Key note lecture

New Chemistry and Mathematics

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Chemistry has changed and mathematics plays now an important role in this discipline. Chemical systems provide paradigmatic examples of nonlinear pattern formation described by reaction-diffusion models. Recent experimental developments in heterogeneous catalysis and biochemistry show that in some important cases one must go beyond this class of models. Chemical reactions in soft matter are coupled to structural phase transitions in such condensed systems and physical interactions between reacting particles should therefore be incorporated into a model. Enzymes and other proteins of a living cell form a network of molecular machines whose collective dynamics can show coherence and mutual synchronization, similar to the behaviour found in large populations of coupled nonlinear oscillators.

Invited lectures

Parity breaking bifurcations in reaction-diffusion systems with nonlocal coupling

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We study two component activator-inhibitor reaction-diffusion models with nonlocal coupling. It can be viewed either as a simple model of a pattern forming chemical reaction subject to a feedback with an intrinsic length scale (range of the nonlocal coupling) or as a limit case for a model describing the interaction of a single activator species with two inhibitor species, one with slow and the other with fast reaction dynamics. The nonlocal coupling introduces the possibility of an oscillatory instability with finite wavenumber (wave bifurcation) in addition to the well known stationary bifurcation with finite wavenumber (Turing bifurcation) and an oscillatory instability with zero wavenumber. The linear stability analysis is performed and gives analytic criteria for the occurence of wave bifurcation as well as various codimension-2 bifurcations, out of which we focus on the simultaneous wave and Turing bifurcation.

Amplitude equations of Ginzburg-Landau type are derived for the codimension-2 Turing-wave point and enable us to predict the nonlinear behavior with respect to the appearance of traveling or standing waves and the competition of waves and Turing patterns. Under conditions of bistability of these structures, we observe a new type of pattern, cellular drifting states that represent a finite width inclusion of waves in a Turing pattern background that travels in the direction opposite to the constituting waves. This structures are unstable in the amplitude equations and appear only in a finite distance to the onset of pattern formation. The drifting states have been studied in detail by extensive numerical studies. Their structure is similar to phenomena occuring in experimental and theoretical studies of parity breaking instabilities in stationary cellular patterns. We compare our results to alternative approaches based on symmetry considerations. Further away from the wave or Turing bifurcation, the models exhibit bistable behavior including the parity breaking front bifurcations studied in pure reaction-diffusion systems. Thus, we have studied the transition from this regime of large amplitude localized coherent structures (pulses, fronts) to a regime closer to onset where periodic patterns described by amplitude equations appear. In the intermediate regime, complex traveling patterns and patches of periodic stationary patterns are observed.

Nano-scale Pattern Formation in Electrodissolution

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We investigate the pattern formation dynamics of two important electrodissolution processes-electropolishing and anodization. Electropolishing is a century-old technology that exploits the stability of free-surface dissolution dynamics to smooth metal surfaces. An applied voltage is used to preferentially dissolve microscopic bumps on the metal surface in an electropolishing electrolyte. In contradiction to this old wisdom, our group discovered 5 years ago that electropolishing is actually unstable at submicron scales. More interestingly, using Atomic Force Micrography, this new instability is found to produce very regular hexagonal and stripe patterns beneath the hydrodynamic boundary layer. In the past three years, we have deciphered the electrochemical origin of this new free-surface instability and have derived a simple evolution equation for its dyanamics. Numerical simulation of the equation and bifurcation analyses of the resulting amplitude equations provide the windows of applied voltage and electrolyte concentrations when both regular patterns appear. They are in good quantitative agreement with our measured experimental data. We can potentially use these results to generate nanoscale self-assembly very cheaply.

Anodization is another electrodissolution process that involves the break down of a metal oxide layer under an applied voltage. It is the fundamental mechanism behind pitting corrosion of protective aluminum oxide layers–a major problem in pollution. We have formulated this free-surface problem into a fingering process. The eletric potential satifies the Laplace equation within a thin strip around the growing finger and nonlinear curvature-dependent boundary conditions at the strip boundary due to field-assisted dissolution and double-layer cuvature effects. Using matched asymptotics and numerical conformal mapping techniques, we obtained the critical voltage for the onset of fingering and the finger dimensions as functions of the electrolyte chemistry. The results are favorably compared to our experimental data. They allow us to determine when corrosion of a metal oxide layer occurs and the size of the resulting pits. We can also generate hexagonally arranged pits that, if properly shrunk in dimension, can be used to house quantum dots.

Mathematical issues in modeling chemically reacting systems

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The research area of spatiotemporal dynamics in systems involving chemical mechanisms and their mathematical models (e.g. reaction-diffusion PDEs) is characterized by a fruitful interplay between mathematics and experimentation. The development of spatially and temporally resolved observation and analysis techniques, beyond model validation, provide constant motivation for exploring certain types of PDEs and their solutions. The purpose of this chapter is to chart this interaction, provide a summary of recent cases of cross-fertilization between chemistry and mathematics, and discuss future directions for this research area.

We start by discussing certain classical reaction-diffusion and reaction engineering paradigms (the chemistry involved in the BZ and related reactions as well as the chemistry of catalytic surface reactions like CO oxidation; the CSTR and the tubular reactor as well as reverse flow reactors). Certain elements of numerical techniques for the computer-assisted study of such models are also briefly reviewed.

We then proceed to discuss a number of current issues including

- pattern formation on inhomogeneous and anisotropic media (PDEs with spatially varying coefficients and issues of homogenization, both analytical and numerical);
- pattern formation in spatiotemporally forced systems (PDEs with spatiotemporally varying coefficients and their use in elucidating chemical and biochemical mechanisms);
- issues of local versus longer-range (up to global) interactions and their effect on reaction dynamics;
- complex geometry issues;
- dissipativity and its implications in low-dimensionality of the dynamics;
- the extraction of features (like fronts and pulses) and the derivation of asymptotic dynamic equations for their evolution;
- kinetically based models of reacting media and their dynamics;

- molecular level modeling of reaction dynamics and its real time observation;

We conclude by a discussion of current technological developments (ranging from the technology of microreactor fabrication to wavelet based "real-time" homogenization techniques and to issues of computational chemistry) which, we believe, will affect the way chemically reacting systems are modelled and analyzed in the near future.

Long-Time Dynamics of Dissipative PDEs with Applications to Reaction-Diffusion Systems

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In this talk we will discuss the long-time behavior of dissipative evolution PDEs, with emphasize on models of fluid flows and reaction-difussion systems. In particular, we will survey the results concerning determining degrees of freedom in such systems, such as determining modes and nodes, global attractors and invariant inertial manifolds. Later we will introduce a novel numerical method for integrating dissipative PDEs - the post-processing Galerkin method. We will present numerical experiments that will demonstrate the superiority, in computational efficiency, of this method in comparison to the standard Galerkin and nonlinear Galerkin methods. Finally, we will present a numerical criterion for designing a linear feed-back controller for stabilizing unstable steady states of parabolic dissipative equations including reaction-difussion systems.

Contributed talks

Existence and mass conservation for the coagulation-fragmentation equation

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We prove existence theorem and reveal the cases when the coagulation-fragmentation equation possesses the mass conservation property. From physical point of view the mass conservation is usually treated as the absence of gel point.

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The mathemetical model for the combined oxydational treatment of water with phosphorus-organical pesticidic contaminants

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The scheme of free-radical reactions in water surroundings was proposed for a decomposition of organical contaminants under the combined treatment of ozone, UV and H_2O_2. A system of quadratic ordinary differential equations was built, which describes this system kinetics, an investigation of dynamics and characteristic of its peculiar points was installed. On basis of experimental data the problem of identification was stated and solved, also the problem of system optimal control was stated:

$$\frac{dc_i}{dt} = A_i c + c^T B_i c + U_i, \quad i = \overline{1,n}$$

$$J(U) = \sum_{l=1}^{k} \alpha_l c_l^2(T) \to \min U$$

$$\int_0^T U_2(t)dt = a, \quad \int_0^T U_3(t)dt = b,$$

$$c(0) = c_0.$$

- $c = (c_1, c_2, \ldots, c_n)^T$ -vector of substance's concentrations in water surrounding
- $A_i = (a_1 i, a_2 i, \ldots, a_n i)$ -coefficients vector for the linear part of kinetics differntial equation with number i
- B_i -the matrice of quadratic form, which describes the non-linear part of kinetics differential equation with number i
- $U = (0, U_2, U_3, 0, \ldots, 0)$-the control vector (we can only control concentrations of ozone and H_2O_2)
- $k, (k < n)$ -quantity of pollutants in water surrounding
- T -the final time point of oxydatinal process
- a -quantity of ozone
- b -quantity of H_2O_2
- c_0 - vector of starting conditions

The solving of optimal control problem was conducted by means of Hamilton-Pontryagin's method.

Kinetic scheme reduction in first order differential equation models

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Modelling reaction kinetics in a homogeneous medium usually leads to stiff systems of ordinary differential equations the dimension of which can be large. A well-known approach to reduce the dimension of such systems is the quasi-steady state assumption (QSSA): the derivative of fast variables is assumed to be zero. This procedure requires some knowledge of the underlying chemistry, moreover the corresponding differential system must be explicitely given. In this paper we shall describe and justify a procedure for a local reduction of the dimension of state space which does not require chemical insight as well as an explicit knowlegde of the system in a singularly perturbed form. The mathematical justification is based on the theory of invariant manifolds.

Continuation of Waves in Reaction-Diffusion-Convection Systems via Heteroclinic and Homoclinic Orbits

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Numerical methods for tracking of homoclinic and heteroclinic orbits provide a powerful tool in analysing bifurcation problems of practical importance, such as a sudden appearance (or extinction) of large amplitude oscillations in stirred tank reactors or occurrence of travelling waves in tubular reactors. Typically, such methods are formulated as a special boundary value problem.

We formulate efficient methods for numerical tracking and continuation of homoclinic/heteroclinic orbits with a nonoscillatory approach to the steady state(s) in systems of ordinary differential equations. The method allows for studying waves in spatially one-dimensional systems upon the moving frame coordinate transformation. In the case of homoclinic orbits, the method provides velocity-parameter curves for solitary pulse waves. Tracking of heteroclinic orbits can be used for finding one-parameter families of front waves. A computer implementation of the methods included in a continuation program CONT is applied to two problems: the first one deals with pulse and front waves in a reaction-diffusion system and the second one focuses on shock waves in adsorption columns.

Posters

Bifurcations to mixed mode oscillations in chemical systems

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Mixed mode oscillations consist of alternating large and small amplitude oscillations and are a common phenomenon in oscillatory chemical systems. To study this phenomenon I analyze an extended version of the Boissonade de Kepper model which is a three dimensional Van der Pol Duffing like oscillator and shares some qualitative properties with models derived by first principles from chemical mechanisms. The model is analyzed by means of geometric singular perturbation theory and an adapted version of Melnikov's method, which allows to derive approximating formulas for some of the codimension one and two bifurcation curves.

5.2 Chemotaxis, Cross-Diffusion and Blow-Up

Organizer : Angela Stevens

Invited lectures

On a reaction–diffusion system modelling chemotaxis

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Chemotaxis as the oriented migration of organisms under the influence of chemical substances can be modelled by partial differential equations of reaction–diffusion type of the form

$$\frac{\partial U}{\partial t} = \Delta U - \chi \nabla \cdot (U \nabla V), \quad \frac{\partial V}{\partial t} = \alpha \Delta V - \beta V + \delta U.$$

Here U denotes the population density, V the density of the chemotactic agent. The system is considered for positive initial values U_0, V_0 under homogeneous Neumann conditions on $\mathbb{R}_+ \times \Omega$ where $\Omega \subset \mathbb{R}^2$ is a piecewise smooth domain; α, β, δ, χ are positive constants. The system has the Lyapunov function

$$F(U,V) = \int_\Omega \left\{ \frac{\chi}{2\delta} \left(\alpha |\nabla V|^2 + \beta V^2 \right) + U(\log U - \chi V) \right\} dx$$

i.e. this functional decays along solutions as time increases. If

$$\frac{\delta \chi \|U_0\|_{L_1}}{4\alpha\Theta} < 1\,,$$

where Θ is the smallest interior angle formed by the boundary curves of Ω, then the Lyapunov function remains bounded from below for all times. Under this smallness condition for the initial values there is a (possibly nontrivial) asymptotic state $(U^\star, V^\star)$. For large initial values the system can show blow–up.

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Hyperbolic Models For Chemotaxis

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Parabolic models for chemotaxis (e.g. Keller-Segel type models) are well known and widely studied in the literature. They describe experiments well but are under discussion for several reasons. First they allow for infinite speed of propagation; secondly the relevant parameters (motility and chemotactic sensitivity) are not directly related to the individual movement of the investigated species.

In this talk I will present hyperbolic models and transport equations for chemotaxis. The general transport model (Alt's equation [1]) is based on the individual movement of the particles, and the way they respond to external stimuli. I will present results on local and global existence of solutions of hyperbolic chemotaxis models in one dimension ([2], [3]). In the case of global existence, the asymptotic behavior of solutions is described by parabolic models (*parabolic limit*). Finally, some numerical simulations show interesting behavior like pattern formation and finite time blow up (see [2]) and the formation of shocks (see [3]).

The results on the 1-D model are joint work with Angela Stevens (Leipzig) and Christian Rohde (Freiburg).

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Behavior of Solutions to a System Related to Chemotaxis

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We will talk about blow-up of solutions of the following system related to chemotaxis

$$
\text{(KS)} \qquad \begin{cases} u_t = \nabla \cdot (\nabla u - \chi u \nabla v), & x \in \Omega,\ t > 0, \\ \tau v_t = \Delta v - \gamma v + \alpha u, & x \in \Omega,\ t > 0, \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0, & x \in \partial\Omega,\ t > 0, \\ u(\cdot, 0) = u_0, v(\cdot, 0) = v_0, & x \in \Omega. \end{cases}
$$

Here Ω is a bounded domain in $\mathbf{R}^2$ with smooth boundary $\partial\Omega$, χ, γ, α and τ are positive constants, and u_0 and v_0 are non-negative smooth functions on Ω.

There exists a unique solution (u, v) to (KS) defined on a maximal interval of existence $[0, T_{max})$, which is smooth in $x \in \overline{\Omega}$ and $0 < t < T_{max}$. If $u_0 \not\equiv 0$ in Ω, the solution satisfies that $u(x,t) > 0$, $v(x,t) > 0$ for $(x,t) \in \Omega \times (0, T_{max})$(see [6]). If $T_{max} < \infty$, we can observe that $\lim_{t \nearrow T_{max}} \|u(\cdot,t)\|_{L^\infty(\Omega)} = \infty$, which we mean that the solution blows up in finite time.

We refer to the system (KS) as Keller-Segel model. Keller-Segel model is a classical model to describe the initiation of chemotactic aggregation of cellular slime molds. On the system (KS), the existence of the solutions has been studied by [1,2,6] and the blow-up of the solutions has been studied by [3].

Particularly, Herrero and Velázquez [3] constructed a radially symmetric solution with u which forms a delta function singularity in a finite time. Such a phenomenon is referred to as the chemotactic collapse.

We will talk about some results concerning the chemotactic collapse.

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Systems of Reaction Diffusion Equations Modelling Chemotaxis and Angiogenesis

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In this paper we present a class of reaction diffusion equations based on the theory of reinforced random walks which are being developed to model problems of chemotaxis and tumour angiogenesis.

Results concerning local existence are presented together with a qualitative analysis of global existence and blow-up. We also present a class of exact similarity solutions.

Contributed talks

Numerical Blow-Up Time Convergence for Discretizations of Reaction-Diffusion Equations

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Convergence of the numerical blow-up times of finite-difference discretizations on a uniform mesh of a $1D$ reaction-diffusion model is analyzed and special attention is devoted to symmetric solutions. Also nonsymmetric solutions are considered when the reaction term $f(u)$ is such that $f(0) = 0$. Sufficient conditions for blow-up in such discretizations are established and upper bounds of the blow-up time, which depend on the maximum norm of the initial conditions are provided. These results, although considering a simple situation of blow-up, extend similar ones by Chen [1] and contribute to a better understanding on how to approximate the blow-up time of solutions of reaction-diffusion equations [2].

References

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Some blowup results for the Keller-Segel model

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In this talk we consider the so-called Keller-Segel model

$$\left.\begin{array}{rcll} a_t(x,t) &=& \nabla\cdot(\nabla a(x,t) - \chi a(x,t)\nabla c(x,t)), & x\in\Omega\subset \mathbb{R}^2, \quad t>0 \\ c_t(x,t) &=& k_c\Delta c(x,t) - \gamma c(x,t) + \alpha a(x,t), & x\in\Omega, \quad t>0 \\ \partial a/\partial n &=& \partial c/\partial n \;=\; 0, & x\in\partial\Omega, \quad t>0 \\ a(0,x) &=& a_0(x), & x\in\Omega \\ c(0,x) &=& c_0(x), & x\in\Omega \end{array}\right\} \tag{1}$$

which has been introduced in [1] and [2]. We prove the existence of radialsymmetric initial data for which the solution of (1) blows up in finite or infinite time. For the proof we assume that Ω is a disk, $\gamma = 0$ and

$$\alpha\chi \int_\Omega a_0(x)dx > 8\pi k_c,$$

where $(\alpha\chi \int_\Omega a_0(x)dx)/k_c$ is not equal to a multiple of 8π.

In a second result we show that if the solution blows up in the nonsymmetric case, the blowup must happen at the boundary of Ω, provided that $\Omega \subset \mathbb{R}^2$ is a smooth domain and

$$8\pi k_c > \alpha\chi \int_\Omega a_0(x)dx > 4\pi k_c.$$

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Long-wave unstable thin film equations — blow-up and steady-states

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We consider long-wave unstable parabolic PDEs of the type

$$h_t = -(h^n h_{xxx})_x - B(h^m h_x)_x$$

where $B > 0$, and n and m are positive exponents. Such equations arise in modelling the flow of thin liquid films, driven motion in a Hele-Shaw cell, the aggregation of aphids, and models of Type-II superconductors.

Bertozzi and I conjectured that if $m < n + 2$ then solutions remain L^∞ bounded for all time, if $m \geq n + 2$ and $m \leq n/2$ the solutions can grow at most exponentially in time, and if both $m > n + 2$ and $m > n/2$ then there is initial data whose solution has L^∞ norm becoming infinite in finite time. We prove the subcritical case $m < n + 2$ and discuss a specific critical and super-critical case $n = 1$, $m \geq n + 2 = 3$.

Laugesen and I have classified a large class of nontrivial steady states of PDEs of the above type. Here, we present the steady states, analytical and numerical results on the linear and nonlinear stability of these steady states, and numerical simulations of the initial value problem.

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5.3 Industrial Applications

Organizer : Andreas Schuppert

Key note lecture

Dynamic Systems in Electronics Industry

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Globalization in industrial competition speeds up an increase in systems complexity at reduced innovation cycle times. Systems modeling gained a key role in keeping pace in this race. Increasing system complexity therefore enforces to keep track with scaling-up problems of model evaluation algorithms. Furthermore it is still not possible to fully cover industrial systems analysis and design at abstraction levels describing systems dynamics by discrete modeling paradigms. This leads to a key challenge to deal with continuous models, typically modeled by sets of differential equations, tightly coupled within a discrete model environment, e.g. control structures. Another source of problems originates of top-down design methodologies, trying to design a system by small modifications of already existing designs or by combination from available modules and components. This enforces a component or block-based modeling concept leading to problems in dynamic systems analysis not yet satisfactorily solved. We discuss these core problems in a number of different electronic systems application fields of our company, e.g. microelectronics design, industrial plant design, automotive electronics and railway traffic applications.

Invited lectures

An Efficient Reduced SQP Strategy for Large-Scale Dynamic Process Optimization

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Efficient and robust techniques for the optimization of dynamic chemical processes are presented. In particular, we address the solution of large, multistage optimal control and design optimization problems for processes which can be described by differential-algebraic equation (DAE) models of index one. A new simultaneous solution strategy – based on multiple shooting and partially reduced sequential quadratic programming (PRSQP) – has been developed to fully exploit the structure of large sparse DAE process models. Unlike other simultaneous strategies based on collocation, direct use is made of existing advanced, fully adaptive DAE solvers. Since the integrations on different multiple shooting intervals are completely decoupled, the approach lends itself well to parallel computation.

