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Abstracts

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Main lectures

Swarming and the Aggregation Equation

Andrea Bertozzi * [1]

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The past ten years has seen a great deal of activity related to the analysis of and understanding of dynamics related to swarming and aggregation. I will review this work for pairwise interaction potentials. I will discuss finite time blowup phenomena and weak solutions, pattern formation, and long term dynamics of these problems.

*Speaker

Complex dynamics, bifurcations, and arithmetic

Laura DeMarco * [1]

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Evanston, United States

I will give an overview of some connections between the study of complex dynamical systems and arithmetic geometry. I will emphasize the analytic ingredients, centered around the study of bifurcations in families of dynamical systems. As an example application, I will present a classical theorem of Mordell and Weil from the 1920s, on rational points of elliptic curves. Time permitting, I will explain a new result about elliptic curves, joint with X. Wang and H. Ye, with its dynamical and potential-theoretic proof.

*Speaker

Small eigenvalues and mean transition times for irreversible diffusions

Barbara Gentz * [1], Nils Berglund [2]

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The spectral theory of reversible diffusions in the small-noise limit is well understood. The small eigenvalues of the generator have been analyzed by a number of different methods, including large deviations, semiclassical analysis and potential theory. The study of the irreversible case, which involves a non-selfadjoint generator, is substantially more difficult. We will discuss an approach based on Laplace transforms of hitting times for Markov chains with continuous state space. These Markov chains arise from random Poincaré maps.
The proposed approach provides information on the exponentially small eigenvalues of the generator, and on mean transition times between attractors. As an illustration, we will present a detailed analysis of the asymptotic behaviour of the first-passage time of a planar diffusion through an unstable periodic orbit in the small-noise limit.

*Speaker

Regularisation by noise in PDEs

Massimiliano Gubinelli * [1,2]

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[2] Institut Universitaire de France (IUF) – Ministère de l'Enseignement Supérieur et de la Recherche Scientifique – Maison des Universités, 103 Boulevard Saint-Michel, 75005 Paris, France

We talk about good properties of bad functions. In the first part of the talk we review progresses in the analysis of situations where the presence of stochastic noise improves the theory of certain classes of PDEs. In the second part we propose a notion of *irregularity* for deterministic signals and use it to analyse the regularising effect of such signals on PDEs. In particular, in certain situations, we show in a quantitative way that the more the perturbation is irregular the more the properties of the equation are better. Examples include linear stochastic transport equations and non-linear modulated dispersive PDEs.

*Speaker

Deterministic Noise (beyond averaging and fluctuations)

Carlangelo Liverani * [1]

[1] Dipartimento di Matematica, University of Rome "Tor Vergata" (Tor Vergata) – Via della Ricerca Scientifica, 00133 Rome, Italy

No real dynamical system is totally isolated. Indeed, to completely describe all the possibly relevant degrees of freedom of a system is an impossible task. A rather successful way to account for the discarded degrees of freedom is to model their cumulative effect by some (possibly very small) external noise. Yet, several crucial questions remain:

1. in which precise technical sense the deterministic action of the discarded degrees of freedom can be modelled by a stochastic process;
2. is such an approximation good for long (possibly very long) times;
3. are there phenomena for which the dynamical origin of the "noise" plays a significant role?

In my talk I will discuss a super simple (but far from trivial) deterministic model of the fast-slow type in which such questions can be addressed and answered precisely. In particular, I will compare the behaviour of such a system with the behaviour of a Wentzell-Freidlin type stochastic process. (Work in collaboration with Jacopo de Simoi)

*Speaker

Dynamics near the subcritical transition of the 3D Couette flow

Nader Masmoudi * [1]

[1] New York University – United States

It is well known that the 3D Couette flow is stable for the Navier-Stokes dynamics. However, the main question is to find the size of the allowed pertubation depending on the viscosity. In this talk, we will discuss the dynamics of small perturbations of the plane, periodic Couette flow in the 3D incompressible Navier-Stokes equations at high Reynolds number. For sufficiently regular initial data, we determine the stability threshold for small perturbations and characterize the long time dynamics of solutions below this threshold. The primary stability mechanisms are an anisotropic enhanced dissipation effect and an inviscid damping effect of the velocity component normal to the shear, both a result of the mixing caused by the large mean shear. After detailing these linear effects, we will discuss some of the important steps in the proof, such as the analysis of the weakly nonlinear (potential) instabilities connected to the non-normal nature of the linearization. Joint work with Jacob Bedrossian and Pierre Germain.

*Speaker

Analysis of "Integrate and Fire" models for neural networks

Benoit Perthame * [1]

[1] Laboratoire Jacques-Louis Lions (LJLL) – Université Pierre et Marie Curie - Paris 6 – France

Neurons exchange informations via discharges, propagated by membrane potential, which trigger firing of the many connected neurons. How to describe large networks of such neurons? How can such a network generate a spontaneous activity? We tackled using nonlinear integro-differential equations. These are now classically used to describe neuronal networks or neural assemblies. Among them, the Wilson-Cowan equations are the best known and describe spiking rates in different brain locations.
Another classical model is the integrate-and-fire equation that describes neurons through their voltage using a particular type of Fokker-Planck equations. Several mathematical results will be presented concerning existence, blow-up, convergence to steady state, for the excitatory and inhibitory neurons, with or without refractory states. Conditions for the transition to spontaneous activity (periodic solutions) will be discussed.
One can also describe directly the spike time distribution which seems to encode more directly the neuronal information. This leads to a structured population equation that describes at time t the probability to find a neuron with time s
elapsed since its last discharge. Here, we can show that small or large connectivity leads to desynchronization. For intermediate regimes, sustained periodic activity occurs. A common mathematical tool is the use of the relative entropy method.

*Speaker

Stability problems for the Einstein equations

Igor Rodnianski * [1]

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The talk will survey the results and progress made in understanding problems of asymptotic stability in General Relativity, specifically as it applies to Minkowski space and black holes, as well as the stable singularity formation in the Big Bang scenario.

*Speaker

Effective models for Ginzburg-Landau vortices

Sylvia Serfaty * [1,2]

[1] Laboratoire Jacques-Louis Lions (LJLL) – Université Pierre et Marie Curie - Paris 6 – France
[2] Courant Institute (New York University) – USA

Ginzburg-Landau type equations are models for superconductivity, superfluidity, Bose-Einstein condensation, etc. A crucial feature is the presence of quantized vortices, which are topological zeroes of the complex-valued solutions. This talk will review some results on the derivation of effective models to describe the statics and dynamics of these vortices, with particular attention to the situation where the number of vortices blows up with the parameters of the problem. We will start with results in collaboration with Etienne Sandier on mean-field models for minimizers of the Ginzburg-Landau equation and the description of their finer microscopic vortex patterns. We will finish with the derivation of effective models for the dynamics of many vortices starting from the parabolic Ginzburg-Landau equation or the Gross-Pitaevskii (=Schrodinger Ginzburg-Landau) equation.

*Speaker

Data Assimilation – New Challenges in Random and Stochastic Dynamical Systems

Andrew Stuart * [1]

[1] Mathematics Institute, University of Warwick – United Kingdom

The proliferation of data, together with carefully crafted mathematical models, means that in many areas of science and engineering the data and model should be considered in conjunction. Taking this perspective leads to new and interesting challenges in mathematical analysis. Since the data is often noisy, and in some cases the model is uncertain, interaction with probability arises naturally. This talk will be devoted to studying the conjunction of data and model in the context of time-evolving problems, a subject frequently referred to as data assimilation. Predicting the state of a chaotic dynamical system whose initial condition is uncertain is difficult even in short time intervals. However in the presence of partial and noisy observations of the system, the question arises as to whether the initial uncertainty can be kept small in the infinite time horizon. We show that studying this problem leads to interesting questions relating to nonlinear Markov chains in discrete time. We also describe continuous time limits, leading to new nonlinear stochastic PDEs, such as families of damped-driven interacting Navier-Stokes equations, coupled through their empirical covariance. Theorems and numerical illustrations concerning this subjects will be presented.

*Speaker

Examples of incompressible flows and some model equations

Sverak Vladimir * [1]

[1] University of Minnesota – United States

We will discuss some interesting scenarios for the incompressible Euler and Navier-Stokes equations, and will also describe some model problems related to these equations.

*Speaker

On chaotic dynamics in non-holonomic mechanical systems

Dmitry Turaev * [1]

[1] Department of Mathematics - Imperial College, London – SW7 2AZ, United Kingdom

We notice three important properties of a typical system of non-holonomic mechanics: such systems are time-reversible, have an energy integral, and the flow in the energy level does not preserve the phase volume. We demonstrate that such systems can be characterised by a peculiar type of chaos, which corresponds to a robust merger of the attractor and the repeller. We discuss basic dynamical constructions which can be behind this phenomenon and show that they can lead to chaos of ultimate diversity and richness. We argue that the effects can persist when a small dissipation and energy pumping are added.

*Speaker

Toward a smooth ergodic theory for infinite dimensional systems

Lai-Sang Young * [1]

[1] Courant Institute of Mathematical Sciences [New York] (CIMS) – 251, Mercer St New York, NY 10012, United States

Focusing on settings that are consistent with semi-flows defined by dissipative parabolic PDEs, I will discuss some first steps toward building a dynamical systems theory, in particular a theory of chaotic systems, for maps and semi-flows in Hilbert and Banach spaces. I will survey known results and present recent progress, including theorems on Lyapunov exponents, periodic solutions and horseshoes, entropy formula and SRB measures, and a notion of "almost every" initial condition that is natural to the underlying dynamics. Technical differences between finite and infinite dimensions will also be discussed.

*Speaker

Existence and stability of a solution with a new prescribed behavior for a heat equation with a critical nonlinear gradient term

Hatem Zaag * [1]

[1] Laboratoire Analyse, Géométrie et Applications (LAGA) – Université Sorbonne Paris Cité (USPC), Institut Galilée, université Paris 13, CNRS : UMR7539 – Institut Galilée, Université Paris 13, 99 avenue Jean-Baptiste Clément, F-93430, Villetaneuse, France, France

We consider the nonlinear heat equation with a nonlinear gradient term:

$$\partial_t u = \Delta u + \mu|\nabla u|^q + |u|^{p-1}u,\ \mu > 0,\ q = 2p/(p+1),\ p > 3,\ t \in (0,T),\ x \in \mathbb{R}^N.$$

We construct a solution which blows up in finite time $T > 0$. We also give a sharp description of its blow-up profile and show that it is stable with respect to perturbations in initial data. The construction relies on the reduction of the problem to a finite dimensional one, and uses the index theory to conclude. The stability is a by-product of the existence proof, thanks to the interpretation of the finite dimensional problem in terms of the blow-up time and point.
The blow-up profile does not scale as $(T-t)^{1/2}|\log(T-t)|^{1/2}$, like in the standard nonlinear heat equation, i.e. $\mu = 0$, but as $(T-t)^{1/2}|\log(T-t)|^{\beta}$ with $\beta > 1/2$. We also show that u and ∇u blow up simultaneously and at a single point, and give the final profile. In particular, the final profile is more singular than the case of the standard nonlinear heat equation. This is a joint work with Slim Tayachi from the University of Tunis El Manar.

*Speaker

Minisymposia

MS1 Celestial Mechanics: M. Guardia, V. Kaloshin

On the continuation of orbits "after infinity" in Newtonian dynamics

Alain Albouy [a] [1]

[1] Observatoire de Paris – CNRS : UMR8028 – 77, avenue Denfert-Rochereau, 75014 Paris, France

Consider a hyperbolic motion of the n-body problem. The asymptotic expansion of each cartesian coordinate of each body begins with a linear term in t. The next term is in $\log t$. If we eliminate the time t and express the coordinates of each body as a function of one of them, there are infinitely many terms with a log, except in a remarkable case. If the limiting configuration is a central configuration, there are no term in log. Chazy discovered this property and related it with the possibility of an analytic continuation of the orbit after infinity, which he claimed to be possible only when the limiting configuration is central. In the example $n = 2$, the orbit is a hyperbola. The continuation after infinity makes it a closed orbit, an ellipse in the projective space. This continuation is clear if we know a remark due to Appell: The Kepler problem in the plane corresponds by central projection with the Kepler problem on the sphere. In the second problem, "after infinity" means below the equator. The hyperbolas project on closed spherical ellipses. We will state precisely Chazy's remarks and prove them by using Appell's central projection, i.e., by the principles of projective dynamics.

[a]Speaker

Billiards in the three body problem

Sergey Bolotin [a] [1]

[1] Moscow Steklov Mathematical Institute and University of Wisconsin – Russia

We consider the plane three body problem with two of the masses much smaller than the third one. Solutions with near collisions of the small bodies shadow trajectories of a billiard type system with the discrete Lagrangian determined by the classical Lambert's problem. In the limit of many revolutions between near collisions the billiard system admits relatively simple description. The approach can be extended to other problems of celestial mechanics.

[a]Speaker

KAM Theorem applied to the plane planetary problem

Thibaut Castan [a] [1]

[1] Institut de Mécanique Céleste et de Calcul des Ephémérides (IMCCE) – Université Lille I - Sciences et technologies, Université Pierre et Marie Curie (UPMC) - Paris VI, INSU, Observatoire de Paris, CNRS : UMR8028 – 77 av Denfert-Rochereau 75014 Paris, France

In 1963, Arnold showed that some version of Kolmogorov's invariant torus theorem can be applied to the 3-body problem, assuming that 2 bodies (the planets) have masses much smaller than the other one (the Sun). We will recall Arnold's scheme of proof and show how to obtain explicit bounds for the masses of the planets. This goes through analyzing the complex singularities of Kepler's equation and of the Newtonian potential, the loss of width of analyticity of the Hamiltonian due to normalizing transformations and using a very precise version of Kolmogorov's theorem strong enough to deal with the degeneracies of the problem.

[a] Speaker

An analytic approach for central configurations

Kuo-Chang Chen [a] [1]

[1] National Tsing Hua University – Taiwan

Self-similar solutions for the n-body problem, whose configurations are called central configurations, are of special importance in celestial mechanics. Many mathematical tools have been applied to this ancient problem in the hope to understand their geometric properties, stability, finiteness, and the existence of certain classes of central configurations. In this talk we propose another analytic approach, which we hope to provide a different perspective for the problem.

[a] Speaker

Oscillatory orbits in the restricted elliptic planar three body problem

Tere M-Seara [a] [1], Marcel Guardia [1], Pau Martin [1]

[1] Universitat Politècnica de Catalunya [Barcelona] (UPC) – Universitat Politècnica de Catalunya C. Jordi Girona, 31. 08034 Barcelona, Spain

The restricted planar elliptic three body problem models the motion of a massless body under the Newtonian gravitational force of two bodies evolving in Keplerian ellipses.The possible motions the massless body can perform were already known by Chazy (1922), who gave a complete classification of all possible states that the body $q(t)$ can approach as time tends to infinity. The possible final states are reduced to four:

- Hyperbolic: $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to c > 0$ as $t \to \pm\infty$.
- Parabolic: $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to 0$ as $t \to \pm\infty$.
- Bounded: $\limsup_{t\to\pm\infty} \|q\| < +\infty$.
- Oscillatory: $\limsup_{t\to\pm\infty} \|q\| = +\infty$ and $\liminf_{t\to\pm\infty} \|q\| < +\infty$

Examples of all these types of motion, except the oscillatory ones, were already known by Chazy. In this talk, we prove the existence of oscillatory motions for any value of the masses of the primaries, assuming they move in ellipses whose exectricity is small enough. The key idea is to look at the restricted elliptic three body problem as a small perturbation of the circular case. Using a previous result about the transversal intersection of the stable and unstable manifolds of infinity in the circular case, the elliptic one can be seen as “a priory chaotic”. Studing the corresponding scattering map with classical perturbative arguments, one can show the existence of transition chains of periodic orbits with bounded angular momentum, and a suitable shadowing lemma provides the existence of oscillatory motions.

[a]Speaker

Quasi periodic coorbital motions in the three body problem

Laurent Niederman [a] [1], Philippe Robutel [2]

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[2] Institut de Mécanique Céleste et de Calcul des Ephémérides (IMCCE) – Observatoire de Paris, CNRS : UMR8028 – 77 av Denfert-Rochereau 75014 Paris, France

The motions of the satellites Janus and Epimetheus around Saturn are among the most intriguing in the solar system since these satellites exchange their orbits every four years. We give a rigorous proof (and up to our knowledge, the first one) of existence of this kind of orbits over infinite times in the three body problem thanks to KAM theory.

[a]Speaker

Feedback Optimal Control of Low-thrust Orbit Transfer in Central Gravity Field

Ashraf Owis [a] [1]

[1] Astronomy, Space and Meteorology Department-Cairo University – Cairo Unicersity St, 12613 Giza, Egypt

Low-thrust trajectories with variable radial thrust is studied in this paper. The problem is tackled by solving the Hamilton Jacobi-Bellman equation via State Dependent Riccati Equation(STDE) technique devised for nonlinear systems. Instead of solving the two-point boundary value problem in which the classical optimal control is stated, this technique allows us to derive closed-loop solutions. The idea of the work consists in factorizing the original nonlinear dynamical system into a quasi-linear state dependent system of ordinary differential equations. The generating function technique is then applied to this new dynamical system, the feedback optimal control is solved. We circumvent in this way the problem of expanding the vector field and truncating higher-order terms because no remainders are lost in the undertaken approach. This technique can be applied to any planet-to-planet transfer; it has been applied here to the Earth-Mars low-thrust transfer.

[a]Speaker

Perihelia reduction in the planetary problem

Gabriella Pinzari [a] [1]

[1] Universita di Napoli Federico II – Naples, Italy

In the XIX century, Jacobi discovered that the motions of three bodies in gravitational interaction may be described by a system of the eighth order, rather than eighteenth. Some degrees of freedom might be neglected by the invariance of the problem by translations and rotations. After him, Radau wrote these equations in Hamiltonian form: he introduced a Hamilton function and four couples of canonical coordinates. The reduction of order by Jacobi and Radau has been extensively applied in the literature, and, for about one century and one half, it appeared as the only available one. In 1963, V.I.Arnold wrote that the lack of a generalization of it to the case of more bodies was an obstacle to the extension of his theorem of stability of planetary motions. In 1982-1983 F. Boigey and A. Deprit extended Jacobi-Radau's reduction to the general case. The coordinates by Boigey and Deprit were next rediscovered by the author and applied to the problem [Chierchia and Pinzari, 2011], allowing for a direct proof of Arnold's statement. Important feature of JRBD reduction are: (i) they are not defined for the problem constrained in the plane and (ii) no symmetry in the Hamiltonian appears, relatively to the invariance of the problem by reflections. We present an alternative reduction based on the perihelia of instantaneous orbits that takes into account items (i) and (ii). Next, we shall show how these items allow for a more global formulation of Arnold's statement.

[a]Speaker

Noncollision Singularities in a Planar Four-body Problem

Jinxin Xue [a] [1]

[1] Department of Mathematics [Chicago] – 5734 S. University Avenue Chicago, Illinois 60637, United States

In this talk, we show that there is a Cantor set of initial conditions in a planar four-body problem such that all the four bodies escape to infinity in finite time avoiding collisions. This proves the Painlevé conjecture for the four-body case. This work is based on an ideal model of Gerver and a joint work with Dmitry Dolgopyat.

[a] Speaker

MS2 Continuation Methods and Applications: B. Krauskopf, J. Sieber

Direct computation of failure boundaries

Hinke Osinga [a] [1]

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Predicting the behaviour of a structure when subjected to an earthquake is an important problem from Civil Engineering. Here, we consider a planar post-tensioned frame, which can be modelled as a two-degree-of-freedom system that is equivalent to the analytical model of a tied rocking block on an elastic foundation. The frame remains structurally sound as long as the tilt angle of the frame does not exceed a certain maximal angle. A standard approach to studying the dynamics would be to run simulations, where it is assumed that the earthquake is a pure sine wave with varying frequency and amplitude. Such a brute-force approach establishes a region in the frequency-amplitude plane for which the structural stability of the frame eventually fails. We propose a novel approach that calculates the failure region in a much more efficient way by determining the failure boundary directly. Our method is based on continuation of a suitable two-point boundary value problem. Our computations demonstrate that the failure boundary is only piecewise smooth and the results highlight further interesting details of how the dynamics is organised in the frequency-amplitude plane.

[a] Speaker

New asymptotics of homoclinic orbits near Bogdanov-Takens bifurcation points.

Willy Govaerts [a] [1], Bashir Al-Hdaibat [1], Yuri A. Kuznetsov [2], Hil G. E. Meijer [3]

[1] Ghent University – Belgium
[2] Utrecht University – Netherlands
[3] University of Twente – Netherlands

We derive explicit asymptotics for the homoclinic orbits near a generic Bogdanov-Takens (BT) point, with the aim to continue the branch of homoclinic solutions rooted in the BT point in parameter and state space. We present second-order homoclinic predictors using a generalization of the Poincaré-Lindstedt (P-L) method. The P-L method leads to the same homoclinicity conditions as the classical Melnikov technique [1], the branching method [2] and the regular perturbation method (R-P)[3]. However, the proposed method does not lead to the strange behavior near the saddle point, namely the appearance of a "parasitic turn" in the solution by the R-P method. Also, use these asymptotics to calculate the initial homoclinic cycle to continue homoclinic orbits in two free parameters. The new homoclinic predictors are implemented in the Matlab continuation package MatCont. We show several examples in the case of multidimensional state spaces.