A practical implementation is provided within the modular optimal control package MUSCOD-II (Leineweber, 1999). The advanced BDF code DAESOL (Bauer *et al.*, 1997) is used for the efficient calculation of the required directional sensitivities. As a typical application, we discuss the optimization of a batch distillation process. This example involves a sparse nonlinear DAE model with several hundred states, leading to a nonlinear programming (NLP) problem with several thousand variables. It is shown that such problems can still be efficiently solved on a standard workstation using the simultaneous strategy presented. Hence optimization strategies of this type are expected to find widespread practical use in the near future.

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Mathematics in industry—the balance between science and economy?

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Although mathematical methods have a significant impact on industrial practice, the acceptance of mathematical science in industry is comparably poor. This is on the one side due to a lack in mathematical education of engineers and economists, on the other side there seems to be a gap between the structure of mathematical research and research and development structures in industry.

In this presentation on the example of mathematical applications in process industry it will be shown how the develpopment of mathematical methods and industrial R&D structures can be brought together.

The scope of the presentation is

- to clarify how R&D projects are structured in industry,
- to show how benefits of mathematical development can be estimated even in an economic sense

- to stimulate the discussion how to improve dynamical systems applications.

Contributed talks

Two-dimensional modelling of pollutant transport in a shallow basin

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Shallow basins are often used for treatment processes such a sedimentation, waste stabilisation, etc. In this paper, a two dimensional numerical model was developed to simulate the sediment and pollutant transport in a shallow basin. The developed model consists of two modules: Hydrodynamic module and sediment/pollutant transport module. A numerical hydrodynamic module based on the St-Venant equations, is resolved by a MacCormack numerical scheme and is used to simulate the circulation pattern in the basin. The obtained flow circulation is used as input to the sediment/pollutant transport module to simulate the transport and dispersion of a pollutant emitted into the basin. To calibrate the numerical model, the distorted scale model of the Windermere Basin (JIAN WU and TSANIS, 1994) was used. In this physical model, the flow visualisation and pollutant transport experiments provide a good calibration. The simulated results were found to be in good agreement with the experimental measurements and the JIAN WU results. With the aid of the validated model, the influence of the construction of dikes on the residence time distributions in the basin were examined.

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On the non-isothermal formation of polymeric fibers

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The focus of this presentation is the nonlinear hyperbolic system governing the one-dimensional formation of viscous or viscoelastic polymeric fibers under non-isothermal flow conditions ("melt-spinning"). Two issues will be discussed in detail (cf. references):

1. We will address the fundamental question of well-posedness of the system in the viscous and viscoelastic regime. The viscoelastic flow will be described through appropriate quasi-linear fluid models in differential form. The problem of well-posedness will be tackled through suitable energy methods.

2. We will furnish a rigorous proof that the stability of the steady-state solution for the linearized equations in the exemplary viscous regime is determined by the eigenvalues of the associated semigroup generator. Since the underlying equations are hyperbolic, this result has to precede any numerical resolution of the spectrum of the semigroup generator.

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Equality of two variable means: reduction to differential equations

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Let Φ be a strictly monotonic continuous function and F be a positive function on an interval I. Then for any $\mathbf{x} = (x_1, \ldots, x_n) \in I^n$ the quantity

$$M_{\Phi\, F}(\mathbf{x}) = \Phi^{-1}\left(\frac{\sum_{i=1}^{n} \Phi(x_i)F(x_i)}{\sum_{i=1}^{n} F(x_i)}\right)$$

lies between the maximum and minimum of the x_i's. It is called *quasiarithmetic mean weighted by the weightfunction F*. The equality problem for two such means:

$$M_{\Phi\, F}(\mathbf{x}) = M_{\Psi\, G}(\mathbf{x}) \quad (\mathbf{x} \in \mathbf{I^n})$$

was solved by Bajraktarević [1] if $n \geq 3$ is *fixed* and the functions are twice differentiable. Aczél and Daróczy [2] solved the same functional equation, assuming that $n\,(\geq 2)$ is *arbitrary* and the functions are continuous.

The aim of this paper is to solve the $n = 2$ *variable equality problem*, assuming, that the functions involved are *six times differentiable.*

We can easily rewrite this problem in the form

$$E(x,y) := \frac{\varphi(x)f(x)+\varphi(y)f(y)}{f(x)+f(y)} - \varphi\left(\frac{g(x)x+g(y)y}{g(x)+g(y)}\right) = 0 \quad (x, y \in J)$$

where $J = \{\Psi(x) \,:\, x \in I\}, g = G \circ \Psi^{-1}, f = F \circ \Psi^{-1}, \varphi = \Phi \circ \Psi^{-1}$. First we obtain a system of differential equations from $E_{xx}(x,x) = 0, E_{xxyy}(x,x) = 0, E_{xxxyyy}(x,x) = 0$ (here the subscripts denote partial derivatives, the first, third and fifth order derivatives of E do not give independent equations).

These derivatives were calculated by the software package MapleV. Eliminating from our system the functions g, f we end up with the equation

$$4U'''U^2 - 18UU'U'' + 15(U')^3 = 0$$

where $U(x) = 4\frac{\varphi'''(x)}{\varphi'(x)} - 6\left(\frac{\varphi''(x)}{\varphi'(x)}\right)^2$. Finding U is relatively simple. To determine φ we have to solve, among others, four Riccati equations. Then we can find g, f. The functions so obtained are also solutions of the functional equation, provided that we make suitable restrictions on their domains. The functional equation has 33 families of solutions and 32 of them are new, compared to the case $n \geq 3$.

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Non-Newtonian Fluids

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The investigation of the motion of viscoelastic fluids has been largely studied by many authors but the rigorous mathematical analysis of compressible viscoelastic fluids has been only recently considered.There exists a proof of the existence of the slightly compressible fluids of the steady motion given by Talhouk [6] and the existence of classical solution of the steady motion of compressible viscoelastic fluids of White-Metzner type in a bounded domain was showed by Sy [5].

The aim of this lecture is to show the existence of classical solution of the steady motion of viscoelastic compressible fluids of Oldroyd type in an exterior domain. Further, the existence and asymptotic behaviour of steady motion of Maxwell compressible fluids will be shown as well as the existence of weak solution of the unsteady motion of Oldroyd type. This is a joint work with A.Sequeira, J.H. Videman, see [1–4].

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5.4 Mechanics

Organizers : Jerrold E. Marsden, Jürgen Scheurle

Key note lecture

Dynamical Systems, the Three Body Problem, and Space Mission Design.

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This talk concerns the application of dynamical systems techniques to the problem of heteroclinic connections and resonance transitions in the planar circular restricted three-body problem. These related phenomena have been of concern for some time in topics such as the capture of comets and asteroids and with the design of trajectories for space missions such as the *Genesis Discovery Mission.*

The main new technical result reported on in this talk is the numerical proof of the existence of a heteroclinic connection between pairs of periodic orbits, one around the libration point L_1 and the other around L_2, with the two periodic orbits having the same energy. This result is applied to the resonance transition problem and to the explicit numerical construction of interesting orbits with prescribed itineraries.

The point of view developed is that the invariant manifold structures associated to L_1 and L_2 as well as the aforementioned heteroclinic connection are fundamental tools that can aid in understanding dynamical channels throughout the solar system as well as transport between the "interior" and "exterior" Hill's regions and other resonant phenomena.

Using these tools, a new technique for constructing missions, such as a petit grand tour of the moons of Jupiter will be given. Other issues such as the use of variational integration algorithms and optimal control techniques for low thrust missions will also be discussed.

Invited lectures

Quasi-Periodic Motions of the Perturbed Euler Top

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Let the (rotational) motion of a rigid body, fixed at one point, be perturbed by a weak external force field. A normal form approach yields an integrable approximation, and the 2-torus action induced by the unperturbed system allows to reduce to one degree of freedom. This reduced system describes the (approximate) motion of the angular momentum in space. In the same way that the domains of rotational and librational motions of the angular momentum are separated by stable and unstable manifolds of hyperbolic equilibria and shrink down to elliptic equilibria, the invariant 3-tori of the integrable approximation are organised by the invariant 2-tori. This extends to the Cantor families of persistent invariant tori of the original perturbed system.

The Hamiltonian structure of the dynamics of ideal liquid bridges

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A *liquid bridge* is a fluid drop that is trapped between two plates. The state of the bridge is determined by the position of the free boundary Σ of the drop and its velocity field v. In [2] and [5] a Hamiltonian formulation was introduced for the dynamics of an ideal inviscid liquid bridge moving under the influence of surface tension and adhesion forces. In this model the contact lines of Σ with the two plates are allowed to move freely along the plates. The Hamiltonian formulation has been used in [3] to study the stability of rigidly rotating liquid bridges with cylindrical shape and bifurcations from the family of rotating liquid cylinders which occur upon variation of the angular velocity of the bridge or its angular momentum, respectively. In [1] a model is introduced to describe axisymmetric potential flow of ideal inviscid liquid bridges with *fixed* contact lines. In this talk we present a Hamiltonian formulation of the equations of motion of an ideal inviscid liquid bridge with fixed contact lines that moves under the influence of surface tension acting on its free boundary Σ. We do not assume that the velocity field of the fluid comes from a potential or that the velocity field is axisymmetric. The configuration space of the liquid bridge in the Lagrangian description has the structure of an infinite-dimensional Hilbert manifold (compare [4]). Using the methods of [3] and [6] we study the stability of rigidly rotating liquid bridges with respect to perturbations that fix the contact lines of the bridge and bifurcations from the family of rigidly rotating liquid cylinders.

References

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Singular relative normal modes for nonlinear Hamiltonian systems

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An estimate on the number of relative periodic orbits around a relative equilibrium in a Hamiltonian system with continuous symmetry is given. This estimate depends in general on the topology of the singular reduced spaces, except for a few particular cases that are also analyzed.

Symmetric Hamiltonian Systems

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Singular reduction tools to study the dynamics induced by a G–invariant Hamiltonian are introduced. Several local and global approaches to the reduction problem are presented and compared. The so–called *reconstruction equations* are discussed in order to provide a local picture of the equations that govern G–Hamiltonian dynamics.

References

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Relative Equilibria of Molecules

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This talk will describe joint work with Igor Kozin (Warwick) and Jonathan Tennyson (UCL) on the role that relative equilibria can play in the prediction and interpretation of molecular spectra. The presentation will concentrate on the example of the molecular ion H_3^+. The global relative equilibrium bifurcation diagram for this example will be described and 'harmonic' quantization and induced representation theory applied to predict how the energies and symmetries of the lowest lying quantum states change as angular momentum increases. A brief description of the possible spectral manifestations of Hamiltonian Hopf bifurcations of relative equilibria will also be given.

References

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Examples of Relative Equilibria

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Analyzing the dynamics of a mechanical system usually starts with computing equilibria or, in other words, steady state solutions and studying their stability properties. If a system has symmetries in the sense that there exists a group of transformations in phase space under which the equations of motion are covariant, then the class of most basic solutions extends to so-called relative equilibria. The talk focuses on recent existence, stability and bifurcation results for relative equilibria of examples for both, dissipative and conservative mechanical systems.

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Contributed talks

On the influence of the kinetic energy on the stability of equilibria of natural Lagrangian systems

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We consider natural Lagrangian systems (T, Π) on $\mathbb{R}^2$ described by the equation $\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} = -\frac{\partial \Pi}{\partial q}$, where $T(q, \dot{q})$ is for all $q \in \mathbb{R}^2$ a positive definite quadratic form in $\dot{q}$ and $\Pi(q)$ has a critical point at 0. We are concerned with the question, whether the kinetic energy can influence the stability properties of the equilibrium 0. We show that there exist a C^∞ potential energy Π and two C^∞ kinetic energies T and $\widetilde{T}$ such that the equilibrium $q(t) \equiv 0$ is stable for the system (T, Π) and unstable for the system $(\widetilde{T}, \Pi)$. We conclude that for C^∞ natural systems the kinetic energy can influence the stability. In the analytic category this isn't true.

References

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On the stability of relative equilibria of tethered satellite systems

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We consider the stability problem of the orbital motion of a system of two satellites joined by a flexible continuous elastic tether, which can have a length up to 100 km. The system moves under the action of the gravitational field of a perfectly spherical Earth

[1]. This stability problem is quite complicated, because an infinite dimensional system, described by a set of coupled nonlinear ordinary and partial differential equations, must be analysed. Analytical solutions may be expected only in special cases.

One special case is given by a circular orbit of the system's center of mass. Due to the $O(3)$-symmetry there exist relative equilibria, which correspond to equilibria in properly rotating frames. These states are of great practical interest. Their stability will be investigated by means of the reduced energy momentum method [2] since by symmetry, besides energy, also the angular momentum is conserved.

We will distinguish between two different types of relative equilibria, existing for the considered system. One are the so-called trivial relative equilibria, for example, the practically most important state where the tether is in its radial configuration. The nontrivial (wavy) tether configurations require active control of the motion of at least one of the satellites. By means of the application of the reduced energy momentum method we show that the trivial radial relative equilibrium state is stable provided the tether is not too long. For the nontrivial relative equilibria we formulate a bifurcation problem, where we vary the position of one of the satellites relative to the other and check the stability of the resulting configuration of the tether, which is supposed to be longer than the straight distance between the two satellites.

Numerical methods have to be applied, first, in the calculation of the relative equilibria by Finite Differences and, further, in the evaluation of the stability condition following from the reduced energy momentum method. This is done by formulating an eigenvalue problem.

References

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Evolution of rotations of a rigid body under the action of perturbed moment

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The authors investigated perturbed rotational motions of a rigid body, similar to regular precession in the Lagrange case, under the action of the moment that is slowly changed in time. The average system of equations of motion is obtained in the first approximation. Examples are considered. The qualitative features of the motion are noted. Perturbed rotational motions of a rigid body, similar to regular precession in the Lagrange case, when the restoring moment depends on the rotation angle are investigated. The averaged system of equations of motion is obtained and investigated in the first and second approximations. Specific mechanical models of the perturbations are considered.

Nonlinear stability of periodic and relative periodic orbits in Hamiltonian systems

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The classical Dirichlet criterion on the stability of equilibria in Hamiltonian systems is generalized to handle the orbital stability of periodic orbits. The result obtained is adapted to the study of the stability of relative periodic orbits in Hamiltonian systems with symmetry.

References

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Posters

An Explicit Smale Horseshoe for Generic Quadratic Maps

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The aim of this work is to present some new results regarding the distribution of homoclinic and periodic points in generic quadratic mappings. We show that it is possible to obtain asymptotic formula for the coordinates of such points. The main feature of these formula is that they depend only on the eigenvalue of the fixed point and give the coordinates in terms of integers.

5.5 Models in Biology, Medicine and Physiology

Organizer : Philip Maini

Key note lecture

From microscopic models of individuals to continuum chemotaxis equations

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A characteristic feature of living systems is that they sense the environment in which they reside and respond to it. The directed motile response of organisms to chemical cues is called *chemotaxis*. Frequently this individual response is manifested by the collective movement of a population up a gradient of concentration of some *chemoattractant*. Smaller organisms such as bacteria frequently rely on some form of *kinesis*, which involves modulation of either the speed or the duration of movement in response to environmental conditions. Both taxes and kineses are often described by macroscopic chemotaxis equations of the form

$$\frac{\partial \rho}{\partial \tau} = \nabla \cdot (D\nabla\rho - \rho\chi(c)\nabla c), \tag{1}$$

where ρ is the density of 'particles' and c is the attractant density, even though the stochastic processes that describe movement at the single-organism level may be quite different. Chemotaxis equations have been derived from a variety of standpoints, and we will discuss several of these. In particular, we show how these can be obtained either from reinforced random walks on a lattice [1], or from a transport equation that describes the velocity jump stochastic process [2].

The simplest version of the mathematical model describing the velocity jump process leads to the equation

$$\frac{\partial}{\partial t}p(x,v,t) + v\cdot\nabla p(x,v,t) = -\lambda p(x,v,t) + \lambda\int_V T(v,v')p(x,v',t)dv', \tag{2}$$

where $p(x,v,t)$ denotes the density of particles at spatial position $x \in \Omega \subset \mathbb{R}^n$, moving with velocity $v \in V \subset \mathbb{R}^n$ at time $t \geq 0$. We first show how to derive a diffusion equation in the diffusion limit of (2), using only a generalized Perron-Frobenius property of the integral operator. In the diffusion limit the diffusion matrix D is not diagonal in general, and we discuss necessary and sufficient conditions under which D reduces to a scalar times the identity. We also show that to derive chemotaxis equations such as (1) we must require suitable conditions on the strength of the bias due to the external field, we give an estimate for the accuracy of the limit equation, and we show how to obtain higher order approximations.

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Invited lectures

Bifurcation analysis of an orientational aggregation model

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We consider the following integro-differential equation for the development of a function f on the circle $S^1 = \mathbb{R}/\mathbb{Z}$ over time.

$$\dot{f}(\theta) = -\int_{S^1} W[f](\theta,\psi) f(\theta)\, d\psi + \int_{S^1} W[f](\psi,\theta) f(\psi)\, d\psi \tag{1}$$

Here W is a functional mapping functions on S^1 to functions on $S^1 \times S^1$. We choose the following form for W.

$$W[f](\theta,\psi) = \int_{S^1} h(\phi-\theta)\, G_\sigma(\psi-\theta-v(\phi-\theta))\, f(\phi)\, d\phi\,, \tag{2}$$

where G_σ is a periodic Gaussian of deviation σ, $v : S^1 \longrightarrow S^1$ is an odd function, and $h : S^1 \longrightarrow \mathbb{R}_{\geq 0}$ is an even function of total mass $\int_{S^1} h = 1$.

Equations (1, 2) describe orientational aggregation of particles (e.g., acting filaments) by a jump process, where a particle located at an angle θ interacting with another particle located at an angle ϕ changes its orientation instantly to an angle ψ. This new angle is chosen relative to the locations of the interaction partners; its preferred value is given by v, and the accuracy of the movement is measured by σ. The rate (or probability) of interaction depends on the relative angle $\phi-\theta$; it is given by h.

We study the bifurcation behaviour of the model (1, 2) in some detail. In particular, we show that when there is a generic bifurcation from the homogeneous distribution (i.e., exactly one of the eigenmodes changes stability), there is an analytic branch of stationary solutions emanating from the bifurcation point whose power series expansion can be computed explicitly. Moreover we show that our model can exhibit a wide variety of types of dynamical behaviour, for example travelling waves, periodic solutions not of travelling wave type, and solutions combining characteristics of both.

Modelling the cellular response to multiple guidance cues

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The chemotactic response of a cell population to a single chemical species has been characterized experimentally for many cell types and has been extensively studied from a theoretical standpoint. However, cells frequently have multiple receptor types and can detect and respond chemotactically to more than one chemical. How these signals are integrated within the cell is not known, and a macroscopic phenomenological approach is therefore often adopted. In this talk, I present chemotactic models based on partial differential (chemotaxis) equations for cell movement in response to multiple chemotactic cues. The derivation generalizes the approach of Othmer and Stevens, [1], who have recently developed a modeling framework for studying different chemotactic responses to a single chemical species. The importance of such a generalization is illustrated by the effect of multiple chemical cues on the chemotactic sensitivity and the spatial pattern of cell densities in several examples. The model can be shown to generate the complex patterns observed on the skin of certain animal species, and used to model a variety of biological phenomena, including patterning in bacteria populations and axonal guidance during development of the nervous system.

References

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Contributed talks

Bifurcations of traveling wave solutions in population models with taxis

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We study the conceptual polynomial model of population with auto-taxis:

$$P_t = F(P) + (P_r/Fi_1(P)/Fi_2(S)S_r)_r, \qquad (A)$$
$$S_t = T_1(P)T_2(S)$$

where P, S are correspondingly density of population and substance produced by population, functions $F(P)$ and $T(P,S) = T_1(P)T_2(S)$ describe the kinetics of population and its substance, constant $D > 0$ is a diffusion coefficient and function $F(P,S) = /Fi_1(P)/Fi_2(S)$ is cross-diffusion coefficient. Under condition of $/Fi_2(S)T_2(S) = const$ (see Othmer, 1997) we describe all principal wave solutions $P(r,t) = P(r+ct) = p(x)$ and corresponding wave regimes of the model arising with non-linear autotaxis for linear, quadratic and cubic local kinetic function $F(P)$. We describe also rearrangements of such regimes with change of parameters of function F and the velocity c of wave propagation.

Investigation of the model (A) is based on transition to corresponding wave ODE system and systematic using of bifurcation diagrams of normal forms ODE to which wave systems are reduced by linear transformations.

Some possible applications of obtained results to study of dynamics of ameba populations, forest insect outbreaks, etc. are discussed.