[1] J. Guckenheimer and P. Holmes. Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields. Springer-Verlag, New York, 1983.

[2] W.-J. Beyn. Numerical analysis of homoclinic orbits emanating from a Takens-Bogdanov point. IMA J. Numer. Anal., 14(3):381-410, 1994.

[3] Yu. A. Kuznetsov, H. G. E. Meijer, B. Al-Hdaibat, and W. Govaerts. Improved homoclinic predictor for Bogdanov-Takens bifurcation. International Journal of Bifurcation and Chaos, 24(04):1450057, 2014.

[a] Speaker

Interactions between noise and rate-induced tipping

Paul Ritchie [a] [1]

[1] College of Engineering, Mathematics and Physical Sciences [Exeter] – Prince of Wales Road Exeter, Devon EX4 4SB, United Kingdom

A non-autonomous system is defined to pass a tipping point when gradual changes in input levels cause the output to change suddenly. We study a prototypical model for rate-induced tipping, the saddle-node normal form subject to parameter drift and noise. We determine the most likely time of escape by finding the optimal path of escape. This is a variational optimisation problem that can be transformed into a second order boundary value problem. This is solved using continuation techniques in AUTO, which generates contours for the optimal time of escape in a two parameter plane.

The overall probability of escape can be approximated using the instantaneous eigenmodes of the also non-autonomous Fokker-Planck equation. Combining the timing and probability of escape can potentially give us an additional early-warning indicator for noise and rate-induced tipping.

[a]Speaker

Nonlinear tuning of microresonators for dynamic range enhancement

Mehdi Saghafi [1], Harry Dankowicz [a] [1], Walter Lacarbonara [2]

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[2] Department of Structural and Geotechnical Engineering, Sapienza University of Rome (Sapienza) – via Eudiossiana 18, Rome 00184, Italy

This talk investigates the application of continuation methods to the design of micromechanical devices, specifically a novel formulation for the nonlinear tuning of nano/microresonators. Path-following techniques are employed to explore the dependence of the resonator dynamic range on system parameters, with emphasis on the geometry of bilayer microbeams. The analysis relies on a combination of the Matlab-based Computational Continuation Core (COCO) with a finite-element discretization of a geometrically exact, nonlinear mechanical model of the transverse and longitudinal dynamics of multilayer microbeams, which also takes into account rotary inertia effects. The system performance is here characterized by the resonator's response to distributed harmonic excitation in the presence of nonlinearities. Second-order perturbation analysis is used to verify the computational scheme for free vibrations of a single-layer model, confirming the presence of a zeroth-order-in-amplitude frequency shift with expected order of convergence. An inverse problem is then proposed for the continuation of the critical amplitude at which the transition to nonlinear response characteristics occurs. COCO's task embedding paradigm enables an efficient approximation of the corresponding cusp bifurcation of limit cycles in terms of three coupled periodic-orbit continuation problems.

[a]Speaker

Continuation and codimension-two bifurcations for a delay-differential equation with two state-dependent delays

Tony Humphries [a] [1]

[1] McGill University – Montréal QC, Canada

We consider the model equation

$$\varepsilon\dot{u}(t) = -\gamma u(t) - \kappa_1 u(t - a_1 - cu(t)) - \kappa_2 u(t - a_2 - cu(t)),$$

with two delays which has no nonlinearity except for state-dependency of the delays, and for which the delays are merely linearly state-dependent. We investigate the bifurcations that arise in both the singularly perturbed $\varepsilon \to 0$ and regular ($\varepsilon = 1$) cases. In the singular limit an algebraic construction allows us to construct candidate large amplitude solutions and find fold and cusp-like bifurcations. Numerically computed bifurcation diagrams using DDE-Biftool confirm the location of these bifurcations and that the bifurcation structures found also persist for $\varepsilon > 0$, as well as allowing us to locate other bifurcations including period-doubling and Hopf bifurcations.

Double-Hopf bifurcations give rise to invariant tori, including at least one stable torus, when $\varepsilon = 1$, that we investigate in some detail. We propose a method for computing the normal form of the double Hopf bifurcation that gives rise to the torus, and show that the resulting normal form corresponds to the bifurcation curves computed numerically. We believe this to be the first normal form computation for a double-Hopf bifurcation in a DDE with state-dependency in the nonlinearity. We also consider the eventual break-up of the torus.

[a] Speaker

Control-based continuation: bifurcation and stability analysis for physical experiments

David Barton [a] [1], Ludovic Renson [2]

[1] University of Bristol – United Kingdom
[2] Universite de Liege – Belgium

Control-based continuation is technique for tracking the solutions and bifurcations of nonlinear experiments. The basic idea is to apply the method of numerical continuation to a feedback-controlled physical experiment. Since in an experiment it is not (generally) possible to set the state of the system directly, the control target is used as a proxy for the state. The challenge then becomes to determine a control target such that the control is non-invasive, that is, it stabilises a steady-state (or periodic orbit) of the original open-loop experiment without altering it otherwise. Once implemented, control-based continuation enables the systematic investigation of the bifurcation structure of a physical system, much like if it had been a numerical model. However, stability information (and hence bifurcation detection and classification) is not readily available due to the presence of feedback control. This paper uses methods from the system identification community to extract stability information in the form of Floquet multipliers from the closed-loop experiment, thus enabling the direct detection of bifurcations. In particular, it is shown that a periodic auto-regressive moving-average model with exogenous inputs (ARMAX) can be constructed that approximates the time-varying linearisation of the experiment around a particular periodic orbit. This method is demonstrated using a physical nonlinear tuned mass damper.

[a] Speaker

Continuation for situations with noisy data

Jens Starke [a] [1]

[1] Technical University of Denmark (DTU) – Matematiktorvet, Bdg. 303 B, 2800 Kgs. Lyngby, Denmark

We investigate the approach of numerical continuaton and bifurcation analysis for situations with noisy data. This is motivated by bifurcation analysis in experiments and equation-free analysis (coarse analysis) for multi-scale systems. Both have in common that the function of interest which defines the zero problem for the continuation cannot be evaluated directly, but only indirectly via observations. In experiments, the noise in these observations is due to measurement errors and in multi-scale systems due to remaining fast scales when in the observed time the system behavior did not converge close enough to the slow low-dimensional manifold representing the macroscopic behaviour. Specialized algorithms are presented to address challenges that arise due to the presence of the noise. The results include continuation of stationary (flow) states, periodic orbits as well as traveling waves. The methods are demonstrated for examples from control-based continuation in a mechanical experiment of a periodically driven impact oscillator and equation-free analysis of particle models for car traffic and pedestrian flow. This is joint work with Frank Schilder, Emil Bureau, Ilmar Santos and Jon Juel Thomsen.

[a] Speaker

MS3 Coupled Oscillator Systems and their Mean-Field Dynamics: Ch. Kuehn, M. Wolfrum

Mean-field limit and fluctuations for interacting oscillators with singular spatial interaction

Eric Lucon [a] [1]

[1] Mathématiques appliquées Paris 5 (MAP5) – CNRS : UMR8145, Université Paris V - Paris Descartes – UFR de Maths et informatique 45 rue des Saints Pères 75270 PARIS CEDEX 06, France

There has been recently a growing interest in generalizations of mean-field models (e.g. Kuramoto or FitzHugh-Nagumo oscillators) to the case where interactions between particles are no longer uniform or do not follow the complete graph. We analyze here the large population behavior of mean-field interacting diffusions $(\theta_i(t))_{1\leq i\leq N}$ with spatial geometry: each oscillator θ_i is at a fixed position x_i on a regular lattice and the interaction between particles i and j depends on a spatial kernel $\Psi(x_i, x_j)$. Interesting examples include P-nearest-neighbor interactions or interactions with polynomial decay.
We first address the convergence as $N \to \infty$ of the empirical measure of the particles to a deterministic process of McKean-Vlasov type. We then discuss the finite-size fluctuations of the system around its limit, with a special consideration given to the influence of the geometry on the fluctuations. In the case of polynomial decay, the system exhibits a phase transition: for a strong interaction, the fluctuations are Gaussian, solutions to a linear SPDE as $N \to \infty$ whereas for weak interaction the fluctuations are deterministic. The main difficulty of the analysis lies in the singularity of the kernel Ψ and requires the introduction of an auxiliary process capturing the dependence of the system in the space variable.
This is joint work with W. Stannat (TU Berlin).

[a] Speaker

A spectral theory of linear operators on a Gelfand triplet and its application to the dynamics of coupled oscillators

Hayato Chiba [a] [1]

[1] Kyushu University – Japan

The Kuramoto model is a system of ordinary differential equations for describing synchronization phenomena defined as a coupled phase oscillators. In this talk, an infinite dimensional Kuramoto model is considered, and Kuramoto's conjecture on a bifurcation diagram of the system will be proved. A linear operator obtained from the infinite dimensional Kuramoto model has the continuous spectrum on the imaginary axis, so that the usual spectrum does not determine the dynamics.To handle such continuous spectra, a new spectral theory of linear operators based on Gelfand triplets is developed.In particular, a generalized eigenvalue (resonance) is defined.It is proved that a generalized eigenvalue determines the stability and bifurcation of the system.

[a] Speaker

Neural Field Models Which Include Gap Junctions

Carlo Laing [a] [1]

[1] Massey University, Auckland – New Zealand

Neural field models are normally derived under the assumption that connections between neurons are synaptic rather than via gap junctions. I will show how to derive a neural field model from a network of quadratic integrate and fire neurons with both synaptic and gap junction connectivity. The derivation is exact in the limit of an infinite number of neurons.

[a]Speaker

Complete synchronization of classical and quantum oscillators

Seung Yeal Ha [a] [1]

[1] Seoul National University (SNU) – Department of Mathematical Sciences, Seoul National University, Seoul 151-747, Korea, South Korea

In this talk, we will report a recent progress on the complete synchronization for classical and quantum oscillators whose dynamics are governed by the Kuramoto and Lohe models. For the complete synchronization, we adopt a Lyapunov functional approach and provide several sufficient conditions for the complete synchronization. This is a joint work with H. Kim, J. Park and S. Ryoo (SNU, Korea) and Sun-Ho Choi (KAIST, Korea).

[a]Speaker

Landau damping in the Kuramoto model

Giambattista Giacomin [a] [1]

[1] Université Paris Diderot – Université Paris Diderot - Paris 7 – France

I will review some recent results on the Kuramoto model.

[a]Speaker

Graph Limits and Dynamics of Large Networks

Georgi Medvedev [a] [1]

[1] Drexel University – 3141 Chestnut Street, PA 19104, United States

The continuum limit is an approximate procedure, by which coupled dynamical systems on large graphs are replaced by an evolution integral equation on a continuous spatial domain. This approach has been useful for studying dynamics of diverse networks in physics and biology. We use the combination of ideas from the theories of graph limits and nonlinear evolution equations to develop a rigorous justification for using the continuum limit for dynamical models on deterministic and random graphs. As an application, we discuss stability of spatial patterns in the Kuramoto model on certain Cayley and random graphs.