Homoclinic bifurcation in a Hodgkin-Huxley Model of Thermally Sensitive Neurons

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Bursting and irregular spiking activity of neurons is often observed in electrophysiological experiments. Recently, several studies give strong arguments that this irregular behavior can be explained in terms of low-dimensional dynamics. To investigate the spiking activity of thermally sensitive neurons we study global bifurcation phenomena, in particular homoclinic bifurcations, in a modified Hodgkin-Huxley model. This model has been proposed by Braun et al. [1] and mimics the action potential observed in electroreceptors from dogfish [1], catfish [2], from facial cold receptors [3] and hypothalamic neurons of the rat [4]. Using the temperature as the control parameter, a large region of chaotic behavior is identified. In a certain temperature region we obtain a sudden increase in the Poincaré return time. We provide numerical evidence that this increase can be explained as a result of a homoclinic bifurcation related to a fixed point of saddle-focus type in the four-dimensional state space. The transition is accompanied by intermittent behavior which obeys the universal scaling law for the average "laminar phase" with the distance from the homoclinic bifurcation. Furthermore, we discuss the influence of noise on the homoclinic bifurcation in the modified Hodgkin-Huxley model to compare our results with observations from electrophysiological experiments, which are always contaminated by noise. In such experiments, the only measurable quantity are interspike intervals which can be regarded as a direct measurement of the Poincaré return time. We present experimental results of interspike interval measurements taken from the crayfish caudal photoreceptor, which qualitatively demonstrate the same bifurcation structure.

References

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General models of size-dependent population dynamics with nonlinear growth rate

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We shall investigate the size structured population models having the birth and aging functions of general type with the growth rate depending on the individual's size and total population. The models are described as the following initial boundary value problem with nonlocal boundary condition:

$$\begin{cases} u_t + (V(x, P(t))u)_x = G(u(\cdot,t))(x), \quad x \in [0,l),\ 0 \le t \le T, \\ V(0, P(t))u(0,t) = C(t) + F(u(\cdot,t)), \quad 0 \le t \le T, \\ u(x,0) = u_0(x), \quad x \in [0,l), \\ P(t) = \int_0^l u(x,t)dx, \quad 0 \le t \le T, \end{cases}$$

where the unknown function $u(x,t)$ stands for the density of the population of size x at time t and $l \in (0,\infty]$ is the maximum size, and so $P(t)$ is the total population at time t. The mappings F and G correspond to the birth and aging functions respectively. The function $V(x, P(t))$ represents the growth rate depending on the size x and the total population $P(t)$ at each time. The function C describes the inflow of zero-size individuals from an external source.

Our aim is to generalize the results of A. Calsina and J. Saldaña [1], where they treated the birth and aging functions of the Gurtin-MacCamy type:

$$\begin{aligned} F(u(\cdot,t)) &= \textstyle\int_0^l \beta(x, P(t))u(x,t)dx, \\ G(u(\cdot,t))(x) &= -m(x, P(t))u(x,t). \end{aligned}$$

This assumption is essential in their arguments. We deal with the birth and aging functions of general type as treated in [2] by G. Webb in the age dependent case (which is equivalent to the case $V \equiv 1$) and we show the existence of the solution.

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[1] A. Calsina and J. Saldaña, A model of physiologically structured population dynamics with a nonlinear individual growth rate, *J. Math. Biol.*, **33**:335–364, (1995).

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Modeling of heterogeneous populations

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Individual-based models of dynamics of heterogeneous population with distributed values of parameter and initial conditions of variables are suggested. The resulting model represents the system of ODE (note that under standard approaches the dynamics of heterogeneous population is described usually by partial differential equations). The formulae for calculation of current average values and dispersions of phase variables and parameters with the help of auxiliaries variables satisfying to a system of equations in variations are obtained. Two main examples are considered: 1) the phenomenon of formation of patterns (known as gap, locus, cenon) in tree populations from initial heterogeneous subpopulation of trees; 2) the models of number dynamics of population with distributed values of a reproduction factor and/or environment parameters. The interesting new effect was founded even for simple population model by Alle' type. Namely, even small non-zero dispersion of environment parameter can result to probably short-lived, but having large amplitude number outbreaks. The research was supported by grants RFFI N 98-04-48502 and 99-04-00234.

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Nonlinear mathematical model of pulsed-therapy of heterogeneous tumors

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The impulsive models arise, generally, in the description of phenomena in the applied sciences subjected to abrupt external changes, where the time of the change can be neglegted, and can be considered as a jump in the simulation of the phenomena under study.

More specifically, we consider the following heterogeneous tumor model

$$\dot{x} = r_1(x,y)\,x \tag{1}$$

$$\dot{y} = r_2(x,y)\,y \tag{2}$$

$$x\left(t_n^+\right) = \eta_n\left(D, x\left(t_n\right), y\left(t_n\right)\right) \tag{3}$$

$$y\left(t_n^+\right) = \theta_n\left(D, x\left(t_n\right), y\left(t_n\right)\right) \tag{4}$$

where D is the drug dose administered, x and y are, respectively, the biomass of sensitive cells and drug resistant cells. The values $r_1(x,y)$, $r_2(x,y)$ are, respectively, the growth rates of sensitive and drug resistant cells. The values $\eta_n(D, x(t_n), y(t_n))$ and $\theta_n(D, x(t_n), y(t_n))$ are, respectively, the biomass of sensitive and resistant cells which survive after the nth drug dose D administered at the moment t_n, the sequence (t_i) is strictly increasing.

Travelling wave solutions for reaction diffusion equations

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As it is well-known, the following general reaction diffusion equation

$$\frac{\partial u}{\partial t} + \frac{\partial h(u)}{\partial x} = \frac{\partial}{\partial x}\left[d(u)\frac{\partial u}{\partial x}\right] + g(u) \tag{1}$$

frequently occurs in many biological and chemical models (see i.e. [2] and [3]). It takes into account both a non-linear convection effect introduced by the term $\frac{\partial h(u)}{\partial x}$ and a density dependent diffusion given by the function $d(u)$.

In [1] we provide results on the existence of *travelling wave solutions*, that is solutions $u(t,x)$ of equation (1) such that $u(t,x) = u(z)$ with $z = x + ct$. The constant c stands for the *speed* of the wave. To be physically realistic $u(z)$ has to be bounded for all z and non-negative. Therefore, the existence of bounded travelling fronts for equation (1) essentially reduces to the study of the following boundary value problem on the real line

$$\begin{cases} ((d(u)u')' - ((c + h'(u))\,u' + g(u) = 0 \qquad \left(' = \frac{d}{dz}\right) \\ u(-\infty) = 0, \quad u(+\infty) = 1 \end{cases} \tag{2}$$

where we stress that also the real constant c is an unknown of the model.

Using lower and upper solutions techniques, we get an existence and uniqueness result of monotone solutions of (2) for any speed $c \geq c^*$, and provide an estimate for the threshold value c^* .

Main Result *Assume that h', g, d are continuous in [0,1], with* $\min d(u) > 0$. *Then, if $D^+(dg)(0) < +\infty$, there exists a constant c^* with*

$$2\sqrt{D_+(dg)(0)} - h'(0) \leq c^* \leq 2\sqrt{\sup_{u\in]0,1]} \frac{d(u)g(u)}{u}} - \min_{u\in[0,1]} h'(u)$$

such that (2) admits monotone solutions if and only if $c \geq c^*$. Moreover, the solution is unique (up to a translaction of the origin).

Whereas, if $D_+(dg)(0) = +\infty$, problem (2) does not admit monotone solutions for any speed c.

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The Basic Reproduction Number for STD's and Linear Chains

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In models for infectious diseases, the basic reproduction number is the crucial parameter which determines the stability of the uninfected situation, i.e. the possibility of an outbreak. In simple situations it depends in a monotone way on the infectivity. Non-monotone behavior may occur in diseases where infectivity depends on time since infection and where transmission depends on social structure [1]. A typical application is the HIV infection where transmission rates depend on existing pair bonds and infectivity changes drastically over time.

A class of epidemic models with pair formation and infectivity depending on time since infection is defined within the framework of hyperbolic PDE's. The reproduction number is derived and leads to a non-autonomous cyclic linear reaction chain

$$x_1 \to x_2 \leftrightarrow x_3 \leftrightarrow \cdots \leftrightarrow x_n \leftrightarrow x_1$$

where additionally there is an outflow from each state with a rate $\mu(t)$. Note that no transition is possible from state two to state one. Let furthermore $\alpha_1(t)$ be the transition rate from state one to state two. If we choose as initial values $x_1(0) = 1$ and $x_i(0) = 0$ for $i = 1, .., n$, the reproduction number can be represented as the number of transitions from state one to state two,

$$R_0 = \int_0^\infty \alpha_1(\tau) x_1(\tau)\, d\tau.$$

The monotonicity of R_0 with respect to the infectivity function is equivalent to the the monotonicity of $\int_0^\infty \alpha_1(\tau)x_1(\tau)\, d\tau$ with respect to an increase of the transition rates from the states i to states $i + 1$, and a decrease of the transition rates from the states $i + 1$ to states i.

The crucial idea of the proof of this monotonicity theorem is to reformulate the solution of the ODE as an integral of sequences, which mimic a stochastic random walk. We do not directly use the stochastic process but a construction that has similar properties, such that it is possible to apply methods from the theory of stochastic particle systems [2].

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Model reduction by extended quasi-steady-state approximation

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The quasi-steady-state assumption (QSSA) can be justified mathematically by the existence of an exponentially attracting integral manifold for the corresponding system of singularly perturbed differential equations. We extend the QSSA with respect to the class of differential systems as well as with respect to the order of approximation. Especially, we extend it to the class of singular singularly perturbed systems where the associated system has a first integral. That means, we may introduce slow pool variables by means of a (nonlinear) coordinate transformation. As an application we prove that the trimolecular autocatalator can be approximated by a fast bimolecular reaction system. Finally we describe a class of singularly perturbed systems for which the first order QSSA can easily be obtained.

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Periodicity versus chaos in population dynamics

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Using recent results on the asymptotics of families of one-dimensional mappings we investigate two standard models in population dynamics, namely the Ricker family

$$R_{\lambda,\beta}(x) = \lambda x e^{-\beta x} \qquad \lambda > 1, \quad \beta > 0$$

and the Hassell family

$$H_{\lambda,\beta}(x) = \frac{\lambda x}{(1+x)^{\beta}}, \qquad \lambda > 1, \quad \beta > 1.$$

These models seek to describe certain single-species population dynamics, where λ is the intrinsic reproduction rate, and β controls the damping for large populations due to the environmental effects. Lately the work by Gyllenberg, Hanski and Lindström shows that models such as these also can be derived from more complicated models with structured competition processes.

Our focus is on generic behavior seen for many parameters and many initial conditions. There is an almost dichotomy between maps with stable cycles and strongly chaotic maps. Strongly chaotic maps here means maps with global, exponential sensitivity to initial conditions and an invariant probability measure, with integrable density, describing the asymptotics of almost all initial conditions.

The following generic results are applicable to the families above: (i) Each map has a unique metric attractor; (ii) Maps with stable cycles are dense in parameter space; (iii) Strongly chaotic maps appears with positive frequency in parameter space; (iv) Asymptotic dynamics is almost independent of initial conditions, and can in most cases be described by an invariant probability measure; (v) Various (in)stability properties of these asymptotic distributions.

Posters

Parameter estimation and model selection in ODEs: the lipid metabolism

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Modelling biological non-linear dynamical systems is often done using ordinary differential equations (ODE). To analyze these systems and the underlying mechanisms with help of data it is crucial to evaluate different modelling approaches and to estimate parameters of the models. To get numerically stable estimates of the parameters we apply the so-called multiple-shooting method [1], [2]. To identify the model which fits the data best we apply likelihood-ratio-tests which have to be adjusted to the occuring non-standard conditions. This concept is exemplified with a data set from human lipoprotein metabolism where different compartment models are compared.

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5.6 Molecular Modelling

Organizer : Folkmar Bornemann

Key note lecture

Conformational Dynamics: How to Compute Almost Invariant Structures in the Complicated Motion of Molecular Ensembles

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The function of many important biomolecules comes from their dynamic properties and their ability to switch between different *conformations*. In a conformation, the large scale geometric structure of the molecule is understood to be conserved, whereas on smaller scales the system may well rotate, oscillate or fluctuate. In [2], it has been demonstrated that conformations can be understood as almost invariant sets of the Hamiltonian system governing the molecular dynamics. Moreover, it has been proposed to follow Dellnitz and Junge [1] in computing these almost invariant sets via certain eigenvectors of the Frobenius Perron operator of the system. We will shortly review this approach in its application to molecular dynamics and discuss its potential drawbacks in this particular case. Analysis of these drawbacks will lead us to the insight that the Frobenius-Perron operator must be replaced by some specific Markov operator T that correctly describes the statistical fluctuations within the molecular ensemble [3]. This reformulation leads to nonlinear state space Markov chains that replace the deterministic Hamiltonian systems of the original approach. The conformations are identified using eigenvectors of T corresponding to the eigenvalue cluster near the maximal eigenvalue 1. Following [4], we illustrate and justify this approach by discussing statistical and spectral properties of T and the associated Markov chain. In order to approximate the desired eigenvectors of T, we discretize the corresponding eigenvalue problem by means of a specific Galerkin procedure. This results in a stochastic discretization matrix T_N [5] for which, even for large molecules, the eigenvectors can be computed efficiently. Finally, it is demonstrated how this leads to approximate conformational subsets. Our approach allows for the first time to identify dynamical conformations of molecular ensembles including their stability life spans and the rate of transitions between them.

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Invited lectures

Molecular Propagation through Crossings and Avoided Crossings of Electron Energy Levels

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Most information about molecular propagation is obtained from the time–dependent Born–Oppenheimer approximation. This approximation fails to apply if two electron energy levels cross or approach close to one another for some configuration of the nuclei. In this talk we describe the time–dependent Born–Oppenheimer approximation and discuss what happens when a molecular system propagates through a generic level crossing or avoided crossing.

Adiabatic Evolution of Systems with Infinitely many Eigenvalue Crossings

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We formulate an adiabatic theorem adapted to models that present an instantaneous eigenvalue experiencing an infinite number of crossings with the rest of the spectrum. We give an upper bound on the leading correction terms with respect to the adiabatic limit. The result requires only differentiability of the considered spectral projector, and some geometric hypothesis on the local behaviour of the eigenvalues at the crossings.

Enhanced Quantum–Classical Molecular Dynamics: Models and Numerical Integrators

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In recent years, *mixed quantum–classical models* have attracted much interest as approximation to a full quantum dynamical description in molecular dynamics. The attention is based on the hope to overcome the obstacles of the widely used Born-Oppenheimer approximation. In this particular model, the dynamics of light particles (i.e., electrons) is *adiabatically* coupled to the classical dynamics of heavier nuclei. It has been assumed that the so–called QCMD model, which consists of a *singularly perturbed* Schrödinger equation nonlinearly coupled to Newtonian equations, might describe the — in many applications crucial — *nonadiabatic effects* as well. The approximation properties of QCMD were thoroughly analyzed [1,2]. It could be shown that QCMD is a valid approximation under certain assumptions and in mildly nonadiabatic scenarios only. A novel and much more promising approach for nonadiabatic systems leads to the Quantum–Classical–Liouville equation [3] implying some kind of QCMD surface–hopping [4].
But not only the modelling should gain attention. To efficiently obtain a reliable numerical solution of the model equations the development of numerical integrators for the case of a singularly-perturbed PDE coupled to a Hamiltonian ODE is required. Due to the great variation of application problems, there are many different ways to discretize the system: using structure conserving [5], long–time–step [6] and averaging methods [7,8].
The first part of the talk is dedicated to an overview of the approximation properties of

some quantum–classical models. In the second part of the talk numerical algorithms to solve the derived model equations will be considered.

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Contributed talks

Electronic structure calculations via reduced density matrices and semidefinite programming

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Joint work with Shidong Jiang, Madhu Nayakkankuppam, Michael L. Overton and Jerome K. Percus (Courant Institute) and François Oustry (INRIA-Grenoble). The ground state variational problem for a many-electron system may be formulated in terms of the one-body and two-body reduced density matrices instead of the complete wavefunction [1-2]. A central issue is the "representability problem" of characterizing (through convex inequalities) the class of functions that can arise as density matrix of a many-electron system. The calculation of ground-state properties then reduces to a linear optimization problem subject to the representability constraints, of which the simplest ones are a finite set of linear equalities and bounds on eigenvalues, as in semidefinite programming. Significant work on this application was done in the 1970s [3], but interest has waned since, partly because of the unsettled status of the representability problem and partly because of the computational cost of solving the associated optimization problem. Recent advances in the application of interior-point and related methods to semidefinite programs [4] have encouraged us to revisit the reduced density matrix approach to electronic structure calculations. Our research follows two tracks. On one hand we are using analytical and numerical approaches to develop new representability conditions. Our new conditions extend fundamentally the known "diagonal" conditions developed in the 1970s [5], which correspond to the well-studied problem of characterizing the Cut Polytope [6]. While a complete solution of this problem appears intractable, we hope that our approach will bring the status of the complete representability problem to the same level as has been achieved for the diagonal problem. On the other hand, we are exploring numerically the strength of the known representability conditions by calculations on model systems, in which we compare the ground state energy found by the optimization approach with the ground state energy found using full configuration interaction.

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5.7 Patterns

Organizer : Paul C. Fife

Key note lecture

The Viscous Nonlinear Dynamics of Twist and Writhe

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Motivated by diverse phenomena in cellular biophysics, including DNA transcription and replication, and bacterial flagellar motion, I describe recent work on four aspects of the overdamped nonlinear dynamics of a an elastic filament in the low Reynolds number regime. (i) A geometrically exact and gauge-invariant intrinsic formulation, utilizing the "natural" frame of space curves, that yields coupled nonlinear PDES for the time evolution of the complex curvature and twist density, and thereby describes the dynamic interplay of twist and writhe. (ii) A class of problems in the dynamics of forced elastic filaments (and related experimental work) that reveals the basic principles of elastohydrodynamics, and in the case of rotational forcing, the competition between twist injection, twist diffusion, and writhing instabilities. (iii) A mechanistic explanation for "chirality inversions" observed in helical bacterial flagella in external flow fields. (iv) Preliminary experiments with optical trapping techniques on the elasticity of supercoiled bacterial fibers that reveal for the first time the bending modulus of a bacterial cell.

Invited lectures

Modelling Bioconvection at Low and High Volume Fractions of Swimming Bacteria

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Modelling the dynamics of suspensions containing swimming micro-organisms requires equations governing (1)conservation of organisms, (2)conservation of advected and diffusing passive scalars which may be emitted or consumed, and which may affect the probability density functions for the organisms' swimmng velocity, (3)conservation of mass, and (4)conservation of momentum, the Navier-Stokes equations. Conservation of mass conventionally leads to divU=0, where U is the fluid velocity. When organisms can swim relative to U, and when their volume fraction becomes large, e.g 0.1, this equation must be augmented. The variability of fluid density, due to swimming, must also be accounted for in the D(momentum)/Dt term of the Navier Stokes equations. The experimental motivation for this research, observations of "microturbulence" in concentrated bacterial suspensions, will be described.

Spatio-Temporal Phase Patterns in Forced Oscillatory Systems

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Periodic forcing of an oscillatory system produces frequency locking bands within which the system frequency is rationally related to the forcing frequency. We study extended oscillatory systems that respond to uniform periodic forcing at one quarter of the forcing frequency (the 4:1 resonance). These systems possess four coexisting stable states, corresponding to uniform oscillations with successive phase shifts of $\pi/2$.

Using an amplitude equation approach near a Hopf bifurcation to uniform oscillations, we study front solutions connecting different phase states. These solutions divide into two groups: π-fronts separating states with a phase shift of π and $\pi/2$-fronts separating states with a phase shift of $\pi/2$. We find a new type of front instability where a stationary π-front "decomposes" into a pair of traveling $\pi/2$-fronts as the forcing strength is decreased. The instability is degenerate for an amplitude equation with cubic nonlinearities. At the instability point a continuous family of pair solutions exists, consisting of $\pi/2$-fronts separated by distances ranging from zero to infinity. Quintic nonlinearities lift the degeneracy at the instability point but do not change the gross effects of the instability. We conjecture the existence of similar instabilities in higher $2n$:1 resonances ($n = 3, 4, ..$) where stationary π-fronts decompose into n traveling π/n-fronts.