[a] Speaker

Dynamics of multifrequency communities of coupled oscillators

Arkady Pikovsky [a] [1]

[1] University of Potsdam – Germany

We generalize the Kuramoto model of globally coupled oscillators to multifrequency communities. After discussing general derivation of the governing equations, we consider three setups: non-resonantly coupled comminities, two subpopulations closed to a resonance 2:1, and three resonantly coupled subpopulations. In all cases complex regimes of collective dynamics are observed.

[a] Speaker

Chimera states in systems with control

Oleh Omel'chenko [a] [1]

[1] Weierstrass Institute for Applied Analysis and Stochastics (WIAS) – Mohrenstrasse 39, 10117 Berlin, Germany

Chimera states are self-organized patterns of coherence and incoherence observed in systems of nonlocally coupled oscillators. In the limit of large system size their properties can be inferred from the analysis of corresponding continuum limit equation. In this talk we will show how the information provided by this analysis can be used to design new systems of coupled oscillators demonstrating new types of chimera states that are inaccessible in conventional simulations or experiments. This is joint work with M. Wolfrum and J. Sieber.

[a] Speaker

MS4 Delay Differential Equations: L. Pujo Menjouet, J. Wu

Modelling medication: Phase resetting for a simple delay differential equation

Michael Mackey [1,2], Marta Tyran-Kaminska [3], Hans-Otto Walther [a] [1]

[1] Justus-Liebig-Universitaet Giessen – Germany
[2] McGill University, Montreal – Canada
[3] University of Silesia, Krakow – Poland

Consider the equation

$$x'(t) = -\gamma x(t) + f(x(t - \tau))$$

for the density of a population of blood cells, say, neutrophils. Suppose a periodic solution corresponds to a disease, with local minima potentially dangerous for the patient. Medication is modelled by an increase of the production function f during a certain period of time. It is successful if the minimum after medication is larger than that of the periodic solution. We begin with the simplest case where f is a one-step function. This enables us to compute solutions explicitly. For suitable γ and f there is a stable and attracting periodic orbit. We calculate solutions in case of medication and find that the effect of medication depends on the phase in which the periodic solution is at the onset of medication. Among others we obtain phase intervals in which medication is useless. For a model with a two-step function, which should be a bit closer to reality, we obtain that medication when administered in the wrong phase even lowers the next minimum and so has an adverse effect.

[a]Speaker

Understanding, treating and avoiding hematological disease: Better medicine through mathematics?

Michael Mackey [a] [1,2]

[1] McGill Univ, Montreal – Canada
[2] Centre for Applied Mathematics in Bioscience and Medicine (CAMBAM) – 3655 Drummond Montreal, Quebec, Canada

This talk will trace the development of physiologically realistic mathematical models for the regulation of mammalian blood cell production–framed as differential delay equations of varying complexity–and the use of these models in furthering out understanding of human hematological disease, and of treating it effectively. Recent efforts of using these models to understand the side effects of chemotherapy will be also discussed.

[a]Speaker

Spreading speeds for a fox rabies model with infection-age dependent diffusion

Horst Thieme [a] [1], Hao Liu, Yang Kuang

[1] School of Mathematical and Statistical Sciences, Arizona State University (SoMSS, ASU) – United States

An epidemic outbreak is considered for rabies in a spatially distributed fox population where the susceptible foxes do not move but infected foxes diffuse with diffusion coefficients that depend on their infection-age (time since infection). This takes into account that foxes in an early phase of the latency period would hardly diffuse while foxes with full-blown rabies may diffuse considerably. Since an outbreak situation is considered, the population turnover of the fox population is ignored. This allows to transform a system consisting of an ODE for the susceptible foxes and an age-dependent diffusion equation for the infected foxed to a single space-time integral equation of Volterra form for the cumulative number of infected foxes and to find an implicit formula for the spreading speed of the rabies epidemic. As implicit as it is, it still allows to study the dependence of the spreading speed on the diffusion coefficients, the length of the latency period, the per capita infection and disease death rates and other demographic or epidemiologic parameters.
Liu, Hao, Spatial Spread of Rabies in Wildlife, Diss., ASU, Dec. 2013
Thieme, H.R., Asymptotic estimates of the solutions of nonlinear integral equations and asymptotic speeds for the spread of populations. J. R. A. Math. 306 (1979), 94-121
Thieme, H.R., X.-Q. Zhao, Asymptotic speeds of spread and traveling waves for integral equations and delayed reaction-diffusion models. JDE 195 (2003), 430-470

[a] Speaker

PDE formulation and delay equation formulation of a cyclin structured cell population model

Sílvia Cuadrado [a] [1], Ricardo Borges [1], Angel Calsina [1], Odo Diekmann [2]

[1] Universitat Autònoma de Barcelona (UAB) – UAB Campus 08193 Bellaterra (BARCELONA) SPAIN, Spain
[2] Utrecht University - UU (NETHERLANDS) – Netherlands

We present two models for a cell population divided into proliferative and quiescent cells. The first one is a nonlinear cyclin content structured pde model for which, under suitable hypotheses, we show existence and uniqueness of a steady state by using positive linear semigroup theory. We also show, for particular values of the parameters, the existence of solutions that do not depend on the cyclin content. For the general case, we make numerical simulations obtaining, for some values of the parameters convergence to the steady state, but for others oscillations of the population. For the second model we use the delay equation formulation of structured population dynamics to derive a system of two renewal equations from individual-level assumptions concerning a cyclin structured population. Nonlinearity arises from the assumption that the rate at which quiescent cells become proliferating is determined by feedback. We characterize steady states and establish the validity of the principle of linearized stability.

[a] Speaker

Periodic solutions of a differential equation with a queueing delay

Tibor Krisztin [a] [1], István Balázs [1]

[1] Bolyai Institute, University of Szeged – Aradi vértanúk tere 1 6720 Szeged, Hungary

We consider a differential equation with a state-dependent delay motivated by a queueing process. The state-dependent delay is a queueing delay, it is implicitly defined by the length of the queue. The length of the queue satisfies a discontinuous differential equation. We formulate a suitable framework for the problem, and prove existence, uniqueness and continuous dependence of the solutions. The main result guarantees the existence of slowly oscillating periodic solutions.

[a]Speaker

Dynamics of the two-delay Belair-Mackey equation

Gergely Röst [a] [1]

[1] University of Szeged – Hungary

We study the dynamics of a differential equation with two delayed terms, representing a positive and a negative feedback, that was proposed by Bélair and Mackey for mammalian platelet production, and the same equation arises from three-stages single species populations as well. By combining various techniques, we prove delay dependent and absolute global stability results for the trivial and for the positive equilibrium, providing new mathematical results as well as novel insights for the related applications. We
show that, somewhat surprisingly, the introduction of a removal term with fixed delay in population models can simplify the otherwise complex dynamics of the equation, and investigate the bifurcations created by such terms. This is a joint work with Alfonso Ruiz-Herrera and Hassan El-Morshedy.

[a]Speaker

Age-structured and delay differential-difference model of hematopoietic stem cell dynamics

Mostafa Adimy [a] [1,2], Abdennasser Chekroun [1,2], Tarik-Mohamed Touaoula [3]

[1] INRIA Lyon (Dracula team) – INRIA – CEI-2, 56, Boulevard Niels Bohr, 69603 Villeurbanne, France
[2] Institut Camille Jordan (ICJ) – Université Claude Bernard - Lyon I (UCBL) – Bât. Jean Braconnier 101 43 Bd du 11 novembre 1918 69622 VILLEURBANNE CEDEX, France
[3] Department of Mathematics [Tlemcen] – Département de Mathématiques, B.P 119 Université de Tlemcen, Algeria

In this paper, we investigate a mathematical model of hematopoietic stem cell dynamics. We take two cell populations into account, quiescent and proliferating one, and we make the difference between dividing cells that enter directly to the quiescent phase and dividing cells that return to the proliferating phase to divide again. The resulting mathematical model is a system of two age-structured partial differential equations. By integrating this system over the age and using the characteristics method, we reduce it to a delay differential-difference system, and we investigate the existence and stability of the steady states. We give sufficient conditions for boundedness and unboundedness properties for the solutions of this system. By constructing a Lyapunov function, the trivial steady state, describing cell's dying out, is proven to be globally asymptotically stable when it is the only equilibrium. The stability analysis of the unique positive steady state, the most biologically meaningful one, and the existence of a Hopf bifurcation allow to determine a stability area, which is related to a delay-dependent characteristic equation. Numerical simulations illustrate our results on the asymptotic behavior of the steady states and show very rich dynamics of this model. This study may be helpful in understanding the uncontrolled proliferation of blood cells in some hematological disorders.

[a]Speaker

On a logistic equation with delayed positive and negative feedbacks

Istvan Gyori [a] [1]

[1] Department of Mathematics - University of Pannonia (DM) – University of Pannonia Department of Mathematics Egyetem u. 10 Veszprém H-8200, Hungary

We consider qualitative properties of a logistic equation with two discrete delays - positive and negative delayed feedbacks. In the parameter plane we characterize the boundedness and global stability of the positive equilibrium. It is shown that there exists an unbounded exponential solution for the nonlinear problem. From the parameter plane analysis we can deduce an idea how positive and negative delayed feedbacks influence the dynamics. For a very special case we completely characterize the stability and the boundedness. It is shown that there exists a parameter set such that the solution is locally stable, but it is not globally stable due to the existence of a blow-up solution. We also consider the existence of positive (real) roots from which we can deduce a part of the instability region for the two delay problem.
This is a joint work with Yukihiko Nakata and Gergely Rost.

[a]Speaker

Local stability analysis of differential equations with state-dependent delay

Eugen Stumpf [a] [1]

[1] Universität Hamburg - Department Mathematik – Bundesstraße 55, 20146 Hamburg, Germany

In this talk, we discuss some aspects of the local stability analysis for a class of abstract functional differential equations. This is done under smoothness assumptions which are often satisfied in the presence of a state-dependent delay. Apart from recapitulating the two classical principles of linearized stability and instability, we deduce the analogon of the Pliss reduction principle for the class of differential equations under consideration. This reduction principle enables to determine the local stability properties of a solution in the situation where the linearization does not have any eigenvalues with positive real part but at least one eigenvalue on the imaginary axis.