The instabilities designate transitions from stationary two-phase patterns to traveling $2n$-phase patterns. As an example, we demonstrate with a numerical solution the collapse of a four-phase spiral wave into a stationary two-phase pattern as the forcing strength within the 4:1 resonance is increased. Comparisons of theoretical results with experimental data obtained by Lin and Swinney on the periodically forced Belousov-Zhabotinsky reaction are discussed.

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Complex Pulse Dynamics in Nonlinear Dissipative Systems

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Pulse dynamics is one of the typical patterns in dissipative systems, which conveys information from one place to another. Like a propagating pulse in nerve axon, such waves in general annihilate upon collision and it has been regarded as a characteristic distinction from solitons in nonlinear dispersive systems. A different class of pulse waves, however, has been found for several reaction diffusion systems including the Gray-Scott model: bouncing pulse waves upon collision or self-replicating patterns. In this talk firstly we discuss a *skeleton structure of self-replicating patterns* from global bifurcational point of view. It turns out that the *aftereffect* of hierarchy structure of saddle-node bifurcation is responsible for self-replicating process. Also pulse interaction approach is quite useful to understand the manner of splitting asymptotically. Secondly we discuss about the *self-destruction* phenomena arising in the Gray-Scott model, which appears typically after the self-replicating process, namely the whole domain is filled by the localized islands through self-replication, then several groups of islands disappear followed by the invading process from the surroundings with replication, and this process repeats over and over again in a chaotic way. The saddle-node structure of the Turing branch of the highest mode as well as the instability of the critical point is crucial to understand this complex dynamics.

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Localized waves: worms in electroconvection

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A wide range of physical systems undergoes bifurcations to spatial and temporal patterns when they are driven sufficiently strongly. Of great recent interest have been systems in which the appearing patterns do not extend over the whole system but become localized in only a small part of it, although the system is translationally invariant. In contrast to fronts that connect two stable states these localized patterns are not topologically stable, and their stability depends on the dynamics of the system.

Usually, localized structures arise in systems that exhibit a bistability between the feature-less state and the spatially extended pattern. Recently, however, localized waves ('worms') have been observed in experiments on electroconvection of nematic liquid crystals in the absence of a bistability between the basic state and the spatially extended pattern [1]. We show that this can be understood if the waves are coupled to a slowly decaying large-scale field [2]. We discuss a Ginzburg-Landau-type model for this coupling that captures various aspects of the experimental observations, e.g. the shape and stability of the worms.

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Spinodal Decomposition for the Cahn-Hilliard Model

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Some forty years ago J.W. Cahn and J.E. Hilliard introduced the fourth-order parabolic partial differential equation

$$\begin{aligned} u_t &= -\Delta(\varepsilon^2 \Delta u + f(u)) \quad \text{in} \quad \Omega\,, \\ \frac{\partial u}{\partial \nu} &= \frac{\partial \Delta u}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega \end{aligned}$$

as a model for several physical phenomena occurring in binary alloys. One particularly intriguing phenomenon is spinodal decomposition: If a homogeneous high-temperature mixture of two metallic components is rapidly quenched to a certain lower temperature, then a sudden phase separation sets in. The mixture quickly becomes inhomogeneous and forms a complicated, fine-grained structure, more or less alternating between the two alloy components.

In this talk I present recent mathematical progress in explaining spinodal decomposition for the Cahn-Hilliard equation. This includes both a description of the main characteristics of the generated patterns, as well as an explanation of the surprising mechanism underlying the phase separation. I will also address the case of multi-component alloys, which is modeled by a system of Cahn-Hilliard equations.

These results were obtained in collaboration with Stanislaus Maier-Paape, Evelyn Sander, and Barbara Stoth.

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Contributed talks

Spinodal decomposition for the stochastic Cahn-Hilliard equation

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The talk addresses the spinodal decomposition for the stochastic Cahn-Hilliard equation

$$u_t = -\epsilon^2 \Delta^2 u + \Delta(u^3 - u) + \xi$$

in a domain Ω with Neumann and no flux boundary conditions. $\Omega \subset \mathbb{R}^d$, $d = 1, 2, 3$ is a domain with sufficiently smooth boundary and ξ a Gaussian white noise process. Based on the work of Maier-Paape and Wanner [1,2] for the Cahn-Hilliard equation, it is shown, that with high probability solutions of the linearized equation starting at $u = 0$ will leave a neighborhood of 0 along a strongly unstable manifold, provided ϵ small enough. This will give solutions with a characteristic wavelength, as discussed in [1]. This linear result is then extended to the nonlinear stochastic equation near the origin.

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Bifurcation for reaction-diffusion systems with nonstandard boundary conditions

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Let us consider a reaction-diffusion system of activator-inhibitor type

$$\begin{array}{l} u_t = \sigma_1(s)u_{xx} + f(u,v) \\ v_t = \sigma_2(s)v_{xx} + g(u,v) \end{array} \quad \text{on } (0,+\infty)\times(0,1) \tag{1}$$

with boundary conditions

$$u(0)=\overline{u},\ v(0)=\overline{v},\quad u_x(1)\in -m_1(u(x_1)), v_x(1)\in -m_2(v(x_2)) \tag{2}$$

where $\sigma : \mathbb{R} \to \mathbb{R}^2_+$ is a smooth curve, $s \in \mathbb{R}$ is a bifurcation parameter, $\overline{u}, \overline{v}$ are real numbers, $f, g : \mathbb{R}^2 \to \mathbb{R}$ are real differentiable functions such that $f(\overline{u},\overline{v}) = g(\overline{u},\overline{v}) = 0$ (i.e. $[\overline{u},\overline{v}]$ is a steady state of (1), (2)), $x_1, x_2 \in (0,1]$ and $m_1, m_2 : \mathbb{R} \to 2^{\mathbb{R}}$ are multivalued functions of a certain type.

The multivalued conditions in (2) with certain m_1 and m_2 describe for example semipermeable membranes on the part of the boundary with sensors in x_1 and x_2, respectively. Particularly, the sensors can be the interior of the domain and therefore at different points than the sources.

There exists a bifurcation point $s_I \in \mathbb{R}$ of (1), (2) at which stationary spatially nonconstant solutions (spatial patterns) bifurcate from the branch of trivial solution $[\overline{u},\overline{v}]$. Moreover, s_I lies in the region of stability of $[\overline{u},\overline{v}]$ as a solution to a corresponding stationary problem to (1) with classical boundary conditions

$$u(0)=\overline{u},\ v(0)=\overline{v},\quad u_x(1)=v_x(1)=0, \tag{3}$$

particularly in the region, where bifurcation for (1) with (3) is excluded.

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Lagrangian turbulence and the dynamo effect in Magnetohydrodynamics

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The generation and maintenance of magnetic fields by the motion of electrically conducting fluids are the subject of dynamo theory. It is a prevalent assumption that this phenomenon can be modeled by the magnetohydrodynamic (MHD) equations which consist of the coupled system of Navier-Stokes equations (NSE) and induction equation.

The general idea is that the mechanical energy pumped into the fluid by heating or other mechanisms is transferred into the induction equation producing the dynamo. We report on bifurcations of the MHD equations which are driven by an external forcing of the Roberts type [1,2]. It drives a flow which serves as a model for the convection in the outer core of the Earth [3] and is realized in an ongoing laboratory experiment aimed at demonstrating a dynamo effect. The symmetry group of the problem is determined and we study in detail the first symmetry breaking transition from the nonmagnetic state. The appearance of the magnetic field is spacially correlated with the location of stagnation points. We observed that the magnetic field is stongly amplified in the neighborhood of the fixed points of the velocity field which have an one-dimensional stable and a two-dimensional unstable manifold. The Lagrangian chaos of passive tracers in this region gives numerical evidence that intersections of stable and unstable manifolds of fixed points of different type are responsible for the creation of the dynamo in this model.

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Asymptotic profiles of nonlinear diffusion problems

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We are describing the existence, uniqueness and profile structure as $\lambda \to +\infty$ of positive solutions to the logistic type problems,

$$\begin{array}{rcll} -\Delta_p u &=& \lambda u^{p-1} - u^q - f(u) & x \in \Omega \\ u &=& 0 & x \in \partial\Omega, \end{array}$$

$\Omega \subset \mathbb{R}^n$ a smooth and bounded domain, Δ_p the p-Laplacian, $q+1>p>2$ while f is C^1 and both satisfies $f = o(u^{p-1})$, $f = o(u^q)$ respectively as $u \to 0$, $u \to +\infty$. The case Ω radially symmetric will studied in detail while spacial emphasis will be put in the arising, on general domains Ω, of dead cores, i.e. sets $\mathcal{O}_\lambda := \{x : u(x) = \bar{u}(\lambda)\}$ where u is a positive solution and $\bar{u}(\lambda)$ satisfies $\lambda u^{p-1} - u^q - f(u)$ (see [1],[2]).

We are also dealing with a second kind of spatially dependent diffusion problems, namely,

$$\begin{array}{rcll} -\Delta_p u &=& \lambda u^{p-1} - a(x)(u^q + f(u)) & x \in \Omega \\ u &=& 0 & x \in \partial\Omega, \end{array} \tag{1}$$

where $a = a(x) \in C^1(\overline{\Omega})$, $a(x) \geq 0$ but the relevant fact is $D := \text{supp } a \subset \Omega$. We are considering the issues of existence and uniqueness of positive solutions and showing that such solutions ceases to exist when $\lambda > \lambda_1(\Omega \backslash D)$. In fact, we are showing that positive solutions blow-up for λ close $\lambda_1(\Omega \backslash D)$ and we will provide a detailed account on the profile of such solutions when $\lambda \to \lambda_1(\Omega \backslash D)$ (see [3] for earlier results).

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Convective Cahn-Hilliard models for kinetically controlled crystal growth

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We consider solidification into a hypercooled melt in which kinetic undercooling and anisotropic surface energy are present. We allow the anisotropy to be strong enough that equilibrium configurations would contain facets.

In one-dimensional case, the formation of facets is governed by the convective Cahn-Hilliard equation

$$u_t + Duu_x + (u_{xx} + u - u^3)_{xx} = 0,$$

where $u(x,t)$ is proportional to the slope of the interface. Depending on the parameter D, the derived equation describes the coarsening of facets (kinks), development of a steady spatially-periodic surface relief and appearance of a non-periodic attractor (roughening). The bifurcations and stability of spatially-periodic solutions, as well as the rate of coarsening, are studied.

In two-dimensional case, several generalizations of the convective Cahn-Hilliard equation have been derived for different orientations of growing surfaces. The numerical simulation reveals the formation of hill-and-valley structures in the form of coarsening square, triangular and rhombic pyramids and grooves.

Spatial patterns for reaction-diffusion systems with nonstandard boundary conditions

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Reaction diffusion systems of the activator-inhibitor type

$$u_t = d_1 \Delta u + f(u,v),\ v_t = d_2 \Delta v + g(u,v)$$

will be considered under the assumptions guaranteeing the loss of stability of the spatially homogeneous stationary solution (diffusion driven instability) in case of classical boundary conditions. The existence and location of a bifurcation of stationary spatially nonhomogeneous solutions (spatial patterns) will be discussed for the case when classical boundary conditions are replaced by nonstandard nonlinear boundary conditions or supplemented by some additional restrictions in the interior of the domain. The class of conditions under consideration includes a description of unilateral membranes or some regulation on the boundary or in the interior of the domain (e.g. a thermostat type regulation). The simplest example are boundary conditions

$$\frac{\partial u}{\partial n} \in m_1(u),\ \frac{\partial v}{\partial n} \in m_2(v)$$

where m_1, m_2 are multivalued functions of a certain type. The method used can be applied to the case when $f,\ g$ contain jumping nonlinearities. The results can be formulated in the abstract form for general inclusions of the type

$$x - T(\mu)x - G(\mu, x) \in -M(x)$$

where for any μ, $T(\mu)$ is a linear completely continuous operator, G is a small compact perturbation, M is a multivalued mapping of a certain type in a Hilbert space, μ is a bifurcation parameter. The main common feature of the problems under consideration is that they cannot be linearized and therefore the classical approaches to bifurcation and stability cannot be used.

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Existence of bounded trajectories via upper and lower solutions

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The talk deals with the boundary value problem on the whole line

$$(P) \qquad \begin{cases} u'' - f(u,u') + g(u) = 0 \\ u(-\infty) = 0 \quad u(+\infty) = 1 \end{cases}$$

where $g : R \to R$ denotes a continuous non-negative function having support $]0,1[$, and $f : R^2 \to R$ is a continuous function. In [2] we propose a new approach in order to treat (P), based on a combination of lower and upper-solutions methods and phase-plane techniques, by means of which we achieve both an existence and non-existence result:

Main theorem. *Assume that* $f(u,0) = 0$ *for any* $u \in [0,1]$ *. Then, if*

$$\frac{f(u,u')}{u'} \geq 2\sqrt{\sup_{u\in]0,1]} \frac{g(u)}{u}} \quad \textit{for every} \ \ u \in [0,1], u' > 0 \tag{1}$$

problem (P) admits monotone solutions. Moreover, if the function $\dfrac{f(u,u')}{u'}$ *is non-decreasing with respect to* u'*, then the solution is unique (up to translation of the origin). Whereas, if*

$$\limsup_{(u,u')\to(0,0)} \frac{f(u,u')}{u'} < 2\sqrt{D_+ g(0)} \tag{2}$$

then problem (P) does not admit monotone solutions.

As an application of previous result we are able to discuss the existence of travelling wave solutions for a reaction diffusion equation of the general type (see [3])

$$\frac{\partial u}{\partial t} + \frac{\partial h(u)}{\partial x} = \frac{\partial}{\partial x}[d(u)\frac{\partial u}{\partial x}] + g(u)$$

which appears in several biological and chemical models (see i.e. [1] and [4]).

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Asymptotics, existence and stability of solutions with internal layers of "fast" and slow elliptic systems

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Elliptic problems, including systems with "fast" and slow equations are considered. We demonstrate our approach for the system

$$\varepsilon^2 \triangle u = g(u, v, x, \varepsilon), \ \triangle v = f(u, v, x, \varepsilon), \ x \in D \tag{1}$$

with prescribed boundary conditions on ∂D.

We give some sufficient conditions of existence of solutions with internal layers. The proof is based on asymptotic method of differential inequalities. Some relevant problems for integro-differential equations are also disscussed.

More detailed consideration of some of the presented problems in scalar case can be found in the papers [1-2].

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Multibump patterns near a co-dimension 2 point

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The behaviour of marginally unstable small solutions in a reaction-diffusion model problem is studied near a co-dimension 2 point. At this co-dimension 2 point the 'classical' Ginzburg-Landau equation approximation breaks down. In numerical simulations on the reaction-diffusion system asymptotically stable multibump patterns are observed. By applying normal form theory, these multibump patterns are found theoretically.

Stabilizing unstable periodic orbits in reaction diffusion systems by global time-delayed feedback control

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The success of chaos control in high-dimensional spatially extended dynamical systems is examined by applying the extended time-delay autosynchronisation control method. The control technique is applied to a globally coupled reaction-diffusion system which describes charge transport in a bistable semiconductor. In our system, the control signal,

$$\epsilon(t) = K \left((1-R) \sum_{m=1}^{\infty} R^{m-1} y(t-m\tau) - y(t) \right) \qquad 0 \leq R < 1. \tag{1}$$

utilizes a control variable $y(t)$ given by the spatial average $y = \langle a \rangle$ of the field $a(x,t)$. We note that with increasing R orbits of higher period and orbits with larger Lyapunov exponents should be stabilized [1]. Using a damped Newton algorithm [2], we explicitly compute the unstable periodic orbits (UPOs) associated with the spatiotemporal chaotic reaction-diffusion system. A variety of spatiotemporal UPOs of period-one, period-two, or period-four can be stabilized if the product of the period τ and the largest Lyapunov exponent λ is less than 1.9. Even though we can compute spatiotemporal UPOs to within a high precision, we can not stabilize the computed long-period UPOs or UPOs with large Lyapunov exponents if $\lambda\tau > 1.9$. This result is in contrast with the approximate theoretical limit of control for low-dimensional temporal chaos [1] where stabilization is possible for $\lambda\tau \leq 2(1+R)/(1-R)$ using the extended time-delay autosynchronisation method.

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Patterns in an electrically driven conducting fluid layer

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A numerical stability and bifurcation analysis is presented for an incompressible, viscous and electrically conducting fluid sheet, bounded by stress-free parallel planes and driven by an external electric field tangential to the boundaries. For the case of a uniform electrical conductivity it is found that the quiescent basic state, in which the current density is uniform and the magnetic field profile across the sheet linear, remains stable, no matter how strong the driving electric field. Instability is possible, however, for appropriately varying conductivity. We have studied in detail the Harris equilibrium, where the conductivity varies across the sheet in such a way that the current is largely concentrated in a layer centered about the midplane of the sheet and the magnetic field has a hyperbolic-tangent profile. A Squire's theorem could be proven stating that two-dimensional perturbations become unstable first. By varying several parameters of the equilibrium, stability boundaries were determined. The unstable perturbations are tearing modes, characterized by current filaments, magnetic islands associated with them and a fluid motion in convection-like rolls. Restricting the problem to two spatial dimensions, the nonlinear evolution of the tearing modes was followed up to time-asymptotic steady states. These are linearly stable with respect to two-dimensional perturbations, but prove to be sensitive to three-dimensional ones even close to the primary bifurcation point. Again stability boundaries were determind by varying the system parameters. The unstably perturbed states were followed up in their three-dimensional nonlinear evolution.

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Some Properties of Transcendental Integrable Dynamical Systems

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It is well-known that the concept of integrability has the various aspects due to in what functions an integration is made (analytical, smooth, meromorphic etc.) [1,2]. In the given activity the problem of integrability of systems of ordinary differential equations in the class of transcendental functions, i.e. functions, after which prolongation in complex area they have essentially singular points is discussed [2]. Concept of integrability of the class of transcendental functions arises for the reason of availability of this system asymptotic (attracting or repelling) limited sets, i.e. sets that have the neighborhood which consist of diversities of dimensionality above 1.

In this paper, we touch upon some qualitative questions of the theory of ordinary differential equations important for study of dynamical systems. The lots of them are arising in dynamics of a rigid body interacting with a medium. We shall review such problems as existence of the so-called monotonic limit cycles, the existence of closed trajectories contractible to a point along two-dimensional surfaces, the existence of closed trajectories not contractible to a point along a phase cylinder, qualitative problems of the theory of topographical Poincare systems and more general systems for comparison with dynamical systems on a plane, the existence and uniqueness of trajectories which have infinitely remote points as limit sets for systems on a plane.

We are also dealing with the existence and uniqueness of trajectories of dynamical systems on a plane which have infinitely remote points as α- and ω-limit sets. So, on

Riemann or Poincare spheres, limit sets of such trajectories is the north pole. These are key trajectories by definition because an infinitely remote point is always singular [3-5].

Recall that in a phase space a trajectory is Poisson stable if in a finite time it returns to any sufficiently small neighborhood of its point.

Sufficient conditions for the existence of Poisson stable trajectories will be formulated in this work.

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5.8 Semiconductors

Organizer : Herbert Gajewski

Key note lecture

Wigner Transforms,Homogenisation and Dispersive Problems

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It is well known by now that Wigner transforms are the appropriate tool to carry out the semiclassical limit of the Schroedinger equation. The weak limit of the Wigner transform of the wave function (the so called Wigner measure) then satisfies the Liouville phase space equation and limits of physical observables can be calculated by essentially computing moments of the Wigner measure. This limit procedure is not at all influenced by the possible occurance of caustics. In the talk we shall generalize the methodology of Wigner transforms to various applied p.d.e. problems. We consider zero pressure and compressible isentropic quantumhydrodynamics, the incompressible Euler limit from the nonlinear Schroedinger equation and mean field Schroedinger-Poisson coupling.

Also we show how the Wigner transform approach can be used to carry out homogenisation limits of quadratic functions (functionals) of solutions of linear and weakly nonlinear antiselfadjoint (pseudodifferential) initial value problems. As typical examples we discuss electrons in crystal lattices, the Maxwell equations in a periodic medium and the classical limit of the Dirac equation (to the relativistic Vlasov equation). Finally, we present a Wigner-function based method to derive dispersion and Morawetz-type estimates for wave-like equations.

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Invited lectures

Moment equations for carrier transport in semiconductors

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Hydrodynamical models are currently used in order to describe carrier transport in submicron semiconductor devices. Hydrodynamical models are obtained as moment equations of the kinetic Boltzmann transport equation, with appropriate closure assumptions. Lately a class of hydrodynamical models has been introduced with the closure based on the maximum entropy ansatz (in the formulation of Extended Thermodynamics) by Anile, Romano and Russo. Mathematically the models consist of a hyperbolic system of conservation laws with sources, endowed with convex entropy and coupled to the Poisson equation.

The simulation of a realistic device, like the $n^+ - n - n^+$ diode, introduces strong gradients in the density at the junctions and therefore requires, for an accurate solution, high order methods suitable for hyperbolic systems of conservation laws. In this article we report on the numerical solution of these models by using a high accuracy, high-order, centred TVD schemes namely the FLIC (Flux LImiter Centred) method introduced by Toro. The homogeneous part of the system was solved using the FLIC scheme, while various conventional methods were employed for the solution of the forcing terms in the framework of a splitting strategy. One of the important advantages of using the FLIC scheme is that this method can be implemented with ease for non stationary problems in several dimensions and arbitrary geometries. Various classes of models have been investigated (according to the various approximations which can be made when applying the maximum entropy approach) and the results have been compared with those of Monte Carlo simulations.

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About some mathematical questions concerning the embedding of Schrödinger–Poisson systems into the drift–diffusion model of semiconductor devices

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Nanostructures — quantum wells, quantum wires and quantum dots — become ever more important for the performance of semiconductor devices. While Van Roosbroeck's equations provide a good landscape view on an electronic device, Schrödinger–Poisson systems portrait the individual features of a nanostructure within the device. This leads to the embedding of Schrödinger–Poisson systems, often including an effective Kohn–Sham operator, into phenomenological models for semiconductor devices. For connecting the different models boundary conditions for both Van Roosbroeck's equations and the Schrödinger–Poisson systems obviously play an important role. This leads to the question about the regularity of basic quantities as density and current on the interface between regions governed by different models.

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The dynamics of some quantum open systems with short-range nonlinearities

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We consider the simplest model of quantum open systems, which is the case of one-body statistical physics with inflow boundary conditions at infinity. We develop a functional framework for some nonlinear dynamics, where the nonlinearities are (very) short-range perturbations of a given Hamiltonian. In this framework, one can prove the global in time existence and uniqueness of a solution for the time dependent problem and the existence of steady states of which the form can be specified. This approach relies on some tools of quantum scattering theory, namely on Mourre theory and Sigal-Soffer propagation estimates. This problem arises from the modelling of semiconductors and the proposed functional framework especially applies to the models described in [1][2][3] and [4].

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Contributed talks

Mathematical analysis of electrostatics BVP for dielectrics in powerful electrical fields

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It is well known that in powerful electrical fields the behaviour of real dielectrics is essentially non-linear and it is not possible to use the linear model in many cases. For dielecrics with non-linear vector-function of saturation $D = D(x, E)$, the appropriate

BVP is formulated as the variational problem for the scalar electrical potential u such that $E = -\nabla u$. For dielectrics with ideal saturation, where $|D| \leq \lambda < +\infty$, the existence of the limited electrostatic load (such external charges with no solution of BVP) and discontinuous electrical potentials with breaks of the first type has been proved by the author recently [1]. From the physical point of view these effects are treated as the electrical puncture of dielectric.

For estimation of the electrical durability of the dielectrical body $\Omega \in R^3$ the following variational problem must be solved [1]

$$t_* = \inf \left\{ \int\limits_{\Omega} |\nabla u(x)| \lambda(x) \, dx : \ u \in V, \ L(u) = 1 \right\}, \tag{1}$$

$$L(u) = \int\limits_{\Omega} \rho u \, dx + \int\limits_{\Gamma^2} \sigma u \, d\gamma, \qquad \lambda(x) = \liminf_{|E| \to \infty} \frac{\Phi(x, E)}{|E|},$$

$$\Phi(x, E) = E_i \int\limits_0^1 D_i(x, tE) \, dt,$$

where $V = \{u \in W^{1,1}(\Omega) : \ u(x) = U(x), \ x \in \Gamma^1\}$ is the set of admissible electrical potentials, U is the given potential on a portion Γ^1 of the boundary $\partial\Omega$, ρ and σ are the external bulk and surface charges, respectively; $\Gamma^2 = \partial\Omega \setminus \Gamma^1$, $a_2 \geq a_1 > 0$ and $a_0 \in R$ are the constants. The limited electrostatic load is $(\rho_*, \sigma_*) = t_*(\rho, \sigma)$. If $t_* < 1$, then the appropriate electrostatics BVP has no solution.

From the mathematical point of view the variational problem (1) is non-correct and, therefore, needs regularization. We use the partial regularization which is based on the special discontinuous finite-element approximation (FEA) [2]. After this discontinuous FEA the problem (1) is transformed into the non-linear system of algebraic equations which is badly determined. Therefore, for the numerical solution the decomposition method of adaptive block relaxation is used [3,4]. The main idea of this method consists of iterative improvement of zones with "proportional" fields by special decomposition of variables, and separate calculation on these variables.

The numerical results show that, for finding the main parameter t_* of a limited electrostatic load, the proposed technique has the qualitative advantage over the standard continuous finite-element approimations.

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The Small Debye Lenght Limit in the Drift-Diffusion Model for Semiconductors

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We are going to study some aspects of the so called Drift Diffusion Model for semiconductors, which is well known from semiconductor physics. The equations for the electron and hole densities n^λ and p^λ, respectively, and for the electric field E^λ read

$$\begin{aligned} n_t^\lambda &= \mu_n(\Delta n^\lambda + \operatorname{div}(n^\lambda E^\lambda)) \\ p_t^\lambda &= \mu_p(\Delta p^\lambda - \operatorname{div}(p^\lambda E^\lambda)) \\ -\lambda^2 \operatorname{div} E^\lambda &= n^\lambda - p^\lambda - C \\ n^\lambda(t=0,x) = n_0^\lambda(x), &\qquad p^\lambda(t=0,x) = p_0^\lambda(x) \end{aligned}$$

with $x \in \mathbf{R^d}$, $t \geq 0$. The quantities μ_n, μ_p denote the electron and hole mobilities and $C = C(x)$ is the dopingprofile. A complete existence theory for this model was given in [1]. The problem we are going to focus on is the vanishing Debye length limit $\lambda \to 0$ (the superscript λ denotes the $\lambda-$dependence).

On the initial time layer scale $s = \frac{t}{\lambda^2}$ the formal limit equations for the limits n, p and F of n^λ, p^λ and $\lambda^2 E^\lambda$, respectively, are given by

$$\begin{aligned} n_s &= \mu_n \operatorname{div}(nF) \\ p_s &= \mu_p \operatorname{div}(pF) \\ -\operatorname{div} F &= n - p - C \\ p(t=0,x) = n_0(x), &\qquad n(t=0,x) = p_0(x), \end{aligned}$$

where n_0 and p_0 are the limits of n_0^λ and p_0^λ as $\lambda \to 0$. We give a rigorous prove of this fact and discuss the limit $s \to \infty$ in the limiting layer equations.

On the original (slow) time scale t the limit $\lambda \to 0$ is completely different. Formally, for $\mu_n = \mu_p = 1$ and $C = 0$ we obtain quasineutrality $n - p = 0$ and as limit the heat equation for the limit w of $w^\lambda = n^\lambda + p^\lambda$

$$\begin{aligned} w_t &= \Delta w \\ w(0,x) &= n_0(x) + p_0(x). \end{aligned}$$

We show this rigorously using an entropy estimate. The cases for differing μ_n and μ_p and nonvanishing C are studied in [2].

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Rigorous derivation of a hierarchie of macroscopic models for semiconductors and plasmas

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Under special physical conditions, certain parameters in the quasi-hydrodynamic equations for semiconductors and plasmas are small compared to reference values. Neglecting the corresponding terms, one gets simplified equations which are numerically easier to solve and which contain the important physical informations. The study of the asymptotic limits and their mathematical verification is of great importance to understand the validity of the various models.

In this talk we analyze three asymptotic limits: relaxation-time limit, zero-electron-mass limit, and zero-space-charge limit in the hydrodynamic (Euler-Poisson) model and the drift-diffusion equations. In particular, we prove the following:

(i) *Zero-electron-mass limit.* When the kinetic energy of the electrons is small compared to the kinetic energy of the positively charged ions (in plasmas) or holes (in semiconductors), the continuity equation for the electrons can be reduced asymptotically to a nonlinear relation between the electro-static potential and the electron density, such that the Poisson equation becomes nonlinear. This model is known by physicists for 'weakly ionized plasmas'. The limit is studied both in the hydrodynamic and the drift-diffusion equations [1,3].

(ii) *Relaxation-time limit.* In the relaxation-time limit, the hydrodynamic model reduces to the drift-diffusion model. This limit is performed for both weakly ionized and not weakly ionized plasmas in the adiabatic (or isentropic) case [2,4].

(iii) *Zero-space-charge limit.* For small scaled Debye lengths, the electron and ion densities are comparable. We study this limit rigorously for the drift-diffusion equations [5]. In the case of plasmas, this limit is also called quasi-neutral limit.

The proofs of the above results are all based on a priori estimates given by the entropy functional and appropriate compactness methods (compensated compactness, compactness-by-convexity). The entropy estimates are useful not only for the hyperbolic Euler equations, but also for the parabolic drift-diffusion models. Therefore, the entropy method proves to be a very powerful tool also for parabolic systems.

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Asymptotics of bound states and resonances for laterally coupled waveguides

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A problem of quantum waveguides with Dirichlet boundary condition which are coupled laterally through small apertures of width $2a$ is considered. Asymptotics in a of an eigenvalue close to the threshold is obtained. The result is in good correlation with variational estimates of Exner and Vugalter and with the corresponding results for zero-width slit model based on the theory of self-adjoint extensions of symmetric operators (Popov). The cases of two different and two identical waveguides are studied. The asymptotics of the eigenvalue for waveguides coupled through finite number of windows is obtained. It is shown that a measure of "common width of apertures" is a sum of two-dimensional harmonic capacities of the windows. Asymptotics of real and imaginary part of resonances (quasi-bound states) close to $N-$th threshold ($N > 1$) is found. The background for the construction is the method of matching of asymptotic expansions of solutions of boundary problems.

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Simulation of current filamentation in an extended drift-diffusion model

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We simulate the nonlinear spatio-temporal dynamics of current filaments in doped semiconductors in the regime of low temperature impurity breakdown. Charge transport is modeled by a drift-diffusion approach combined with local generation-recombination (GR) kinetics describing the autocatalytic impurity impact ionization which sets in above a certain threshold field. This is shown to result in an S-shaped current density-field relation. The parameters of our model have been determined by a Monte Carlo simulation of the microscopic scattering processes. The transport equations are solved self-consistently with Poisson's equation on two-dimensional spatial domains using a finite element discretization in space and a backward Euler scheme in time. Because of the very different timescales, and the steep spatial gradients between the current density inside and outside the filaments, which differ by several orders of magnitude, our problem poses high challenges to the numerics.

We consider different sample geometries, where we employ Dirichlet boundary conditions at the contacts and Neumann boundary conditions elsewhere. For a rectangular spatial domain with two small circular contacts we find that the nascence of current filaments upon application of a dc bias is induced by two impact ionization fronts originating from both cathode and anode. After their coalescence they form a pre-filament between the two contacts which subsequently grows into a fully developed filament. The action of an external perpendicular magnetic field leads to an asymmetrically broadened or bent filament due to the Lorentz force. For radially symmetric samples with two concentric circular contacts ("Corbino disks") we find spontaneous symmetry breaking which results in a break-up of the initial radial impact ionization front into several streamers. These develop into a number of radial pre-filaments. Due to the global coupling via the external load resistance competition between the pre-filaments occurs, with only a limited number of them surviving as stable filaments depending upon the bias conditions ("winner takes all dynamics"). Our results are in good agreement with experimental investigations of current filamentation in thin n-doped GaAs films at low temperatures for different contact geometries [1].

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5.9 Steady Water Waves

Organizer : Gérard Iooss

Key note lecture

Phenomena beyond all orders and bifurcation of reversible homoclinic connections

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We have developed a general method to obtain an exponentially small equivalent of oscillatory integral when it involves solutions of *nonlinear differential equations*. The method is based on a very precise description of the holomorphic continuation of the solutions of the perturbed equation and in particular of their behavior near the complex singularities of the solutions of the unperturbed system. The method that we use to build step by step the holomorphic continuation of the solutions is a rigorous version for this problem of the formal theory of Matching Asymptotic Expansions (M.A.E.). In particular, the construction of the holomorphic continuation of the solutions near the singularities is based on the study of an "inner system" which is well known in the theory of M.A.E. and which appears to be the relevant part of the equation near the singularities.

This method enables us to study the problem of the persistence of homoclinic connections to 0 for the one parameter families $V(.,\mu)$ of reversible vector fields which admit at the origin a $0^2 i\omega$ resonance, i.e. vector fields for which the origin is a fixed point and for which 0 is a double non semi simple eigenvalues and $\pm i\omega$ are simple eigenvalues of the differential at the origin $DV_u(0,0)$. This problem cannot be solved by a direct Melnikov approach since the Melnikov function is given by an oscillatory integral and is exponentially small. Our "Exponential tools" enable us to prove [2] that generically, vector fields admitting at 0 a $0^2 i\omega$ resonance at the origin do not admit any homoclinic connection to 0, whereas we proved in [1] that they always admit homoclinic connections to exponentially small periodic orbits. One example of such a vector field in infinite dimensions occurs when describing the irrotational flow of an inviscid fluid layer under the influence of gravity and small surface tension (Bond number $b < 1/3$) for a Froude number F close to 1. In this context a homoclinic connection to 0 would be called a solitary wave.

We have also developed a general method to obtain an exponentially small equivalent of bi-oscillatory integrals. In this case, the interaction between the two frequencies, makes the study more intricate and a partial complexification of time is necessary. This last method enables to study the $(i\omega_0)^2 i\omega_1$ resonance, i.e. one parameter families $V(\cdot,\mu)$ of reversible vector fields for which $\pm i\omega_0$ are a double non semi simple eigenvalues and $\pm i\omega_1$ are simple eigenvalues of the differential at the origin $DV_u(0,0)$. Such a resonance occurs for water waves when studying several layers problems and for chains of coupled nonlinear oscillators.

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Invited lectures

Symplecticity and criticality: extending criticality to finite-amplitude vortical steady waves

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At the cornerstone of the theory and practice of open channel hydraulics is the concept of criticality of uniform flows. In the setting of uniform flows, criticality can be defined in a number of different – but equivalent – ways, but the most fundamental definition is generally given in terms of an energy-momentum argument. Considering the fundamental nature of criticality, it is remarkable that it has never been generalized as far as we are aware to finite-amplitude states or any steady state wave other than a uniform flow. On the other hand it is not at all obvious how this might be accomplished. In this talk we show that the classical definition can be interpreted in terms of a symplectic variational principle, and that this definition easily generalises. The theory is then applied to steady water waves with constant vorticity. Much like in the uniform flow case, we show that criticality for finite-amplitude steady water waves is related to the bifurcation of solitary waves, although in this case the solitary waves are biasymptotic to periodic travelling waves of finite amplitude.

Multiplicity of algebraically decaying solitary waves

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It is known that, for appropriate values of the parameters (densities, gravity, surface tension, depths of the layers, propagation speed), two solitary waves decaying algebraically exist at the interface between two finite layers of perfect fluids [1]. The mathematical model consists of a system of partial differential equations with a free boundary. It can be reduced by the centre-manifold theorem to a four-dimensional Hamiltonian system with a centre equilibrium that has two non semi-simple and imaginary eigenvalues. Homoclinic orbits, which correspond to solitary waves, are then obtained by normal form theory and a careful persistence analysis.

Our main result states that there are not only two solitary waves, but in fact infinitely many that decay algebraically. More precisely, either there is a circle of solitary waves or we can establish the existence of so-called "multibump" solutions, in the spirit of [2] where saddle-focus equilibria are considered.

The dynamics near the centre can only be fully understood by taking into account the third and fourth order terms of the Taylor expansion of the Hamiltonian at the equilibrium. In general it is not true that the set of points in the phase space that converge to a centre is a manifold. Thanks to the fact that some coefficient of the normal form at the centre is negative, we can nevertheless establish the existence of two-dimensional stable and unstable manifolds made of orbits converging polynomially to the equilibrium. Moreover the Calculus of Variations allows us to obtain other orbits near the equilibrium that are reminiscent of those provided by the lambda-lemma in hyperbolic dynamical systems.

Then we show that the existence of a homoclinic orbit that is the topological intersection of the stable and unstable manifolds implies the existence of infinitely many "multibump" homoclinic solutions and the topological entropy of the system is positive. In addition, for interfacial waves, if the intersection is not topologically transverse, we can deduce that the stable and unstable manifolds coincide and consist of a circle of homoclinic orbit.

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Interfacial waves in the presence of a free surface

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In a two-fluid system where the lower fluid is bounded below by a rigid bottom and the upper fluid is bounded above by a free surface, two kinds of solitary waves can propagate along the interface and the free surface: classical solitary waves characterized by a solitary pulse or generalized solitary waves with in addition nondecaying oscillations in their tails. The origin of these nonlocal solitary waves can be easily understood from a physical point of view. The dispersion relation for the above system shows that short waves can propagate at the same speed as a solitary wave. The interaction between the solitary wave and the short waves creates a nonlocal solitary wave. In this talk, the interfacial-wave problem is reduced to a system of ordinary differential equations by using a classical perturbation method. The classical as well as the generalized solitary waves which are solutions of this system of model equations are constructed.

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Two- and three-dimensional steady gravity-capillary water waves via Hamiltonian spatial dynamics

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The hydrodynamic problem concerning the irrotational flow of a perfect fluid of constant density in a domain bounded above by a free surface and below by a rigid bottom is known as the water-wave problem. Here we examine the gravity-capillary steady water-wave problem, in which the fluid is acted upon by the forces of gravity and surface tension and all waves are uniformly translating with constant speed. The problem depends upon two dimensionless combinations α and β of the various physical quantities. We consider the two-dimensional version of the problem, in which x is the horizontal direction of uniform translation and y points vertically upward, together with a three-dimensional version in which the waves have an additional periodic dependence upon the horizontal direction z perpendicular to x.

Starting from a variational principle, we show that the two- and three-dimensional steady water-wave problems can be formulated as infinite-dimensional Hamiltonian systems in which x plays the role of the time-like variable. A centre-manifold reduction technique is then used to show that the problem is locally equivalent to an ordinary differential equation whose solution set can be analysed. In the two-dimensional problem we look for the existence of solitary waves, that is solutions to the system which decay to zero at large values of $|x|$. We identify two regions (I and II) of (β, α)-parameter space in which the system has infinitely many solitary waves. In region I these waves are multi-troughed waves of depression with exponentially decaying outskirts; in region II they are modulated waves.

In the three-dimensional problem we identify regions of (β, α)-parameter space in which there are generalised solitary waves, that is waves which decay to a spatially periodic motion at large $|x|$. In particular, we find a family of three-dimensional disturbances which decay to two-dimensional periodic waves at large $|x|$.

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Three-dimensional capillary-gravity waves

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We study analytically the existence of three-dimensional (3D) surface waves, i.e. waves for which the associated free-surface displacement varies in two horizontal directions, the third direction being the vertical direction, as solutions of the full Euler equations. The waves are assumed to travel at a constant speed on the free surface of an inviscid fluid layer of finite depth under the influence of gravity and surface tension. We consider small-amplitude waves bifurcating from the state of rest that are periodic in one of the two horizontal directions.

The method which we apply is based on ideas from the theory of dynamical systems. The water-wave problem is formulated as a dynamical system in which the evolutionary variable is the variable in the unbounded horizontal direction. The bounded solutions are orbits in an infinite-dimensional phase-space consisting of functions living on the bounded cross-section of the domain. Restricting to solutions of small amplitude the corresponding orbits live on a finite-dimensional - center - manifold and are described by a finite-dimensional system of ODEs. The normal form of this system can be obtained. The reversibility and the $O(2)$-symmetry of the system play an important role in the analysis.

Two types of 3D waves are considered: waves which are periodic in the direction transverse to propagation that we call periodically modulated traveling waves, and waves which are periodic in the direction of propagation that we call periodic traveling waves. We have two different formulations of the system depending on which of the two types of waves above we are looking for. In both cases, an infinite sequence of bifurcations of increasing complexity as the surface tension tends to zero is found. In the case of periodically modulated traveling waves, the simplest bifurcation, for the lowest periodic mode, leads to a reduced system of dimension ten, and in the case of periodic traveling waves, to a reduced system of dimension four. Various types of 3D symmetric or asymmetric waves are obtained. Similar waves have been found for a model equation for three-dimensional water-waves in [2] and [3]. For the Euler equations, several types of reversible periodically modulated traveling waves have been constructed in [1] by using a Hamiltonian formulation of the system.