[a]Speaker

Lyapunov-Razumikhin techniques for state-dependent DDEs

Tony Humphries [a] [1], Felicia Magpantay [2,3]

[1] McGill University – Montréal QC, Canada
[2] University of Michigan – United States
[3] University of Manitoba – Canada

We present theorems for Lyapunov and asymptotic stability of steady states for general state-dependent delay differential equations (DDEs) using Lyapunov-Razumikhin methods. The Lyapunov stability result applies to nonautonomous DDEs with multiple discrete state-dependent delays of the form

$$\begin{aligned} &\dot{u}(t) = f\big(t, u(t), u(t-\tau_1(t,u(t))), \ldots, u(t-\tau_N(t,u(t)))\big), && t \geqslant t_0, \\ &u(t) = \varphi(t), && t \leqslant t_0, \end{aligned}$$

and is proved by a contradiction argument adapted from a previous result of Barnea for retarded functional differential equations (RFDEs). Our asymptotic stability result applies to autonomous DDEs with multiple state-dependent discrete delays. Its proof is entirely new, and is based on a contradiction argument together with the Arzelà-Ascoli theorem. This alleviates the need for an auxiliary function to ensure the asymptotic contraction. We apply our results to the state-dependent model equation

$$\begin{aligned} &\dot{u}(t) = \mu u(t) + \sigma u(t-a-cu(t)), && t \geq 0, \\ &u(t) = \varphi(t), && t \leqslant 0, \end{aligned}$$

to directly establish asymptotic stability in parts of the stability domain along with lower bounds for the basin of attraction. We also generalise our techniques to derive a condition for global asymptotic stability of the zero solution to the model problem, and also to find bounds on the periodic solutions when the steady-state solution is unstable.

[a]Speaker

The role of host immunity for structured tick population dynamics

Jianhong Wu [a] [1]

[1] Laboratory for Industrial and Applied Mathematics, York University, Canada (LIAM) – 4700 Keeles Street, York University, Toronto, Ontario, Canada M3J 1P3, Canada

This talk, based on recent work with Xiaotian Wu (University of Montreal), will discuss the model formulation and dynamics analysis for a general stage-structured population model, involving development delay and host immunity. We focus on the mathematical challenge obtaining natural conditions for the boundedness, dissipativeness, and nonlinear oscillation caused by this interface of delay and host immunity.

[a] Speaker

MS5 Dynamics of Chemical, Metabolic, and Gene Regulatory Networks: B. Fiedler, H. Matano, A. Mochizuki

Data-based modeling of on/off switch mechanism arising from biochemical network

Mariko Okada [a] [1]

[1] RIKEN Center for Integrative Medical Sciences – 1-7-22 W518 Suehiro-cho, Tsurumi-ku, Yokohama, Kanagawa, Japan

Signal transduction network is a biochemical system to sense, sort and transfer a variety of extracellular information to transcription factors in the nucleus to regulate gene expression for cell determination. Interestingly, signaling pathways often control this process in a nonlinear manner and, in some cases, analogous graded doses of extracellular stimuli promote digital activation of transcription factors. Spatio-temporal molecular network plays an essential function to realize the response. In this talk, based on experimental data observation and ODE modeling, a mechanism of digital activation of transcription factor NF-κB, will be discussed. In antigen-stimulated BCR response, NF-κB activity is controlled by two positive feedback loops within the signaling pathway to produce a switch-like activation of NF-κB. These feedback loops contribute to determine the threshold for NF-κB-mediated B cell proliferation, suggesting that the mechanism is important for B cell lineage commitment. Digital activation of transcription factors may be beneficial in a noisy cellular environment for accurate control of cell fate decision. Our studies suggest that cellular complexity might arise from combinatorial regulation of binary states of transcription factors.

[a]Speaker

Generic dynamics of coupled cell networks

Romain Joly [a] [1]

[1] Institut Fourier (IF) – CNRS : UMR5582, Université Joseph Fourier - Grenoble I – 100, rue des Maths BP 74 F-38402 Saint-Martin-d'Hères, France

The generic properties of the qualitative dynamics generated by an ODE $\dot{X}(t) = F(X(t))$ are now well understood. For example, for almost all non-linearity F, the periodic orbits of the ODE are hyperbolic. This hyperbolicity ensures the stability of the periodic orbits with respect to small perturbations of the parameters (e.g. perturbations of F, numerical simulation...) and also enables an accurate description of the dynamics as well as the use of tools as Takens' observation theorem.

Coupled cell networks form a subclass of ODEs and the description of their generic dynamics is still very relevant. However, since this subclass has an empty interior, one cannot directly use the generic results concerning general ODEs. In fact, some generic results may even fail if too many symmetries are imposed to the network.

The purpose of this talk is to discuss these questions and to present some recent results.

[a]Speaker

Stability and Instability in Complex Chemical Reaction Networks

Martin Feinberg [a] [1], Daniel Knight [1], Guy Shinar [3,2]

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[3] Javelin Medical – 4 Pekeris St., Rehovot 76702, Israel
[2] Weizmann Institute of Science (WIS) – 234 Herzl Street, Rehovot 76100 Israel, Israel

Across the landscape of all chemical reaction networks, including very complex ones, there is a surprising degree of dull, stable behavior. This appears to be so no matter what might be the details of the kinetics for the individual reactions. Yet there are also reaction networks, in particular certain ones commonly encountered in biochemistry, that exhibit quite rich dynamical behavior. Apparently, then, dull behavior in reaction networks is enforced by certain structural features that are shared widely across the reaction network landscape but not universally so. One of the central problems in chemical reaction network theory is to understand the connections between the structure of a reaction network and its capacity for various kinds of behavior. We will describe some recent results that are wide-ranging and that also indicate just how subtle such connections can actually be.

[a] Speaker

Sensitivity of chemical reaction networks: A structural approach.

Atsushi Mochizuki [a] [1,2], Takashi Okada [2], Bernold Fiedler [3]

[1] CREST JST – Japan
[2] Theoretical Biology Laboratory, RIKEN – Japan
[3] Institut für Mathematik, Freie Universität Berlin – Germany

In living cells a large number of reactions are connected by sharing substrates or product chemicals, forming complex network systems like metabolic network. One experimental approach to the dynamics of such systems examines their sensitivity: each enzyme mediating a reaction in the system is increased/decreased or knocked out separately, and the responses in the concentrations of chemicals or their fluxes are observed. However, due to the complexity of the systems, it has been unclear how the network structures influence/determine the responses of the systems. In this study, we present a mathematical method, named structural sensitivity analysis, to determine the sensitivity of reaction systems from information on the network alone. We investigate how the sensitivity responses of chemicals in a reaction network depend on the structure of the network, and on the position of the perturbed reaction in the network. We establish and prove a general law which connects the network topology and the sensitivity patterns of metabolite responses directly. Our theorem explains two prominent features of network in sensitivity: localization and hierarchy in response pattern. We apply our method to several hypothetical and real life chemical reaction networks, including the metabolic network of the E. coli TCA cycle. The theorem is useful, practically, when examining real biological networks based on sensitivity experiments.

[a] Speaker

Hidden slow-fast dynamics in biochemical models

Ilona Kosiuk [1], Christian Kuehn [2], Peter Szmolyan [a] [2]

[1] Max Planck Institute for Mathematics in the Sciences (MPI-MIS) – Germany
[2] Technische Universität Wien (TU-Wien) – Austria

The analysis of the global dynamics of biochemical systems can be a challenging task even for systems of small or moderate size. Often these difficulties are related to the occurrence of variables and parameters of vastly different orders of magnitudes. Hence, very different mechanisms may dominate the dynamics in certain regions of phase- and/or parameter-space. Identifying and analysing these regimes and the resulting decompositions into subsystems can be utilized in the analysis of the global dynamics. In suitably scaled variables the dynamics within individual regimes may be organized by lower-dimensional slow manifolds. However, often crucial dynamical effects take place at non-hyperbolic parts of these slow-manifolds which prevents the application of standard methods from geometric singular perturbation theory. The blow-up method can be used to overcome these difficulties and to match the individual regimes. In the talk this approach is presented in the context of selected models describing metabolic or signalling processes in cellular biology.

[a]Speaker

Morse decomposition of regulatory networks via its determining nodes

Hiroshi Kokubu [a] [1], Bernold Fiedler [2], Atsushi Mochizuki [3], Gen Kurosawa [3], Hiroe Oka [4]

[1] Kyoto University – Japan
[2] Free University of Berlin – Germany
[3] RIKEN – Japan
[4] Ryukoku University – Japan

The regulatory network is a coupled ODE system associated with a network representing regulation relations among variables. This is a mathematical formulation of a biological regulatory network, given by Fiedler et al. (JDDE 2013).
One of the main theorems of their paper is that the global attractor of a regulatory network can be reconstructed if one monitors all the information of solutions on the negative real line only at a good subset of nodes called the feedback vertex set (abbrev. FVS). This means that one can understand the nontrivial global dynamics of the regulatory network only from its FVS variables. This is, however, not very useful for applications, as one needs to monitor infinitely long time. In this talk, we shall show that, if we restrict attention to only a coarse information of global dynamics, namely its Morse decomposition, it is sufficient to monitor only on a finite time interval, or even at finitely many sample time points, at a FVS. We shall also show a result of numerical computation for Mirsky's circadian rhythm network as a test example.
This is a joint work with Bernold Fiedler, Free University of Berlin, Germany; Gen Kurosawa, RIKEN, Japan; Atsushi Mochizuki, RIKEN, Japan; Hiroe Oka, Ryukoku University, Japan.

[a]Speaker

Balloons in chemical reaction networks

Hiroshi Matano [a] [1]

[1] University of Tokyo (UTokyo) – Graduate School of Mathematical Sciences, University of Tokyo, 3-8-1 Komaba, Tokyo 153-8914, Japan

This talk is concerned with sensitivity analysis of chemical reaction networks. More precisely, we focus on monomolecular networks in equilibrium states and establish a simple criterion for determining regions of influence when any one of the reaction rates is perturbued through sensitivity experiments of enzyme knock-out type. Our argument is largely based on a recent work of Fiedler and Mochizuki, and it gives a certain interpretation of their results.

[a] Speaker

MS6 Fluid Dynamics: J. Bedrossian, H. Nussenzveig Lopes

Nonlinear, Nonstationary, Self-Organized States in Vlasov Plasmas and Connections to Long Lived Nonlinear States in 2D Euler Flows

Bedros Afeyan [a] [1]

[1] Polymath Research Inc – United States

We will describe the long time evolution of new states of self-organization in Vlasov plasmas, externally driven for short times, starting from equilibrium. An analogy between pair plasmas described by the Vlasov-Poisson system and 2D Euler vorticity equations will be exploited to describe new non-damped states and their impact for shear flows. Concepts such as Landau Damping, BGK Modes, Plasma Echoes, Quasilinear Theory and Strong Turbulence models will be reviewed and generalized to Kinetic electrostatic Electron (and Positron) Nonlinear Waves (KEEN and KEEPN waves). Further generalizations to multiple interacting KEEN and KEEPN wave states will also be described as a route to statistical mechanical models where geometric shapes (vortices) interacting on myriad scales necessitate memory effects to be taken into account. Connections to Stochastic Loewner Evolution (SLE), conformal invariance and multifractal analysis using geometric measure theory and multiresolution analysis of strong turbulence will be included.
B. Afeyan, F. Casas, N. Crouseilles, A. Dodhy, E. Faou, M. Mehrenberger, E. Sonnendrücker, Simulations of Kinetic Electrostatic Electron Nonlinear (KEEN) Waves with Two-Grid, Variable Velocity Resolution and High-Order Time-Splitting, Eur. Phys. J. D, 68 11, p. 295, (2014).
*Work supported by DOE FES-NNSA Joint program on HEDLP and AFOSR.