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Gravity and Capillary-gravity traveling waves for superposed fluid layers, one being deep

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The mathematical study of 2D traveling waves in the potential flow of two superposed layers of perfect fluid, with free surface and interfaces (with or without surface tensions) and with the bottom layer of infinite depth, is formulated as a *ill-posed reversible evolution problem*, in a suitable function space, where the horizontal coordinate plays the role of a "time". We give the structure of the spectrum of the linearized operator L_μ near equilibrium (μ denotes the set of parameters). This spectrum consists in a set of isolated eigenvalues of finite multiplicities, a small number of which lie near or on the imaginary axis, and of the *entire real axis* which constitutes the *essential spectrum* where there is no eigenvalue, except 0 in some cases.

A *normal form analysis in infinite dimensional space* is performed. We split the space into i) an invariant subspace belonging to the continuous spectrum (which crosses the imaginary axis) and to eigenvalues far from the imaginary axis and ii) a complementary invariant subspace belonging to eigenvalues near or on the imaginary axis, including 0, if this one is an eigenvalue. This splitting and normal form analysis are made possible thanks to the explicit knowledge of the resolvent of the linear operator L_μn near 0, and to good estimates of the resolvent on the imaginary axis.

Concerning periodic waves, we give a general constructive proof of bifurcating solutions, adapting the Lyapunov-Schmidt method to the present (reversible) case where 0 (which is "resonant") belongs to the continuous spectrum. In particular we give the results for the generic case and for the 1:1 resonance case.

For the 1:1 resonance case, (with an additional 0 eigenvalue embedded in the continuous spectrum in some cases), in the case of the two reversible homoclinic orbits for the reduced normal form, we can show that they do persist for the full system. They correspond to *traveling solitary waves*, with oscillations *algebraically damped* at infinity (the non exponential damping being due to the continuous spectrum).

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Existence and Non-existence of Solitary Waves in Water

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This talk concerns permanent capillary-gravity waves on the free surface of a two-dimensional incompressible inviscid fluid of finite depth with small surface tension. Assume that the flow is irrotational and a single hump wave, called solitary wave, moving with a constant speed will be studied. The existence of solitary waves with small oscillations at infinity, called generalized solitary waves, is considered and the relationship between the amplitude and the phase shift of the oscillation is fully discussed. There are two approaches to obtain the generalized solitary waves, assuming the magnitude of the amplitude first or fixing the phase shift first. It will be shown that the generalized solitary waves obtained from these two approaches are actually connected. By letting the amplitude be a function of the phase shift δ and the long-wave parameter ϵ, it is obtained that there exists a curve in δ-ϵ plane. When δ and ϵ are near this curve, the amplitude can become arbitrary large. Otherwise, the amplitude is exponentially small. For some surface tension close to $1/3$, the amplitude never become zero for small ϵ, which gives the non-existence of truly solitary waves.

Contributed talks

The wave resistance problem for a surface-piercing body

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One considers a "slender" cylinder semisubmerged in a heavy incompressible and inviscid fluid and moving at uniform, supercritical speed in the direction orthogonal to its generators; the problem is to find the resulting steady two-dimensional flow.

By a hodograph transformation, the problem (originally set up in a domain with a free boundary) reduces to the determination of a function, holomorphic in a fixed domain, satisfying different conditions (one of which, the Bernoulli condition, is nonlinear) on different parts of the boundary; but two separating points between these parts, corresponding to the points of contact fluid-cylinder, are unknown and their determination is part of the problem. One proves the existence of a "smooth" solution for which the free boundary and the cylinder profile (which is assumed convex and reasonably smooth) form a single C^1–streamline; furthermore the free boundary is monotone increasing downstream, monotone decreasing upstream and lies under the level of calm water. This is not the only solution of the problem; for one proves that under additional assumptions, another solution exists, which is singular at the contact points fluid-cylinder; such points appear as stagnation points for the flow.

The linear approximation of the wave resistance problem (called Neumann-Kelvin problem) has been extensively studied (see, e.g. [1] and [2]). The analogous problem for a completely submerged body has been treated in [3] and [4].

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5.10 Unsteady Hydrodynamic Waves

Organizer : John F. Toland

Key note lecture

Boundary Integral Methods for Water Waves

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We discuss numerical methods for time-dependent water waves and the analysis needed to prove the convergence of properly designed versions to the actual solution. Boundary integral methods have been widely used for water waves; they are based on singular integrals arising from potential theory. The water surface is tracked by points which move with the fluid velocity. The velocity is determined from an integral equation on the surface. (For a recent survey, see [2].) We will begin by reviewing the equations of water waves and their qualitative properties. We then present the boundary integral formulation and describe its use as a numerical method, with emphasis on the three-dimensional case. In [1] a version of the boundary integral method for doubly periodic water waves is proved to converge to the actual solution. We will discuss several issues related to the design and analysis of this method. A new approach is used for the computation of singular integrals, such as a single layer potential on a surface; the Green's function is regularized and an efficient local correction is computed for the trapezoidal sum. The numerical stability depends on choosing discretizations so that the errors are governed by equations with structure analogous to that of the full equations. For the stability estimates, mapping properties of the discrete integral operators are used which are found by treating the sums as discrete versions of pseudodifferential operators. The Dirichlet-to-Neumann operator on the surface is a fundamental part of the equations of motion; the wavelike character of the motion depends on the positivity of this operator. Similarly, the stability of the numerical method seems to depend on the positivity of the discrete version of this operator.

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Invited lectures

Nash-Moser Theory for Standing Water Waves

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Unlike progressive (or steady) Stokes waves, standing waves are a truly time-dependent phenomenon in the sense that they can not be regarded as stationary relative to a moving reference frame and we believe that, until now, there has been no existence theory, even for the case of small amplitude waves.

Our purpose is to show how the Nash-Moser iteration method can be used to give a rigorous proof of the existence of certain small-amplitude standing water waves on an incompressible irrotational flow of finite depth over a horizontal bottom under gravity, when surface tension is neglected. The waves in question are solutions of the Euler equations, formulated in Lagrangian coordinates, for an ideal incompressible fluid which satisfy exactly the constant-pressure and kinematic boundary conditions on the free surface. But in addition, the solutions for which existence is established satisfy further constraints which are expressed in terms of the normal component of pressure gradient on the free surface. These constraints are designed to facilitate the mathematical development here, with particular regard to the method for obtaining a priori bounds necessary for the Nash-Moser method. They have no obvious analogue for it in the theory of Stokes waves.

Because the proof is long and quite technical it will be possible only to outline the main result and to sketch the method in the limited time available.

Justification of quasistationary approximation in the problem of evolution of a capillary drop

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The problem of motion of a viscous capillary drop consists in the determination of a bounded domain $\Omega_t \subset R^3$, of the velocity vector field $\vec{v} = (v_1(x,t), v_2(x,t).v_3(x,t))$ and of the scalar pressure $p(x,t)$, $x \in \Omega_t$, satisfying the Navier-Stokes equations

$$\varepsilon(\vec{v}_t + (\vec{v}\cdot\nabla)\vec{v}) - \nabla^2\vec{v} + \nabla p = 0, \qquad \nabla\cdot\vec{v} = 0, \tag{1}$$

initial condition

$$\vec{v}(x,0) = \vec{v}_0(x) \tag{2}$$

in a given domain Ω_0 and the conditions at the free boundary $\Gamma_t = \partial\Omega_t$

$$\vec{v}\cdot\vec{n} = V_n, \qquad T(\vec{v},p)\vec{n} - H\vec{n} = 0. \tag{3}$$

Here ε is a positive parameter proportional to the Reynolds number, V_n is the velocity of evolution of Γ_t in the normal direction $\vec{n}$, T is the stress tensor and H is the doubled mean curvature of Γ_t. Quasistationary approximation consists in setting $\varepsilon = 0$ in (1) (which removes the time derivative $\vec{v}_t$ and the nonlinear term) and in the elimination of the initial condition (2) for $\vec{v}$ (but not for Ω_t). One can expect that the approximating solution is close to $(\vec{v}, p)$ in the case of small ε.

We prove that the problem (1)-(3) is uniquelly solvable on a certain finite time interval $(0, t_1)$ independent of ε and that the difference between the exact and the approximating solutions has the order of magnitude $O(\varepsilon)$ for $t > t_0 > 0$.

The proof is based on a new Schauder type estimate for the linearized problem (1)-(3).

Well-posedness in Sobolev spaces of the full water wave problem

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We consider the motion of the interface separating an inviscid, incompressible, irrotational fluid from a region of zero density in the three dimensional space; we assume that the fluid region is below the region of vacuum and the fluid is under the influence of gravity and the surface tension is zero. Assume that the density of mass of the fluid is one, the gravitational field is $(0, 0, -1)$, and at time $t \geq 0$, the free interface is $\Sigma(t)$, and the fluid occupies the region $\Omega(t)$, the motion of the fluid is described by

$$v_t + v \cdot \nabla v = -(0,0,1) - \nabla p \qquad on\ \Omega(t),\ t \geq 0, \tag{1}$$

$$div\, v = 0 \qquad on\ \Omega(t),\ t \geq 0, \tag{2}$$

$$curl\, v = 0, \qquad on\ \Omega(t),\ t \geq 0. \tag{3}$$

where $v = (v_1, v_2, v_3)$ is the fluid velocity, p is the fluid pressure. Since we neglect the surface tension, the pressure is zero on the interface. So on the interface:

$$p = 0, \qquad on\ \Sigma(t) \tag{4}$$

$$(1, v) \text{ is tangent to the free surface } (t, \Sigma(t)). \tag{5}$$

The above model is a 3-dimensional water wave model. It is generally known that when surface tension is neglected, the motion of the interface between an inviscid fluid and vacuum under the influence of gravity can be subject to Taylor instability [1], [2]. In a previous work [3], we studied the 2-dimensional water wave model, we showed that for 2-D water wave, the sign condition relating to Taylor instability always holds for nonself-intersecting interface, that is, the motion of the interface is not subject to Taylor instability; we showed further that the 2-D full nonlinear water wave problem is wellposed in Sobolev spaces, locally in time, for any initially nonself-intersecting interface.

In this talk, I will show that the results we obtained for the 2-D water wave also hold for 3-D, that is, the motion of 3-D water wave is not subject to Taylor instability and the full 3-D water wave problem is uniquely solvable in Sobolev spaces for any nonself-intersecting initial interface. I will demonstrate that Clifford analysis is an effective tool for 3-D water wave problem.

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Contributed talks

Surface waves for an incompressible ideal fluid

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In this communication, we are concerned with a free boundary problem for the motion of an incompressible ideal fluid under the gravitational field in a two-dimensional domain of infinite extent and of finite depth. Our problem is to find the two-dimensional domain occupied by the fluid which is bounded below by the fixed bottom and above by the free surface together with the velocity vector field and the pressure of the fluid satisfying the system of Euler equations and the initial-boundary conditions. The Euler equations consist of the conservations of mass and momentum under the gravitational field in a downward direction. The purpose of this paper is to establish the temporary local solvability in a Sobolev space.

We also consider the same problem as above taking into account of the surface tension on the free surface. For this problem we prove the temporary local existence theorem irrespective of the direction of the gravitational field, and the convergence of this solution to the cprreponding one as the surface tension coefficient tends to zero if the gravitational field acts in a downward direction.

For the proof, we rely on the methods due to Nalimov and Yosihara for the water waves, and to Iguchi-Tanaka-Tani for the vortical water waves of infinite depth.

F. Further topics

6.1 Delay Equations

Nonlinear Retarded Functional Differential Equations of Arbitrary (Fractional) Orders

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The arbitrary (fractional) order integral operator I^{α} , $\alpha \geq 0$ is a singular integral operator, and the arbitrary (fractional) order differential operator $D^{\alpha} = I^{-\alpha}$, $\alpha \geq 0$ is a singular integro-differential operator, they generalize (interpolate) the integral and differential operators of integer orders. The fractional calculus $\{I^{\alpha} ; \alpha \in R\}$ is enjoying growing interest not only among mathematicians, but also among physicists and engineers. In this lecture we study the existence of monotone solution (in the class $L_1(0,T)$ of Lebesgue integrable functions) of the problems

$$D_a^{\alpha}\, x(t) = F(t, x(m(t))) , \quad x(a) = x_o \geq 0 , \quad \alpha \in (0,1],$$

where the function F is a nonlinear function satisfies the Caratheodory conditions, and the more general one

$$D^{\alpha}x(t) = f(t, x(t), D^{\alpha_1}x(t-r), D^{\alpha_2}x(t-2r)), ., ., ., D^{\alpha_n}x(t-nr)) \ , \ t \in I.$$

$$D^{j}x(t) = 0 \ , \quad for \ \ t \leq 0 \ , \ j = 0, 1, 2, ..., n.$$

Here the function $f(t,U)$ satisfies the Caratheodory conditions i.e., $t \to f(t,U)$ is measurable for every $U \in R^{n+1}$ and $U \to f(t,U)$ is continuous for every $t \in I$ and $\alpha \in (n, n+1]$, $\alpha_k \in (k-1, k]$, $k = 1, 2, ..., n$, and $\alpha_o = 0$.
As special case we also consider the following equations:

$$D^{\alpha}x(t) = a(t) + \sum_{k=0}^{n} g_k(t, x(t)) D^{\alpha_k}x(t-kr) \ , \ t \in I,$$

$$D^{\alpha}x(t) = a(t) + \sum_{k=0}^{n} f_k(t, D^{\alpha_k}x(t-kr)) \ , \ t \in I,$$

with the initial conditions (3), where f_k and g_k are nonlinear functions that satisfy the Caratheodory conditions.

Special slowly oscillating periodic solution of a state dependent delay differential equation

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We consider the following state dependent delay differential equation

$$\dot{x}(t) = f(x(t - r(x(t))))$$

where f is a real smooth function. This equation playes an important role in medicine, economy and so forth... We are interested in the existence of slowly oscillating periodic solutions, in particular special symmetric ones.

6.2 Numerics

On the Mathematical Foundation of the Nonstandard Finite Difference Method

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The finite difference method is one of the oldest, simplest and thus very popular technique for the numerical treatment of differential equations. For most of the equations in mathematical physics finite difference schemes have been designed and investigated from both theoretical point of view (convergence) and practical point of view (consistency, stability). Furthermore, provided the solution is smooth enough, these schemes produce numerical solution with optimal rates of convergence when the step-size approaches zero.

One disadvantage of this standard approach is that qualitative properties of the exact solution are not transferred to the numerical solution. In practice, the limit of the step-size is not reached. What we have is the numerical solution obtained for one or several values of the step-size. Thus, the stated disadvantage might be catastrophic.

The nonstandard approach discussed in this talk preserves essential properties of the exact solution. This is achieved by replacing derivatives by nonstandard finite difference operators with denominators that are suitable function of the step-size, nonlinear expressions being, in general, approximated nonlocally.

Nonstandard finite difference techniques were developed empirically [1] for solving practical problems in applied sciences and engineering. Although they produce results satisfactory to their users, these techniques have not yet been subjected to rigorous mathematical analysis. Some concepts are still unclear.

Our aim is to give rigorous mathematical meaning and justification of some key concepts involved in the design of nonstandard finite difference schemes. We also discuss a certain number of "successful" empiric procedures the mathematical justification of which is still pending.

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Optimal derivative procedure of a nonliear ODE.

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We introduce a certain notion of approximation defined as the optimal derivative of the following nonlinear ordinary differential equation

$$\frac{dx}{dt} = F\left(x\left(t\right)\right), x\left(0\right) = x_0.$$

This approximation is global in the neighborhood of a steady state. That is obtained as a sort of mean value of the derivative of the nonlinear function along trajectories linking the initial value to the origin. The procedure is based on the minimization of a certain functional with respect to a curve starting from an initial value x_0 and going to 0 as t goes to infinity. At each step, it gives a linear map, starting from the Jacobian matrix $DF\left(x\right)$ estimated at the initial value x_0. The optimal Derivation of the nonlinear equation is obtained as a limit of the sequence of linear maps determined by the procedure and can be written as

$$\frac{dx}{dt} = \tilde{A}x\left(t\right), x\left(0\right) = x_0.$$

Our results are in the line of previous work by Vujanovic in (1973) and Jordan et al (1987).

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An Approximate Method of Solution of Some Types of Singular Integro-Differential Equations

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The presentation is devoted to approximate method of solutions of singular integro-differential equations, which is based on the approximation of functions by interpolated polynomials, fundamental functions of which are characteristic functions of differential and integral operators. This method was discussed in the papers [1]. We here spread it on the Cauchy problem for many types of singular and hypersingular integro-differential equations as:

$$a(t)x(t) + b(t)\int_{-1}^{1} \frac{p(\tau)x'(\tau)d\tau}{\tau - t} + \lambda \int_{-1}^{1} p(\tau)h(\tau)d\tau = f(t), \; -1 < t < 1,$$

where $p(t)$ is a weight, $a(t), b(t), h(t), f(t) \in H_\alpha$,

$$\int_{-1}^{1} \frac{p(\tau)x'(\tau)d\tau}{(\tau - t)^2} + \lambda \int_{-1}^{1} p(\tau)h(\tau)d\tau = f(t), \; -1 < t < 1,$$

where $\int_{-1}^{1} x(t)t^{-2}dt$ is hypersingular integral.

For example, let us consider the Cauchy problem for the Prandtl singular integro-differential equation of finite wing:

$$K\Gamma \equiv \frac{\Gamma(t)}{b(t)} - \frac{1}{\pi}\int_{-1}^{1} \frac{\Gamma'(\tau)d\tau}{\tau - t} = f(t), \quad \Gamma(-1) = \Gamma(1) = 0$$

where $-1 < t < 1$, $b(t), f(t) \in H_\gamma$ $(0 < \gamma < 1)$. We obtain the approximate solution in the analytical form

$$\Gamma_n(t) = \sqrt{1 - t^2}\sum_{k=0}^{n} \left(\frac{1}{\gamma_k} \sum_{i=0}^{n} U_i(\mu_k)U_i(t) \right) \frac{f(\mu_k)B(\mu_k)}{(\sqrt{1 - \mu_k^2} + B(\mu_k))}$$

where $\gamma_k = \sum_{i=0}^{n} U_i^2(\mu_k)$ is a Chebyshev polynomial of the second kind, k is its degree and μ_k, $k = 0, 1, \ldots, n$, are the roots of the polynomial $U_{n+1}(t)$.

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Event recognition using a binary discriminant tree

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In this paper, we propose a new design procedure for constructing a multivariate and discriminant binary tree for classifying high energy reactions data.

A binary decision tree is usually generated by repeated splits of the learning event set into two descendant subsets [1]. At each node of the tree, the split is obtained by a cut on the selected variable. Several attempts have been made to construct binary trees otherwise [2-4].

Instead of an univariate design, we suggest to use some linear discriminant functions. These discriminant functions are combinations of discriminant variables. Their choice is imposed by the topology of the events to be classified. Moreover, the computation of the functions' cuts values is based on minimizing the Kolmogorov- Smirnoff distance. Finally, a test of performance using efficiencies and purities of classifications is used.

Experiments on real and simulated test data, containing four classes of high energy reactions events show that the proposed discriminant tree classifier is faster and more efficient than the univariate binary decision tree.

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On the investigation of boundary value problems

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One of the problems which will be considered is the following two-point non-linear boundary value problem:

$$\begin{aligned} dx/dt &= f(t, x, \lambda_1, \lambda_2), \\ Ax(0) + C(\lambda_1)x(T) &= d(\lambda_1, \lambda_2), \qquad x_1(0) = x_{10},\ x_2(0) = x_{20}. \end{aligned}$$

Here, $f \in C([0,T] \times D \times [a_1, b_1] \times [a_2, b_2], R^n)$, $D \subset R^n$ ($n \geq 3$) is closed and connected, and $d \in C([a_1, b_1] \times [a_2, b_2], R^n)$; A and $C(\lambda_1)$ are members of $R^{n\times n}$ such that $\det[k_1 A + k_2 C(\lambda_1)] \neq 0$ for some real $k_1 \neq k_2$ and all $\lambda_1 \in [a, b]$; $\lambda_1 \in [a_1, b_1]$ and $\lambda_2 \in [a_2, b_2]$ are unknown scalar parameters.