[a]Speaker

Appearance of turbulence in the Euler limit with Boundary effects.

Claude Bardos [a] [1]

[1] Laboratoire Jacques-Louis Lions (LJLL) – INRIA, Université Paris VII - Paris Diderot, CNRS : UMR7598, Université Pierre et Marie Curie (UPMC) - Paris VI – B.C. 187 75252 Paris Cedex 05, France

It seems that it is in presence of boundary effects that the classical issues of turbulence ie loss of regularity, the appearance of a non trivial Reynolds stress tensor and anomalous energy dissipation are the more visible.
Up to know the only general result is a theorem of Kato which connect these different effect.
1 I intend to give some observation on these issues using first the notion of wild solution of De Lellis and Székelyhidi .
2 I want to underline the fact the equivalent criteria for the non convergence to a smooth solution, namely:

$$\epsilon = \lim_{\nu \to 0} \partial_{\vec{b}} u_\tau > 0$$

is the basic ingredient in the ansatz proposed by Prandtl and Von Karman for the turbulent layer.

[a]Speaker

Determining wavenumber for fluid equations

Alexey Cheskidov [a] [1], Mimi Dai [1], Landon Kavlie [1]

[1] University of Illinois at Chicago – United States

In this talk we review classical results on determining modes for fluid equations and present a slightly different approach where we start with a time-dependent determining wavenumber defined for each individual trajectory and then study its dependenc on the force. While in some cases this wavenumber has a uniform upper bound, it may blow up when the equation is supercritical. Nevertheless, the average determining wavenumber is uniformly bounded even for the 3D Navier-Stokes and some supercritical SQG equations.

[a]Speaker

Uniformly rotating solutions for incompressible active scalars

Diego Cordoba [a] [1]

[1] Instituto de Ciencias Matematicas - CSIC (ICMat-CSIC) – Spain

A particular kind of weak solutions for a 2D active scalar are the so called sharp fronts, i.e., solutions for which the scalar is a step function. The evolution of such distribution is completely determined by the evolution of the boundary, allowing the problem to be treated as a non-local one dimensional equation for the contour. In this setting we will present several analytical results for the surface quasi-geostrophic equation (SQG) and the vortex patch equation. This is a jointwork with Angel Castro and Javier Gomez-Serrano.

[a]Speaker

Invariant measures for passive scalars in the small noise inviscid limit

Michele Coti Zelati [a] [1]

[1] Department of Mathematics, University of Maryland – Department of Mathematics Mathematics Building University of Maryland College Park, MD 20742-4015, United States

For linear inviscid transport equations, we analyze a class of invariant measures obtained as limits of invariant measures for stochastic viscous drift diffusion problems. By means of a rigidity result, we are able to gather information about the support of such inviscid (gaussian) measures in terms of the spectral properties of the transport operator. In the particular cases of non-degenerate shear flows and relaxation enhancing flows, inviscid measures are unique and their covariance operator can be explicitly computed. Other two-dimensional flows, such as cellular flows, can be treated as well.

[a]Speaker

ODEs with weakly regular velocity fields and applications to the Euler and Vlasov-Poisson equations

Gianluca Crippa [a] [1]

[1] Department of Mathematics and Computer Science, University of Basel – Spiegelgasse 1 CH-4051 Basel, Switzerland

I will show how the approach to ODEs with non-Lipschitz velocity field based on quantitative a priori estimates (introduced a few years ago with De Lellis) can be extended in such a way to obtain well posedness and compactess in a framework not included in the PDE theory of renormalized solutions of DiPerna-Lions and Ambrosio. This framework includes cases in which the derivative of the velocity is a singular integral of an integrable function, or even (for some components) of a measure. As a consequence, I will show how this implies existence and stability of Lagrangian solutions to the Euler and Vlasov-Poisson equations in the L^1 context. The talk will be based on various joint papers with Anna Bohun (Basel) and Francois Bouchut (Paris Est).

[a]Speaker

Finite determining modes for quasi-geostrophic equation

Mimi Dai [a] [1]

[1] Department of Mathematics, Statistics and Computer Science [Chicago] (UIC) – University of Illinois at Chicago, Chicago, IL 60607-7045, USA, United States

We prove that there exists a finite wave number $\Lambda(t)$ such that, on the global attractor of the quasi-geostrophic equation, if two solutions θ_1 and θ_2 coincide on the low frequency part $(\theta_1)_{\leq\Lambda(t)} = (\theta_2)_{\leq\Lambda(t)}$, then the two solutions are identical $\theta_1(t) \equiv \theta_2(t)$.

[a]Speaker

The Taylor model in magnetohydrodynamics (MHD)

David Gerard-Varet [a] [1]

[1] Institut de Mathématiques de Jussieu (IMJ) – CNRS : UMR7586, Université Paris VII - Paris Diderot – 2, place Jussieu 75251 Paris Cedex 05, France

We shall discuss a model introduced by J.B. Taylor in 1963, that comes from a formal asymptotic limit of MHD equations with rotation. This asymptotic model, relevant to the Earth's dynamo problem, should in principle allow for easier numerics. However, its simulation has been unsuccessful so far, due to unclear stability properties. The aim of the talk is to present recent mathematical results on this stability issue (joint work with I. Gallagher, L. Saint-Raymond)

[a]Speaker

Stability of time periodic solutions of the Navier-Stokes-Maxwell system

Slim Ibrahim [a] [1], Pierre Gilles Lemarie-Rieusset [2], N. Masmoudi [3]

[1] University of Victoria – Canada
[2] University of Evry – University of Evry – France
[3] New York University – United States

We study global wellposedness of a time periodic forced system of Magneto-Hydro-Dynamic equations. The system is a coupling of the incompressible Navier-Stokes equations with the Maxwell equations through the Lorentz force and Ohm's law for the current. We first show the existence of global small time-periodic mild solutions. Then, we prove their asymptotic stability. This is a joint work with P. G.-Lemarie and N. Masmoudi

[a]Speaker

Uniform time of existence for solutions of the α-Euler equations

Dragos Iftimie [a] [1], Busuioc Valentina [2], Milton Lopes Filho [3], Helena J. Nussenzveig Lopes [3]

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We consider the α-Euler equations on a bounded domain in dimension three with Navier perfect slip boundary conditions. We show that if the initial data is sufficiently smooth and if α is sufficiently small, then there exists a lower bound uniform in α for the maximal time of existence of the solution. This implies in particular the convergence of the solutions as $\alpha \to 0$ towards the solution of the incompressible Euler equation. This is joint work with V. Busuioc, M. Lopes Filho and H. Nussenzveig Lopes.

[a]Speaker

Onsager's Conjecture

Buckmaster Tristan [a] [1]

[1] Courant Institute – United States

In 1949, Lars Onsager in his famous note on statistical hydrodynamics conjectured that weak solutions to the Euler equation belonging to Hölder spaces with Hölder exponent greater than 1/3 conserve energy; conversely, he conjectured the existence of solutions belonging to any Hölder space with exponent less than 1/3 which dissipate energy. The first part of this conjecture has since been confirmed (cf. Eyink 1994, Constantin, E and Titi 1994). During this talk we will discuss recent work by Camillo De Lellis, László Székelyhidi Jr., Philip Isett and myself related to resolving the second component of Onsager's conjecture. In particular, we will discuss the construction of weak non-conservative solutions to the Euler equations whose Hölder $1/3 - \epsilon$ norm is Lebesgue integrable in time.

[a]Speaker

Almost global existence for the Prandtl boundary layer equations

Vlad Vicol [a] [1]

[1] Department of Mathematics - Princeton University – Department of Mathematics Fine Hall, Washington Road Princeton NJ 08544-1000 USA, United States

We consider the Prandtl boundary layer equations on the half plane, with initial datum that lies in a weighted Sobolev space with respect to the normal variable, and is real-analytic with respect to the tangential variable. The boundary trace of the horizontal Euler flow is taken to be a constant. We prove that if the Prandtl datum lies within ϵ of a stable profile, then the unique solution of the Cauchy problem can be extended at least up to time $T_\epsilon \geq \exp(\epsilon^{-1}/\log(\epsilon^{-1}))$.

[a]Speaker

MS7 Free Boundary Problems and Related PDEs: E. H. Kim, K. Trivisa

Global Existence of weak solutions for some transport equations with nonlocal velocity fields

Hantaek Bae [a] [1]

[1] Ulsan National Institute of Science and Technology – South Korea

In this talk,we study nonlocal and quadratically nonlinear transport equations. Prototypical examples are the surface quasi-geostrophic equation, the incompressible porous medium equation, Stokes equations, magnetogeostrophic equation and their variants. In this talk, we address the global existence of weak solutions of some model equations with rough initial data. To this end, we carefully choose dissipative quantities to minimize conditions of initial data mainly using the entropy.

[a]Speaker

Global existence of weak shocks past a solid ramp

Myoungjean Bae [a] [1]

[1] POSTECH – South Korea

Prandtl (1936) first employed the shock polar analysis to show that, when a steady supersonic flow impinges a solid wedge whose angle is less than a critical angle (i.e., the detachment angle), there are two possible configurations: the weak shock solution and the strong shock solution, and conjectured that the weak shock solution is physically admissible. The fundamental issue of whether one or both of the strong and the weak shocks are physically admissible has been vigorously debated over several decades and has not yet been settled in a definite manner. In this talk, I address this longstanding open issue and present recent analysis to establish the stability theorem for steady weak shock solutions as the long-time asymptotics of unsteady flows for all the physical parameters up to the detachment angle for potential flow. This talk is based on collaboration with Gui-Qiang G. Chen (Univ. of Oxford) and Mikhail Feldman(UW-Madison).

[a]Speaker

Existence of solutions for nonlinear models for tumor growth.