We give a numerical-analytic method of successive approximations to establish existence theorems for a solution $(x^*, \lambda_1^*, \lambda_2^*)$ and construct approximations to the solution.

It is shown that, under certain conditions, the sequence

$$\begin{aligned} x_{m+1}(t, z, \lambda) = z + k_1 H d_1(z, \lambda) &+ \int_0^t f(s, x_m(s, z, \lambda), \lambda_1, \lambda_2) ds \\ &- \frac{t}{T} \int_0^T f(\tau, x_m(\tau, z, \lambda), \lambda_1, \lambda_2) d\tau + \frac{t}{T} H d_1(z, \lambda) ds, \end{aligned}$$

where $m = 0, 1, 2, \ldots$, $x_0(t, z, \lambda) \equiv z + k_1 H d_1(z, \lambda)$, $H = [k_1 A + k_2 C(\lambda_1)]^{-1}$, $d_1(z, \lambda) = d(\lambda_1, \lambda_2) - [A + C(\lambda_1)]z$, $\lambda = (\lambda_1, \lambda_2)$,

$$z = \mathrm{col}(x_{10}, x_{20}, y_1, \ldots, y_{n-2}) = \mathrm{col}(x_{10}, x_{20}, y),$$

uniformly converges.

The function $x^*(t,z,\lambda) = \lim_{m\to\infty} x_m(t,z,\lambda)$ is a solution of the original problem iff the pair (y,λ) satisfies the determining equation

$$\Delta(y,\lambda) := \frac{1}{T}(k_2 - k_1)Hd_1(z,\lambda) - \frac{1}{T}\int_0^T f(t, x^*(t,z,\lambda),\lambda_1)dt = 0.$$

A survey of the investigations concerning the method under consideration for various types of problems can be found in [1, 2].

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Coupling of kinetic and hydrodynamic equations: the case of Carleman model

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The coupling of kinetic equations and their hydrodynamic limits was introduced and studied by Le Tallec and Tidriri [2,3] and Tidriri [4]. This approach was introduced in order to solve several difficulties that occur at the interface between fluid mechanics and kinetic theory. Because of the practical importance of these methodologies, the establishment of their mathematical foundations is of crucial importance. The mathematical theory of such coupling started in [4] and continued in [2,3], where the coupling of two models of hydrodynamical type is considered. In [5,7], the author provided an analysis of the coupling of two models of kinetic type. In [6] the author provided the analysis of the coupling of kinetics equations and their hydrodynamics limits for the Carleman model. In these papers, the author established the existence theory and the asymptotic behaviour of the resulting coupled systems. Moreover, he studied the semi-discretized (with respect to the time variable) coupled systems via the Time Marching Algorithm also introduced in [2,4] and established the convergence theory for the resulting coupled problems. Finally, he provided numerical results confirming the above mentioned mathematical results.

As far as we know the above mentioned results are the first mathematical results about the analysis of coupled systems of kinetic equations or coupled systems of kinetic equations and their hydrodynamical limits.

In our proposed talk, we shall first, present a motivation to our studies by giving some numerical results and experimental evidences that show why such coupling is of crucial practical importance. we shall then give more details about the analysis of the coupling of kinetic equations and their hydrodynamics limits.

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6.3 Ordinary Differential Equations

Uniformly Isochronous Quintic Planar Vector Fields

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In this paper we analyze the center problem, and consequently the isochronicity, for the vector fields $\left(-y + x\sum_{i=1}^{4} H_i(x,y), x + y\sum_{i=1}^{4} H_i(x,y)\right)$, with H_i homogeneous polynomial of the i-th degree, $i = 1, \ldots, 4$, and $H_i H_4 \neq 0$ for some $i = 1$, 2 or 3. To treat such problem basically we use procedure based in normal forms and also characterization of center through the existence of Lie's symmetries of the system (see [**1**]). Besides, we give commutators, null divergence factors and first integrals of some subfamilies. We also include the phase portrait of the centers with polynomial commutators on the Poincare disk. We finish the paper with two open problems.

The vector fields we are studing belong to the family

$$(S) \qquad (-y + xH(x,y), x + yH(x,y)),$$

where H is an analitic function in a neighbourhood of the origin, with $H(0,0) = 0$. As far as we know, some subfamilies of these systems have been studied by several authors. [**4**] analyzes (S) with H an homogeneous polynomial of an arbitrary degree, drawing to the conclusion that if the degree of H is odd the origin is a center, whereas if it is even it's necesary that the integral of H on S^1 must vanish. Moreover, in [**1**] the isochronous condition have been obtained according to the coeficients of the systems. [**7**] research some properties for first integrals and for commutators of the systems studied by Conti with H an homogeneous function. [**6**] provides linearizating changes for the reversibles systems (S) with $H = H_1 + H_2$. Next, [**5**] removes the reversibility condition and shows that the only existing centers are the reversible ones. [**2**] and [**3**] study, in an independent way, the centers of the systems (S) with $H = H_1 + H_2 + H_3$, with $H_i H_3 \neq 0$, $i = 1, 2$. All of them are reversible centers and there are some isochronous ones which don't have polynomial commutator.

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Investigation of Resonances Using Higher Order Normal Forms

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In the present analysis the higher order normal forms technique [2] is applied to the local analysis of the shock adiabat in an elasto-plastic solid and the transient resonance response of a non-linear slow-variant simple degree of freedom system. The solution of these problems is reduced to the investigation of a trajectory dynamic which is governed by the corresponding systems of non-linear ordinary differential equations. The local analysis in the vicinity of a stationary point is provided in the resonance case when the corresponding linear part has a multiple zero eigenvalue. The step to step technique of nearly identity transformations is applied to the analysis of a dynamics on the central manifold.

The shock adiabat in elasto-plastic solid with the smooth convex yield surface is constructed by embedding the Kelvin-Voigt type viscosity element in the rheological model [1], [3]. In the general case of a combined loading in the viscous layer the problem is reduced to the investigation of the following system of ordinary differential equations

$$\frac{du_i}{d\xi} = \Big(L_{1i1p}(\sigma_{\alpha\beta}) - (c^2+\varepsilon)\delta_{ip}\Big)u_p\,, \qquad \frac{d\sigma_{ij}}{d\xi} = -L_{ijk1}(\sigma_{\alpha\beta})u_k$$

here ξ is the coordinate of a viscous layer, u_i and $\sigma_{\alpha\beta}$ are accelerations and stresses, $L_{ijkl}(\sigma_{\alpha\beta})$ are components of the acoustic tensor, c is the sound velocity at the state after the shock, ε is the small nonnegative parameter.

Small intensity shock waves correspond to the heteroclinic type trajectories

$$\lim_{\xi\to\pm\infty} u_i(\xi) = 0\,, \quad \lim_{\xi\to\pm\infty} \sigma_{ij}(\xi) = \sigma_{ij}^{\pm}$$

which satisfy the inequality of an active plastic load:

$$A_{pqk1}F_{pq}(\sigma_{\alpha\beta})u_k \le 0$$

A_{ijkl} are elastic moduli, $F_{ij}(\sigma_{\alpha\beta})$ is the yield surface gradient.

Using the Lax evolutionary conditions the types of bifurcations with respect to a change in the velocity of a shock wave are found. An integration of the normal form is reduced to the investigation of an ordinary differential equation of the first order. The conditions of solvability which follow from the inequality of active plastic load are established. Numerical simulations of heteroclinic orbits are performed.

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Differential Equations of Infinite Order with Subordinate Terms

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Let $G \subset R^\nu$, $\nu > 1$, be a bounded domain and Γ be a boundary of the domain. The solvability of the following Dirichlet problem for nonlinear differential equations of infinite order is being considered

$$L_1(u) + L_2(u) = \sum_{|\alpha|=0}^{\infty} (-1)^{|\alpha|} D^\alpha A_\alpha(x, D^\gamma u) + \sum_{|\alpha|=0}^{\infty} (-1)^{|\alpha|} D^\alpha B_\alpha(x, D^\omega u) = h(x),$$

$$x \in G, \tag{1}$$

$$D^\beta u(x) = 0, \qquad x \in \Gamma, \qquad |\beta| = 0, 1, \dots . \tag{2}$$

The $A_\alpha(x, D^\gamma u)$, $B_\alpha(x, D^\omega u)$ are continuous functions of $x \in G$ and of all possible derivatives $D^\gamma u$ and $D^\omega u$, $|\gamma| \leq |\alpha|$, $|\omega| \leq |\alpha|$. The right side $h(x)$ of equation (1) may be a generalized function with singularity of infinite order.

Definition. The differential operator $L_1(u)$ is called principal in comparison with operator $L_2(u)$ (henceforth referred to as subordinate) if the space, being the domain of $L_1(u)$, is compactly embedded in the space corresponding to $L_2(u)$.

The conditions for compact embedding of the corresponding spaces for the comparison of the operators $L_1(u)$ and $L_2(u)$ has been studied. That allowed us to receive the conditions of solvability of the problem (1), (2), to indicate the methods of its examination. If (1) is the linear equation containing subordinate terms then it is Fredholm. That theory has allowed to receive the solvability of the series of problems which could not be solved earlier. That is demonstrated by examples.

On n-dimensional Riccati extended forms

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We deal with initial value problems for the system of implicit difference equations of the form

$$A^*(n)\Phi(n) - \Phi(n+1)f^*A^*(n)\Phi(n) = 0, \qquad n = 0, 1, \ldots,$$

where the $(m+1)\times(m+1)$ complex matrices $A(n)$ are composed of entries $a_{i,j}(n)$, $i, j = 1, \ldots, m+1$, f is a fixed nonzero vector of $m+1$ complex components. The initial value is prescribed as $\Phi(0) = \Phi_0$.

For the lowest dimension ($m = 1$), the problem reduces to iteration

$$x(n+1) = \frac{\overline{a}_{11}(n)\,x(n) + \overline{a}_{12}(n)}{\overline{a}_{21}(n)\,x(n) + \overline{a}_{22}(n)}, \quad n = 0, 1, \ldots.$$

This iteration with constant real coefficients was studied by Brandt. Later, Kočic and Ladas considered real varying coefficients, too. Recently in the joint paper with Agarwal, we investigated the case with complex, constant coefficients when $m = 2$. We described the equilibria. Some problems of asymptotic stability were addressed there, too.

In general case, we observe that there exists at most one local solution $\Phi(n)$, $n = 0, \ldots, N+1$, of the problem such that $f^*A^*(n)\Phi(n) \neq 0$, $n = 0, \ldots, N$. For this local solution $f^*\Phi(n) = 1$, $n = 1, \ldots, N+1$, holds.

It turns out that the local solution can be given directly. It helps us to formulate a criterion for the the existence of the global solution.

Attention is paid to the constant case. The results obtained for the case $m = 2$ earlier for equilibria and asymptotic stability are extended now, as well.

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On the Solutions of the Second Order Differential Equation with Variable Parameters

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Consider a linear differential equation

$$\frac{d^2x}{dt^2} + p(t)\frac{dx}{dt} + q(t)x = 0, \tag{1}$$

where t is time, $p(t), q(t)$ are continuous functions which are bounded on every compact. In [1] one can find that no solution of (1) can possess a zero of order greater than one. Let t_1 and t_2 denote consecutive zeros of some solution $x(t)$ of the equation (1) $(t_2 > t_1)$.

Theorem. Let $a = \sup\limits_{t_1 \le t \le t_2} |p(t)|,\ b = \sup\limits_{t_1 \le t \le t_2} |q(t)|$. Then

1) $t_2 - t_1 \ge \frac{4}{\sqrt{4b-a^2}}(\frac{\pi}{2} - \arctan\frac{a}{\sqrt{4b-a^2}})$ if $4b > a^2$;

2) $t_2 - t_1 \ge \frac{4}{a}$ if $4b = a^2$;

3) $t_2 - t_1 \ge \frac{2}{\sqrt{a^2-4b}} \ln\frac{a+\sqrt{a^2-4b}}{a-\sqrt{a^2-4b}}$ if $4b < a^2$.

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Some Criteria on the Uniqueness of Limit Cycles in planar Polynomial Systems

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In this article we give criteria on the uniqueness of limit cycles in planar polynomial systems. It is well known that provided a system has an invariant straight line, there can be at most one limit cycle [1]. Also that parabolas, which are particular solutions of quadratic systems can coexist with limit cycles [2]. We prove the uniqueness of some known algebraic limit cycles for quadratic systems. We present also criteria and proofs on the existence and uniqueness of limit cycles under some conditions, as well as open questions on this subject.

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Some unexpected properties of limit cycles of quadratic systems in the plain

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In the generic case every autonomous quadratic system in the plane, which has a finite singular point, can be reduced to a quadratic system with five essential parameters. We consider two of them

$$\frac{dx}{dt} = -(x+1)y + \alpha Q(x,y)\ , \quad \frac{dy}{dt} = Q(x,y)\ ,$$
$$Q = x + \lambda y + ax^2 + b(x+1)y + cy^2\ ; \tag{1}$$

$$\frac{dx}{dt} = 1 + xy\ , \quad \frac{dy}{dt} = \sum_{i+j=0}^{2} a_{ij}x^i y^j\ , \quad \sum_{i+j=0}^{2} a_{ij}(-1)^j = 0. \tag{2}$$

System (1) has the larger number of rotation parameters λ, b, α. For every parameter of system (1) (for example, b) we can define the curve of limit cycles at every point (x, b) on the plane $x = b$ which corresponds to a limit cycle of system (1) passing through the point $(x, 0)$ for given value parameter b. The curve of limit cycles corresponds to a manifold of limit cycles in the space (x, y, b). It is known [1] that for a given rotation parameter (for example, b) the curve of limit cycles corresponds to a unique function of the variable x having a point of Andronov-Hopf bifurcation. We establish that in contrary for parameters a, c corresponding curves of limit cycles can have some branches without points of Andronov-Hopf bifurcation and they can have also turning points. We establish that quadratic system (2) can have around a weak focus $A(1, -1)$ a separatrix cycle with two saddles at infinity and critical stability ($a_n = 0$).

We prove also that it is possible to take as Dulac's function the function $B = |\Psi(x,y)|^{1/k}$, $k = const$, Ψ is a polynomial with respect to y of degree $2m$, such that

$$div(Bf) = \frac{1}{k} sign\Psi\, |\Psi|^{1/k-1}\, P(x), P(x) > 0, x > 0.$$

It means [2], that the number of limit cycle of system (2) around the focus $A(-1,1)$ is not more than m, $m = 1,2,3$; f is the vector field associated with system (2). So we have a posibility to attack the Hilbert problem for quadratic systems.

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Limit sets in planar systems of piece-wise linear differential equations with a line of discontinuity

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We consider planar systems of piece-wise linear differential equations with a line of discontinuity. Systems of this type have applications in science and engineering. They provide mathematical models for mechanical systems with Coulomb friction, or for electrical circuits with a thin triode, or for direct control systems with a two-point relay characteristic, see [1]. We estimate all possible limit sets of such systems. They consist of stationary, periodic, homoclinic and heteroclinic orbits. We study the existence, number and stability of the orbits mentioned above. In our analysis we distinguish between stationary points, which do or do not belong to the line of discontinuity, and orbits with or without sliding motion, see [2].

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On stability and bifurcations in nonlinear systems of differential equations

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We consider the nonlinear system of differential equations

$$\frac{dx}{dt} = A(\mu)x + X(\mu, x), \; X(\mu, 0) = dX(\mu, 0) = 0 \tag{1}$$

continuously depending on a vector parameter μ.

One of the most studied phenomena arising in the neighborhood of the point $x = 0$ is the Hopf's bifurcation. It arises when by $\mu = 0$ the matrix A(0) has a pair of pure imaginary eigenvalues. In this talk the analogous problem connected with the study of higher codimension bifurcation is considered.

Our main concern was with the following problems.

i) Stability of the equilibrium point of the system (1), when at $\mu = 0$ the matrix $A(0)$ has m a pair of purely imaginary eigenvalues. (The results obtained for the problem (i) relate to two different situations: non-resonance case and resonant case.)

ii) Bifurcation of m-dimensional invariant tory, when $\mu \neq 0$ (non-resonance case).

iii) Bifurcation of steady resonance modes.

Lyapunov functions in estimates of dimension of attractors of dynamical systems

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Lyapunov functions are introduced into the estimates of Hausdorff and fractal measures of the sets which are shifted along the trajectories of differential equations. As a result new estimates of attractors also containing Lyapunov functions are obtained. Lyapunov functions in the estimates of Hausdorff dimension of attractors of Lorenz system are constructed. With the help of these functions the validity of Eden conjecture that the dimension of Lorenz attractor is estimated by the Lyapunov dimension of a zero saddle point is proved.

For linear homogeneous continuous and discrete systems frequency-domain estimates of singular numbers of fundamental matrices are received. Here Yakubovich-Kalman and Kalman-Szego theorems about solvability of matrix inequalities are applied. On the base of frequency-domain estimates of singular numbers estimates of Lyapunov dimension of Lorenz and Hennon attractors are obtained. The comparison of these estimates with computer experiments is done.

The connection of these results and the classical theory of motion is discussed. Special attention is paid to the global stability.

On the Stability of a Second Order Linear Differential Equation with Variable Parameters

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Consider an oscillator described by the following differential equation

$$\ddot{x} + f(t)\dot{x} + g(t)x = 0 \tag{1}$$

where the damping and rigidity coefficients $f(t)$ and $g(t)$ are continuous and bounded functions of the time t. Most of the theories examining a stability problem of the zero solution are based on the Lyapunov stability and instability theorems and the corresponding Lyapunov function is assumed as an energy-type function

$$V = \frac{1}{2}c_1(t)\dot{x}^2 + \frac{1}{2}c_2(t)x^2$$

where $c_1(t), c_2(t)$ are time variable functions. In [1], A.P.Merkin considered the case $c_1(t) = c_2(t) = 1$ and stability conditions were obtained only for constant f and g. An extension was done in [2] for periodic functions $f(t)$ and $g(t)$. By means of a Lyapunov function which is a quadratic form with respect to x and $\dot{x}$, V.M.Starzhinsky [3] (assuming that $0 < l \le f(t) \le L,\ 0 < m \le g(t) \le M$) obtained sufficient conditions of asymptotical stability for the solution

$$x = 0, \quad \dot{x} = 0 \tag{2}$$

of equation (1). They are written as restrictions to the constants l, L, m, M.

In this paper sufficient asymptotic stability conditions of the solution (2) are obtained which are close to necessary and sufficient conditions of stability. We suppose that $g(t)$ is continuously differentiable and that the inequalities

$$|f(t)| < M_1, \quad |g(t)| < M_2, \quad |\dot{g}(t)| < M_3$$

hold for $t \in R_+ = [0; \infty)$.

Theorem. If the conditions

$$g(t) > \alpha_1 > 0, \qquad p(t) = \frac{1}{2}\frac{\dot{g}(t)}{g(t)} + f(t) > \alpha_2 > 0$$

are fulfilled, then the solution (2) of the differential equation (1) is uniformly asymptotically stable.

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Decoupling of impulsive differential equations

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Impulsive differential equations provide an adequate mathematical models of evolutionary processes that suddenly change their state at certain moments. The theme of this communication is the dynamical equivalence of impulsive differential system to a system which is simpler than the given one in terms of decoupling and linearization.

Let us consider the impulsive system in the Banach space $\mathbf{U} = \mathbf{X} \times \mathbf{Y}$

$$\begin{cases} dx/dt = A(t)x + f(t,u),\ dy/dt = B(t)y + g(t,u), \\ \Delta x|_{t=\tau_i} = x(\tau_i + 0) - x(\tau_i - 0) = C_i x(\tau_i - 0) + p_i(u(\tau_i - 0)), \\ \Delta y|_{t=\tau_i} = y(\tau_i + 0) - y(\tau_i - 0) = D_i y(\tau_i - 0) + q_i(u(\tau_i - 0)), \end{cases} \tag{1}$$

where $u = (x, y)$, $i \in \mathbb{N}$, f, g, p_i, q_i satisfy Lipschitz conditions with small ε uniformly with respect to t. Let $\Phi(\cdot, t_0, u_0)\colon \mathbb{R} \to \mathbf{U}$ be the solution of the system (1), where $\Phi(t_0 + 0, t_0, u_0) = u_0$.