Donatella Donatelli [a] [1], Konstantina Trivisa [2]

[1] University of L'Aquila (DISIM-UNIVAQ) – via Vetoio, 67100 L'Aquila, Italy
[2] Department of Mathematics [Maryland] – Mathematics Building University of Maryland College Park, MD 20742-4015, United States

We investigate a free boundary problem modeling the growth of tumors cells. The model is given by a multi-phase flow and the tumor is described as a growing continuum Ω with boundary $\partial\Omega$ both of which evolve in time.
The model is given by a multi-phase flow model: the densities of the different cells are governed by a set of transport equations, the density of the nutrient and the density of the drug are governed by rather general diffusion equations, while the velocity of the tumor is given by Brinkman's equation. Global-in-time weak solutions are obtained using an approach based on penalization of the boundary behavior, diffusion and viscosity in the weak formulation. Both the solutions and the domain are rather general, no symmetry assumption is required and the result holds for large initial data.

[a]Speaker

Emergent behaviors of Cucker-Smale flocking particles

Seung Yeal Ha [a] [1]

[1] Seoul National University (SNU) – Department of Mathematical Sciences, Seoul National University, Seoul 151-747, Korea, South Korea

Collective behaviors of interacting particle systems are often observed in our nature, e.g., flocking of birds, swarming of fish and herding of sheeps etc. In previous literature, several phenomenological models were proposed and studied mostly using numerical simulations. Among others, the flocking model proposed by Cucker and Smale in 2007 has been extensively studied analytically. In this talk, we will review recent progress on the flocking dynamics of the Cucker-Smale type flocking models. In particular, we discuss sufficient conditions leading to mono-cluster and bi-cluster flockings.

[a]Speaker

Nonlocal equations for nonlinear electromagnetic surface waves

John Hunter [a] [1], Ryan Halabi [1]

[1] University of California at Davis – United States

Surface plasmon polaritons (SPPs) are electromagnetic surface waves that propagate on an interface between a conductor and an insulator and decay exponentially away from the interface. In the quasi-static, high-wave number limit, these waves have constant linearized frequency and nonlinear effects lead to complex spatial dynamics. We derive asymptotic equations for quasi-static SPPs on a planar interface between isotropic materials with a cubic Kerr nonlinearity. The asymptotic equations are Hamiltonian and couple the projections of the electric field on the interface onto its positive and negative wavenumber components. We discuss the nonlinear dynamics of these nonlocal equations, including the formation of singularities.

[a]Speaker

Spots in the Swift–Hohenberg Equation

Scott Mccalla [a] [1], Bjorn Sandstede [2]

[1] Montana State University – United States
[2] Brown University – United States

The existence of localized radially symmetric two and three dimensional solutions to the Swift–Hohenberg equation is explored both numerically through continuation and analytically through the use of geometric blow-up techniques. The bifurcation structure for these solutions is elucidated by formally treating the dimension as a continuous parameter in the equations. This reveals a family of solutions with an anomalous amplitude scaling that is far larger than expected from a formal scaling in the far field. The existence of these solutions is then established at onset using geometric blow-up techniques. This approach has the advantage that an *a priori* knowledge of this scaling is unnecessary as it naturally emerges from the construction.

[a]Speaker

Some free boundary problems for moving drops: Existence and homogenization.

Antoine Mellet [a] [1], Inwon Kim [2]

[1] University of Maryland – University of Maryland, College Park, MD 20742, USA, United States
[2] University of California Los Angeles (UCLA) – United States

We will present two simplified models describing the motion of a small liquid drop on a solid surface:
the thin film equation and the quasi-static approximation. Existence and homogenization results for these models will be presented. Of particular interest to us is the description of a drop sliding down an inclined plane.

[a]Speaker

The Hele-Shaw free-boundary asymptotics for fluid models of tumor growth

Benoit Perthame [a] [1]

[1] Laboratoire Jacques-Louis Lions (LJLL) – Université Pierre et Marie Curie - Paris 6 – France

The growth of solid tumors can be described at a number of different scales from the cell to the organ scales. For a large number of cells, the 'fluid mechanical' approach has been advocated recently by many authors in mathematics or biophysics. Several levels of mathematical descriptions are commonly used, including only elastic effects, nutrients, active movement, surrounding tissue, vasculature remodeling and several other features. We will focuss on the links between two types of mathematical models. The 'microscopic' description is at he cell population density level and a more macroscopic, description is based on a free boundary problem close to the classical Hele-Shaw equation. Asymptotic analysis is a tool to derive these Hele-Shaw free boundary problems from cell density systems in the stiff pressure limit. This modeling also opens other questions as circumstances in which instabilities develop. This work is a collaboration with F. Quiros and J.-L. Vazquez (Universidad Autonoma Madrid), M. Tang (SJTU) and N. Vauchelet (LJLL).

[a]Speaker

Expansion of an isentropic gas into vacuum

Denis Serre [a] [1]

[1] Unité de Mathématiques Pures et Appliquées (UMPA-ENSL) – CNRS : UMR5669, École Normale Supérieure (ENS) - Lyon – France

Tai-Ping Liu (1996) introduced the notion of "physical solution" of the isentropic Euler system when the gas is surrounded by vacuum. This notion can be interpreted by saying that the front is driven by a force resulting from a Hölder singularity of the sound speed. We address the question of when this acceleration appears or when the front just move at constant velocity.
We know (Grassin 1998, Serre 1997) that smooth isentropic flows with a non-accelerated front exist globally in time, for suitable initial data. In even space dimension, these solutions may persist for all $t \in \mathbb{R}$; we say that they are *eternal*. We derive a sufficient condition in terms of the initial data, under which the boundary singularity must appear. As a consequence, we show that, in contrast to the even-dimensional case, eternal flows with a non-accelerated front don't exist in odd space dimension. Our argument is related to that of Milnor (1978) in his proof of the hairy ball Theorem.
In one space dimension, we give a refined definition of physical solutions. We show that for a shock-free flow, their asymptotics as both ends $t \to \pm\infty$ are intimately related to each other.

[a]Speaker

Analysis of 2+1 Diffusive-Dispersive PDE Arising in River Braiding

Saleh Tanveer [1], Charis Tsikkou [a] [2]

[1] The Ohio State University (OSU) – United States
[2] West Virginia University (WVU) – United States

In the context of a weakly nonlinear study of bar instabilities in a river carrying sediment, P. Hall introduced an evolution equation for the deposited depth which is dispersive in one spatial direction, while being diffusive in the other. In this talk, we present local existence and uniqueness results using a contraction mapping argument in a Bourgain-type space. We also show that the energy and cumulative dissipation are globally controlled in time.

[a]Speaker

On a mechanical model for tumor growth

Konstantina Trivisa [a] [1]

[1] Department of Mathematics [Maryland] – Mathematics Building University of Maryland College Park, MD 20742-4015, United States

Recent advances on a nonlinear model for tumor growth will be discussed.

[a]Speaker

Co-dimension One Self-Assembly

James Von Brecht [a] [1]

[1] California State University Long Beach – United States

I will discuss mathematical models and tools for analyzing physical and biological processes that exhibit co-dimension one characteristics. Examples include the self-assembly of Polyoxometalate (POM) macroions into hollow, spherical structures called blackberries as well as the assembly of surfactant molecules into micelles and vesicles. I will characterize when such structures can arise in the context of isotropic and anisotropic models, and will also discuss insights into physical models of these behaviors.

[a]Speaker

MS8 Inverse Spectral Theory and Applications: D. Damanik, G. Teschl

Universality of Mesoscopic Fluctuations in Orthogonal Polynomial Ensembles

Jonathan Breuer [a] [1]

[1] Einstein Institute of Mathematics – The Hebrew University of Jerusalem Jerusalem, 91904, Israel, Israel

We shall discuss fluctuations on the mesoscopic scale for orthogonal polynomial ensembles and show that these are universal in the sense that two measures with asymptotic recurrence coefficients have the same asymptotic mesoscopic fluctuations (under an additional assumption on the local regularity of one of the measures). The convergence rate of the recurrence coefficients determines the range of scales on which the limiting fluctuations are identical. A particular consequence of our results is a Central Limit Theorem for the modified Jacobi Unitary Ensembles on all mesoscopic scales. This is joint work with Maurice Duits.

[a]Speaker

The isospectral torus of quasi-periodic Schrödinger operators via periodic approximations

David Damanik [1], Michael Goldstein [2], Milivoje Lukic [a] [2]

[1] Rice University – 6100 Main Street, Houston, Texas 77251-1892, United States
[2] University of Toronto – 40 St. George St. - Toronto, Ontario - M5S 2E4, Canada

We study quasi-periodic Schrödinger operators $H = -\frac{d^2}{dx^2} + V$ in the regime of analytic sampling function and small coupling. More precisely, the potential is

$$V(x) = \sum_{m\in\mathbb{Z}^\nu} c(m)\exp(2\pi i m\omega x)$$

with $|c(m)| \leq \epsilon \exp(-\kappa|m|)$. Our main result is that any reflectionless potential Q isospectral with V is also quasi-periodic and in the same regime, with the same Diophantine frequency ω, i.e.

$$Q(x) = \sum_{m\in\mathbb{Z}^\nu} d(m)\exp(2\pi i m\omega x)$$

with $|d(m)| \leq \sqrt{4\epsilon}\exp(-\frac{\kappa}{4}|m|)$. The proof relies on approximation by periodic potentials $\tilde{V}$, which are obtained by replacing the frequency ω by rational approximants $\tilde{\omega}$. We adapt the multiscale analysis, developed by Damanik–Goldstein for V, so that it applies to the periodic approximants $\tilde{V}$. This allows us to establish estimates for gap lengths and distances and Fourier coefficients of $\tilde{V}$ which are independent of period, unlike the standard estimates known in the theory of periodic Schrödinger operators. Starting from these estimates, we obtain the main result by comparing the isospectral tori and translation flows of $\tilde{V}$ and V.

[a]Speaker

Periodic GMP matrices

Benjamin Eichinger [a] [1]

[1] Johannes Kepler University Linz – Austria

Let $E = [b_0, a_0] \setminus \bigcup_{j=1}^{g} (a_j, b_j)$ be a finite system of intervals. Not all sets of this kind are spectra of periodic Jacobi matrices. This is only the case if there exists a polynomial T such that $T^{-1}([-2, 2]) = E$. We define a new class of operators called GMP matrices such that for any finite system of intervals the isospectral torus, i.e.,

$$A(E) := \{A : A \text{ is reflectionless GMP matrix on its spectral set } E\},$$

consits of periodic GMP matrices. Their spectral theory and relation to Jacobi matrices will be discussed. To describe the latter one we introduce a discrete dynamical system on $A(E)$, which we call the Jacobi flow on GMP matrices.

[a] Speaker

Limit-Periodic Operators with Homogeneous Spectra

Jake Fillman [a] [1], Milivoje Lukic [2]

[1] Rice University [Houston] – P.O. Box 1892, Houston, Texas 77251-1892, United States
[2] University of Toronto – 40 St. George St. - Toronto, Ontario - M5S 2E4, Canada

We will discuss the spectra of limit-periodic Jacobi matrices and continuum Schrödinger operators. Specifically, the spectrum of a limit-periodic operator which obeys the Pastur-Tkachenko condition is homogeneous in the sense of Carleson. When combined with work of Gesztesy-Yuditskii, our theorem implies that the spectrum of a continuum Schrödinger operator with Pastur–Tkachenko potential has infinite gap length whenever the potential fails to be uniformly almost periodic.