Definition. Two impulsive differential systems are *dynamically equivalent* if there exists a map $H\colon \mathbb{R} \times \mathbf{U} \to \mathbf{U}$ and function $e\colon \mathbf{U} \to \mathbb{R}_+$ such that
(i) $H(t, \cdot)\colon \mathbf{U} \to \mathbf{U}$ is a homeomorphism.
(ii) $H(t, \Phi(t, t_0, u_0)) = \Psi(t, t_0, H(t_0, u_0))$ for all $t \in \mathbb{R}$, where $\Psi(\cdot, t_0, u_0)\colon \mathbb{R} \to \mathbf{U}$ is the solution of the second system.
(iii) $\max\{|H(t,u) - u|, |H^{-1}(t,u) - u|\} \le e(u)$.

We find sufficient conditions under which the impulsive system (1) is dynamically equivalent to the system

$$\begin{cases} dx/dt = A(t)x + f(t,x,k(t,x)),\ dy/dt = B(t)y, \\ \Delta x|_{t=\tau_i} = C_i x(\tau_i - 0) + p_i(x(\tau_i - 0), k(\tau_i - 0, x(\tau_i - 0))), \\ \Delta y|_{t=\tau_i} = D_i y(\tau_i - 0). \end{cases} \tag{2}$$

The last system splits into two parts. The first of them does not contain the variable y, while the second one is linear. This result allows one to replace the given system by a simpler one. Relevant results concerning decoupling and simplifying of the impulsive differential systems are given also. The proof of the theorems is based on a Green maps and a functional integral equations technique.

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Autonomous Differential Systems with Prescribed Limit Set

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We present a class of non-linear differential systems for which an invariant set can be prescribed. Moreover, we show that a system in this class can be explicitly solved if a certain associated linear homogeneuos system can be solved. As a simple application we construct a plane autonomous system having a given closed curve on polar form as the only limit cycle .

Monotonicity and boundedness of the atomic radius in the Thomas-Fermi theory

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Consider an atom consisting of N electrons and a nucleus of positive charges Z. The ground state density of electrons is a non negative function $\rho \in L^1(\mathbb{R}^3)$ such that $\int_{\mathbb{R}^3} \rho(x)dx = N$. In the semiclassical Thomas-Fermi approximation the consistency for a neutral atom ($N = Z$) can be expressed : ϕ denoting the total electric potential,

(A1) $\phi(x) = Z\ |x|^{-1} - \int \rho(y)\ |x-y|^{-1}\ dy$ (Electrostatic attraction and repulsion) and

(A2) $\rho(x) = [2\sqrt{2}/(3\pi^2)]\ \phi(x)^{3/2}$ (Pauli-principle).

For any $k > 0$, the radius $R_k(N, Z)$ such that k electrons lie outside $B(0, R_k(N, Z))$ is defined by

(A3) $\int_{|x|>R_k(Z)} \rho(x)dx = k; \quad R_k(Z) := R_k(N, Z).$([1])

After some normalizations using the Thomas-Fermi scaled potential ($\phi^{TF}(x) := Z^{4/3}u(z^{1/3}x)$), these equations are rewritten as:

$u'' + \frac{2}{r}u' = u_+^\gamma, \quad r > 0; \quad \gamma = 3/2$

$\rho(r) = u(r)_+^{3/2} \quad \int_{r>R(z)} \{z^{4/3}\ u(z^{1/3}\ r)\}^{3/2} r^2 dr = k.$

After some qualitative investigations of a more general (singular ground state) problem

$$U'' + \frac{a}{r}U' = U_+^\gamma, \quad r > 0; \quad a > 1; \quad \lim_{r\searrow 0} U(r) = +\infty; \quad \lim_{r\nearrow\infty} U(r) = 0 \tag{1}$$

we prove the following main theorem for $a = 2, \quad \gamma = 3/2$ and any $k > 0$:

Theorem A

The problem (S0) has a unique solution $U \in C^2((0,\ \infty))$ such that $U \simeq r^{1-a}$ at 0

and $U \simeq r^{-4}$ at ∞ and
i) any admissible $z \equiv Z$ in (A1) - (A3) satisfies
$z \geq z_k := k/\{\int_0^\infty U(r)^{3/2}\, r^2 dr\}$;
ii) $R_k(z_k) = 0$ and $\{d/dz\}R_k(z) > 0$ in $z \geq z_k$;
iii) $0 < \lim_{z \nearrow \infty} R_k(z) := R_\infty < \infty$.

The monotonicity established here of the radius in Thomas-Fermi theory agrees with the experimental fact that atomes grow in size as one goes down each group in the periodic table. (The radius does not increase with Z throughout the whole periodic table but if one considers a fixed group (e.g. the noble gases), it does).

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6.4 Partial Differential Equations

On the Dirichlet Problem for the Nonlinear Parabolic Equations in Non-smooth Domains

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We study the Dirichlet problem for the parabolic equation

$$u_t = a\Delta u^m + b \bigtriangledown \cdot u^\gamma + cu^\beta$$

with $a, m, \gamma, \beta > 0, b, c \in I\!R$, in a bounded, non-cylindrical and non-smooth domain $\Omega \subset I\!R^{N+1}, N \geq 2$. Existence and boundary regularity results are established. We introduce a notion of parabolic modulus of left-lower (or left-upper) semincontinuity at the points of the lateral boundary manifold and show that the upper (or lower) Hölder condition on it plays a crucial role for the boundary continuity of the constructed solution. The Hölder exponent $\frac{1}{2}$ is critical as in the classical theory of the one-dimensional heat equation $u_t = u_{xx}$.

Problem of Small Denominators in the Theory of Continuous Systems Nonlinear Oscillations

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The problem of nonlinear oscillations of continuous systems is an outstanding one in science and engineering and has received considerable attention in recent years. From a fundamental, as well as design, standpoint it is desirable to calculate periodic solutions in an analytical way. Unfortunately, an exact determination of periodic solutions of nonlinear continuous systems is generally out of the question, even for the simplest case (nonlinear wave equation). The most natural ways for approximate solving of problem under consideration are variational and asymptotic approaches. Here must be noted papers of P.Rabinovitch, H.Bresis (variational approaches), G.Wayne, J.Bourgain, W.Craig (KAM theory), Yu.A.Mitropol'sky et al. (averaging), S.L.Lau et al. (multyscale method). In our paper we propose and analyze two new asymptotic approaches for construction of periodic solutions of nonlinear PDE. The difficulty in the construction is that one must overcome the phenomenon of small denominators that is present in this problem. We think that these novel approaches to small denominators problems are robust, and give an interesting alternative to the classical approach of the KAM theory.

Our first approach is based on the so-called "small δ method", proposed by C.M.Milton et al. The main idea of this approach may be described as follows. Let us consider boundary value problem for nonlinear wave equation

$$u_{tt} = u_{xx} - \varepsilon u^3, \tag{1}$$

$$u(0,t) = u(\pi,t) = 0. \tag{2}$$

We introduce artificial "small parameter δ" in such a way, that

$$u^3 = u^{1+2\delta} = u\left(1 + \delta \ln\left(u^2\right) + 0.5\delta^2 \left(\ln\left(u^2\right)\right)^2 + ...\right). \tag{3}$$

Then we represent solution of boundary value problem (1), (2) in the form of expansion

$$u = u_0 + \delta u_1 + \delta^2 u_2 + ... \tag{4}$$

It is very important that thanks expansion (3) we have only one resonance term in each approximation, and routine Poincaré-Lindstedt technique may be used for it elimination. In our solution we must not suppose parameter small, and we may construct periodic as well as quasi-periodic solutions.

Our second approach is valid for small value of ε and is based on the renormalization procedure with artificial "small parameter" introduction. First of all from the conditions of secular terms absence we obtain infinite system of nonlinear algebraic equations. Then we introduce artificial "small parameter μ" in order to diagonalize this system, and we search solution in the form of expansions in powers of μ. Then we use Padé approximants for truncated series in μ and set $\mu = 1$. Proposed approaches are in some sense complimentary to each other and may be greatly extended.

Chaotic eigenfunctions, maximum norms and random superpositions of plane waves

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The growth of the maximum norms of quantum eigenstates of classically chaotic systems with increasing energy is investigated. The maximum norms provide a measure for localization effects in eigenfunctions. An upper bound for the maxima of random superpositions of random functions is derived. For the random-wave model this gives the bound $c\sqrt{\ln E}$ in the semiclassical limit $E \to \infty$. The growth of the maximum norms of random waves is compared with the growth of the maximum norms of the eigenstates of six quantum billiards which are classically chaotic. The maximum norms of these systems are numerically shown to be conform with the random-wave model. Furthermore, the distribution of the locations of the maximum norms is discussed.

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On differential operators of Mizohata type

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The differential operator of Mizohata

$$\frac{\partial}{\partial t} + it^{2n+1}\frac{\partial}{\partial x}$$

is not locally solvable at the origin of R^2. We call differential operators of Mizohata type the next operators

$$M = \frac{\partial}{\partial t} + ib\,(t)\,\frac{\partial}{\partial x}$$

where $b(t)$ is a real function of class C^∞, satisfying $tb(t) > 0$,$\forall\ t \in R^*$. These operators are also not locally solvable at the origin , because they do not satisfy the Nirenberg-Treves condition (P) ([3])

In this work we study the differential equation

$$Mu = f$$

where f is a regular function in R^2 .

We give necessary and sufficient conditions on f such that this last equation has a C^1 solution in a neighbourhood of the origin. These results generalise the results obtained in the case of the Mizohata equation by other authors (see [4], [1], [2]).

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Goursat problems, multiparameter elliptic operators and related spectral asymptotics

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Partial differential equations with multidimensional time occur in various bran- ches of science, for example, in the theory of probability and Brownian motion, the theory of boundary layers, cosmology and physics.

We introduce a new class of polyparabolic equations with multidimensional time and develop the existence and uniqueness theory for Goursat like problems in the corresponding Sobolev-like scale of spaces.

On our way to the existence and uniqueness theorem for polyparabolic problems we study elliptic boundary value problems with parameters in a bounded domain.

The notion of the ellipticity with parameters for differential and pseudodifferential operator pencils is applied to the resolvent construction, which leads to the definition of the corresponding complex powers and $\zeta-$function, i.e. to a new functional calculus of operators with promising applications to multiparameter spectral theory.

We prove the extension theoremfor the traces of the kernels of the complex powers and the corresponding $\zeta-$function. The last appears to be a meromorphic function in C^m (m is the number of parameters) with polar sets of the first order and the corresponding holomorphic residue-forms.

In particular, for $m = 2$, this investigation gives the opportunity to write a two-parameter spectral asymptotics in terms of these forms using the two-dimensional refinement of the Ikehara Tauberian theorem.

Mathematical analysis of the finite elasticity BVP for materials with ideal saturation

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For materials with ideal saturation the elastic potential $\Phi(x, \nabla u)$ of which has the linear growth in $|\nabla u|$, where $x \mapsto u \in R^3$ is the map, the existence of the limited static load (such external static forces with no solution of BVP) and discontinuous maps with breaks of the sliding type was proved by the author in [1,2]. From the physical point of view these effects are treated as the destruction of material.

For estimation of the mechanical durability of the elastic solid $\Omega \in R^3$ the following variational problem must be solved

$$t_* = \inf \left\{ \int_\Omega |\nabla u(x)| \lambda(x)\, dx : \ u \in V,\ L(u) = 1 \right\}, \tag{1}$$

$$L(u) = \int_\Omega \langle f, u \rangle\, dx + \int_{\Gamma^2} \langle F, u \rangle\, d\gamma, \qquad \langle g, u \rangle(x) = \int_x^{u(x)} g(x, u) \cdot du,$$

$$\lambda(x) = \inf \left\{ \frac{\Phi(x, Q)}{|Q|} : \ Q \in R^{3\times 3} \right\},$$

where $V = \{u \in W^{1,1}(\Omega, R^3) : \ u(x) = u^0(x),\ x \in \Gamma^1\}$ is the set of admissible maps, u^0 is the given map of a portion Γ^1 of the boundary $\partial\Omega$, f and F are the external mass and surface forces, respectively, and $\Gamma^2 = \partial\Omega \setminus \Gamma^1$. The limited static load is $(f^*, F^*) = t_*(f, F)$. If $t_* < 1$, then the appropriate finite elasticity BVP has no solution.

From the mathematical point of view the variational problem (1) is non-correct and, therefore, needs regularization. We use the partial regularization which is based on the special discontinuous finite-element approximation (FEA) [3]. After this discontinuous FEA the problem (1) is transformed into the non-linear system of algebraic equations which is badly determined. Therefore, for the numerical solution the decomposition method of adaptive block relaxation is used [2,4]. The main idea of this method consists of iterative improvement of zones with "proportional" fields by special decomposition of variables, and separate calculation on these variables.

The numerical results show that, for finding the main parameter t_* of a limited static load, the proposed technique has qualitative advantages over standard continuous finite-element approximations.

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L^2–Regularity Theory of Linear Strongly Elliptic Dirichlet Systems of Order $2m$ with Minimal Regularity in the Coefficients

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In this talk we consider the following Dirichlet system of order $2m$:

$$\begin{aligned} L(x,\nabla)u &= f(x) && \text{in } \Omega, \\ \nabla^k u &= 0 && \text{on } \partial\Omega \ (k=0,\ldots,m-1). \end{aligned}$$

Here, Ω is a smooth bounded domain in $\mathbb{R}^n$ and the differential operator satisfies the Legendre–Hadamard condition. From the general elliptic theory we know that for sufficiently smooth coefficients and for $f \in H^{-m+s}(\Omega,\mathbb{R}^N)$, every weak solution $u \in H_0^m(\Omega,\mathbb{R}^N)$ is actually in $H^{m+s}(\Omega,\mathbb{R}^N)$ and satisfies an a–priori estimate of the following form:

$$\|u\|_{H^{m+s}(\Omega,\mathbb{R}^N)} \le C_1 \|f\|_{H^{-m+s}(\Omega,\mathbb{R}^N)} + C_2 \|u\|_{L^2(\Omega,\mathbb{R}^N)}.$$

The latter a–priori estimate is of particular interest in applications to nonlinear PDE's (see e.g. [2] and [3]). There, the coefficients of $L(x,\nabla)$ result from a linearization procedure and consequently they cannot be chosen as smooth as one likes. Therefore, the authors cannot use the famous results stated in [1].

Here, we proove the above regularity result under minimal assumptions on the coefficients of $L(x,\nabla)$ and we give an explicit representation formula for the regularity constants C_1 and C_2.

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On the Solvability of Thermoviscoelastic Contact Problems with Coulomb Friction

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In a physically realistic model for contact problems with friction it is reasonable to include the aspect of heat transport, since friction usually generates a non–negligible amount of heat. In the present contribution we study a viscoelastic contact problem coupled with a heat conduction equation. The equations are coupled by volume terms describing the heat generated by viscous deformation and the deformation created by heat, and by a boundary term modeling the generation of heat by friction. In the contact problem the contact condition is formulated with the displacement velocities. The weak formulation of the problem consists of a variational inequality for the contact problem and a variational equation for the heat conduction problem.

The heat generated by friction can be modeled by a nonlinear non–monotone boundary term with quadratic growth in the variational formulation. If this model is used in a problem with linear constitutive laws for the material properties, then this boundary term dominates in estimates. This up–to–now prevents the proof of the existence of solutions for such a model. Here we consider two different simplifications:

1. In the first model we limit the growth of the heat generated by friction by a linear term and use linear constitutive laws for the viscoelastic material.
2. In the second model we admit the full quadratic growth of the heat generated by friction. In order to control this growth we employ a constitutive law for the viscoelastic material satisfying suitable growth conditions.

In both cases the problem is solved with a fixed point approach. Let $U(\vartheta)$ denote the solution of the contact problem with given temperature field ϑ and $\Theta(u)$ be the solution of the heat conduction problem with given displacement field u. If both problems are uniquely solvable, then the operator

$$\Phi := \Theta \circ U, \quad \Phi(\vartheta) = \Theta(U(\vartheta))$$

is well defined. Any solution of the coupled thermoviscoelastic contact problem is associated with a fixed point of this operator. However, the uniqueness of the solution to the contact problem is still an open problem. Therefore we employ the above described fixed point approach to an approximate problem obtained by the application of the penalty method. The existence of a fixed point for the approximate problems is proved with the fixed point theorem of Banach in the first case and the fixed point theorem of Schauder in the second case. The solvability of the original thermoviscoelastic contact problem follows by passing to the limit of the penalty parameter.

On the integrability of the Pfaffian equation concerning the octahedral web formed by four pencils of spheres

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In this paper, we consider four pencils of spheres whose centers lie on the same plane such that any three pencils cut the fourth pencil under a hexagonal 3-web [1,2]. We then use a system of Pfaffian equations to obtain the condition for the surface web formed by four pencils of spheres to be an octahedral web and show that this condition involves 28 parameters [1].

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Effect of large vibration amplitudes on the mode shapes and natural frequencies of thin circular cylindrical shells. A multi-mode approach

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A theoretical model has been proposed by Benamar et al. for studying the non-linear dynamic behaviour of thin straight structures like beams and plates [1,2,3,4]. The results obtained by these works has been in good agreement with previous theoretical and experimental investigations. The main topic of this paper is to present the mathematical development of a semi-analytical method, based on Hamilton's principle and spectral analysis, by extension of the previous model to the shell-type structures, in order to study the effects of large vibration amplitudes on the mode shapes and natural frequencies of thin circular cylindrical shells. The Donnell's non-linear theory is used, and, the longitudinal, circumferential and transverse displacement functions are expanded in the form of a series of basic functions satisfying the boundary conditions and harmonic motion is assumed. The non-linear dynamic variational problem obtained by applying Hamilton's principle is transformed into a static case by integrating the time functions over a period of vibration. The minimisation of the energy functional with respect to the basic function contribution coefficients has lead to a set of non-linear algebraic equations whose solution leads to the non-linear longitudinal, circumferential and transverse modes shapes, given as a function of the amplitudes of vibration, and their corresponding frequencies. Application of this approach was adapted, to the shells of infinite length, using a multi-mode approach, considering, first; transverse vibrations only [5], second;

coupling circumferential-transversal vibrations[6,7], and recently the simply supported circular cylindrical shells using a single mode approximation[8]. Because of their interest in many branch of engineering (aeronautical, ocean and civil), we will present same results, at multi-mode approach, for the simply supported circular cylindrical shells.

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Gain of Regularity for one nonlinear dispersive equation

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Using the technics of Craig - Kappeler - Strauss, cf.,[1] we study smoothness properties of solutions of some nonlinear equations of evolution dispersive.

We consider the nonlinear dispersive equation

$$\partial_t u + \partial_x f(u) = \epsilon \partial_x \beta(\partial_x u) - \delta \partial_x^3 u \tag{1}$$

with $x \in \mathbb{R}$, $t \in [0, T]$ and T is an arbitrary positive time. The flux $f = f(u)$ and the (degenerate) viscosity $\beta = \beta(\lambda)$ are given smooth functions satisfying certain assumptions. The complete equation arises, at least for the diffusion $\beta(\lambda) = \lambda$ (the so called Korteweg - de Vries - Burgers equation)as a model of the nonlinear propagation of dispersive and dissipative waves in many different physical systems. We studied this case in [2–3]. In this work we obtained under certain additional condition on f and β a C^∞ solution $u(x, t)$ for $t > 0$ if the initial data $u(x, 0)$ decay faster than polinomially on $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$ and have certain initial Sobolev regularity.

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Posters

Path Formulation for a Modal Family

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All persistent bifurcation diagrams in unfoldings of the modal family

$$g(x, \lambda) = \varepsilon x^4 + 2ax^2\lambda + \delta\lambda^2$$

are described using path formulation: each bifurcation problem in the unfoldings of g is reinterpreted as a λ-parametrized path in the universal unfolding of x^4. The space of unfolding parameters for the modal family is divided into regions where bifurcation problems are contact-equivalent and the bifurcation diagrams for these persistent problems are shown.

Bifurcation of Infinitely Many Fine Modes from Radially Symmetric Internal Layers.

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For a system of reaction-diffusion equations of activator-inhibitor type defined on an N-dimensional disk, it is shown that there exists a family of spherically symmetric stationary transition layer solutions when the activator diffuses very slowly compared with the inhibitor. Spectral analyses are also carried out for the linearization of the internal layers, which reveal that the layer solutions are unstable, and more importantly, that there exist infinitely many static bifurcations from the radially symmetric layer as the diffusion rate of the activator goes to zero. The wave length (l) of the bifurcated solutions becomes more fine as the diffusion rate (d) gets smaller. The relation between these two quantities is: $l \propto d^{1/4}$. This result proves the validity of a conjecture given in [1] for the special case where the domain is a disk.

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