[a] Speaker

Blow up for the two-component Camassa–Holm system

Katrin Grunert [a] [1]

[1] Norwegian University of Science and Technology – Norway

The two-component Camassa–Holm system

$$\begin{cases} u_t - u_{txx} + 3uu_x - 2u_x u_{xx} - uu_{xxx} + \rho\rho_x &= 0, \\ \rho_t + (u\rho)_x &= 0, \end{cases}$$

serves as a model for shallow water. Furthermore, it is a generalization of the famous Camassa–Holm equation, which has been studied intensively due to its rich mathematical structure. Thus a huge class of solutions enjoys wave breaking within finite time, but there is also a regularising effect which prevents many solutions form blow up. Hence the aim of this talk is twofolded. On the one hand we want to study this regularising effect in some detail and on the other hand we want to focus on how to predict if a solution enjoys wave breaking in the nearby future or not. This talk is based on joint work with H. Holden and X. Raynaud.

[a]Speaker

Wave phenomena of the Toda lattice with steplike initial data

Johanna Michor [a] [1], Iryna Egorova [2], Gerald Teschl [1]

[1] University of Vienna, Faculty of Mathematics – Oskar-Morgenstern Platz 1, 1090 Wien, Austria
[2] Institute for Low Temperature Physics (ILT) – 47, Lenin Ave, 61103 Kharkiv, Ukraine

We give a survey of the long-time asymptotics for the Toda lattice with steplike constant initial data using the nonlinear steepest descent analysis and its extension based on a suitably chosen g-function. Analytic formulas for the leading term of the asymptotic solutions of the Toda shock and rarefaction problems (including the case of overlapping background spectra) are given and complemented by numerical simulations. We provide an explicit formula for the modulated 2-band solution in terms of Abelian integrals on the underlying hyperelliptic Riemann surface.

[a]Speaker

Direct and Inverse Spectral Relationships

Thomas Vandenboom [a] [1]

[1] Rice University [Houston] – P.O. Box 1892, Houston, Texas 77251-1892, United States

We discuss some consequences of viewing classical inverse results for Jacobi operators through a direct-spectral lens.

[a]Speaker

Elements of J-theory in application to the Killip-Simon and other spectral problems

Peter Yuditskii [a] [1]

[1] Johannes Kepler University Linz (JKU) – Austria

J-theory is a multiplicative theory of J-contractive analytic matrix functions. We will discuss a relation of this theory with the spectral theory for GMP matrices, and, depending on time, with some other spectral problems.

[a] Speaker

MS9 Lattice Dynamical Systems: A. Hoffman, H. J. Hupkes

Pinning and snaking on lattices: orientation dependence and multiple pinning mechanisms

Andrew Dean [a] [1], Paul Matthews [2], Stephen Cox [2], John King [2]

[1] Department of Biology [York] – University of York PO Box 373 York YO10 5PW, United Kingdom
[2] School of Mathematical Sciences [Nottingham] – University Park, Nottingham, NG7 2RD, United Kingdom

We study pinning of fronts and the associated homoclinic snaking on bistable lattices. Bistability leads to the existence of travelling waves comprising a front connecting the two stable states. In the analogous continuous system, the wavespeed varies smoothly and is zero at a single point only. In contrast, a lattice system exhibits a range of values at which the front is stationary, due to the energy barrier imposed by the periodic lattice. Within this range, back-to-back fronts may be glued together forming localised solutions which 'snake' back and forth in parameter space via successive saddle-node bifurcations. We first look at one-dimensional fronts on a planar lattice, and employ the method of exponential asymptotics in the continuum limit to derive the width of the pinning region and the associated snaking bifurcation diagram. In particular, we show that the width depends discontinuously on the orientation ϕ of the front to the underlying lattice, vanishing when $\tan \phi$ is irrational, and is exponentially small otherwise. Standard asymptotic techniques based solely on algebraically small quantities cannot derive such an effect. We then consider a fourth-order, pattern-forming lattice problem in one dimension. In this case there are two periodic structures to which a front may pin: the lattice and the sinusoidal pattern. We investigate how including a third spatial scale in this manner impacts the the snaking bifurcation diagram.

[a]Speaker

Propagation failure in some nonlocal equations

Grégory Faye [a] [1]

[1] Centre d'analyse et de mathématique sociale (CAMS) – CNRS : UMR8557, Ecole des Hautes Etudes en Sciences Sociales (EHESS) – 190-198 avenue de France, 75013 Paris, France

We investigate pinning regions and unpinning asymptotics in nonlocal equations of the form

$$u_t = d(-u + \mathcal{K} * u) + f(u),$$

where $\mathcal{K} * u$ denotes the convolution on the real line. Here, the nonlinearity f is of bistable type and $d > 0$. We show that phenomena are related to but different from pinning in discrete and inhomogeneous media. We establish unpinning asymptotics using geometric singular perturbation theory in several examples. We also present numerical evidence for the dependence of unpinning asymptotics on regularity of the nonlocal convolution kernel. This is joint work with Taylor Anderson, Arnd Scheel and David Stauffer.

[a]Speaker

Random Attractors for stochastic lattice dynamical systems

Xiaoying Han [a] [1]

[1] Auburn University – United States

In this talk I will present some recent results on the existence of pullback random attractors for stochastic lattice dynamical systems(SLDSs) with different type of white noise. In particular, I will first review existing results on first order SLDSs with finite multiplicative white noise, and infinite additive white noise in the weighted spaces of infinite sequences. Then I will introduce the most recent result on SLDSs with infinite multiplicative white noise, i.e., a different noise at each node, which requires different techniques from existing methodologies.

[a]Speaker

Approximations of dynamics of nonlinear lattices on the extended time scale

Dmitry Pelinovsky [a] [1]

[1] Department of Mathematics, McMaster University – 1280 Main Street West, Hamilton, Ontario, Canada, L8S 4K1, Canada

I will address two popular approximations of dynamics of nonlinear lattices. Small-amplitude slowly varying travelling waves in the Fermi-Pasta-Ulam lattices are approximated by solutions of the Korteweg-de Vries equations. Small-amplitude weakly coupled oscillators in the Klein-Gordon lattices are approximated by solutions of the discrete nonlinear Schrodinger equations. For both approximations, I will show how to justify the amplitude equations on the standard and extended time scales by using a priori energy estimates. I will also show how approximations on the extended time scale can be used to prove approximate nonlinear stability of solitary waves and breathers in nonlinear lattices.

[a]Speaker

Generalized dNLS models as normal forms for KG lattices and applications

Tiziano Penati [a] [1]

[1] Department of Mathematics F.Enriques, University of Milan – Via Saldini 50, 20133 Milan, Italy

Generalized dNLS models emerge as resonant normal forms for Klein-Gordon lattices in the small energy regime and anticontinuum limit. In the case of an arbitrary large but finite 1D lattice, the use of discrete symmetries allow to get a sharp dependence of the estimates on the size of the lattice. Results available on the generalized dNLS lattices, like long time stability of breathers, approximation of the Cauchy problem and non existence of vortex-like multibreathers can be transfered to the original Klein-Gordon lattice.

[a]Speaker

Travelling waves in nonlinear metamaterial lattices

Vassilios Rothos [a] [1]

[1] Aristotle University of Thessaloniki (AUTH) – Lab of Nonlinear Mathematics & Dept. of Mechanical Engineering AUTH, Thessaloniki 54124, Greece

Nonlinear Localizations appear generically in nonlinear metallic, and PT metamaterials in the presence of dissipation that is always present in practice. We consider a lattice equation modelling one-dimensional metamaterials formed by a discrete array of nonlinear resonators. We focus on periodic travelling waves due to the presence of a periodic force. The existence and uniqueness results of periodic travelling waves of the system are presented. Employing a Melnikov analysis we study the existence and persistence of such travelling waves, and study their linear stability. Our analytical results are found to be in good agreement with direct numerical computations.

[a] Speaker

On solitary waves in diatomic lattices

Anna Vainchtein [a] [1], Yuli Starosvetsky [2], J. Douglas Wright [3], Ron Perline [3], Aaron Hoffman [4]

[1] University of Pittsburgh [Pittsburgh] – 301 Thackeray Hall, Pittsburgh, PA 15260, United States
[2] Technion - Israel Institute of Technology [Haifa] – Technion City, Haifa 3200003, Israel
[3] Drexel University (DREXEL UNIVERSITY) – 3141 Chestnut Street, Philadelphia, PA 19104, United States
[4] Franklin W. Olin College of Engineering – United States

We consider a mass-spring chain of alternating particles of different masses. At generic mass ratios, pulses propagating through the chain radiate lattice waves traveling behind them, thus precluding formation of genuine solitary waves. However, numerical simulations and some recent work on strongly nonlinear granular chains suggest that under certain conditions there is a sequence of special 'anti-resonance' values of mass ratio at which there is no radiation and solitary waves do exist. Using multiscale asymptotic analysis, we find a Fredholm-type condition for mass ratios approximating such values. The condition is explicit for the diatomic Toda lattice, where the obtained small-ratio approximate values are in good agreement with the numerical results.

[a] Speaker

Breathers in two-dimensional lattices

Jonathan Wattis [a] [1]

[1] School of Mathematical Sciences [Nottingham] – University Park, Nottingham, NG7 2RD, United Kingdom

We use asymptotic techniques to construct approximations to breathers in two-dimensional lattices. We consider scalar Fermi-Pasta-Ulam lattices, that is, where these is only one variable at each lattice node, and there is no onsite potential, only nonlinear nearest neighbour interactions. Through the use of multiple scales, we obtain a nonlinear Schródinger reduction. We are able to solve this system in two cases - (i) stationary breathers in an arbitrary potential, and (ii) moving breathers in a symmetric potential. In this latter case, there is an ellipticity criterion for the existence of breathers. We summarise results for the square lattice, as well as the triangular and honeycomb geometries. If time permits, we may also present preliminary results on the analysis of a vector two-dimensional lattice.

[a] Speaker

MS10 Mathematical Mechanisms Underlying the Biophysics of Neuronal Activity: R. Barrio, H. Osinga

On the effects on cortical spontaneous activity of the symmetries of the network of pinwheels in visual area V1.

Olivier Faugeras [a] [1], Pascal Chossat [1,2], Romain Veltz[1]

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[2] Laboratoire Jean Alexandre Dieudonné (JAD) – Université Nice Sophia Antipolis (UNS), CNRS : UMR7351 – Université de Nice - Sophia Antipolis U.M.R. no 6621 du C.N.R.S. Parc Valrose 06108 Nice Cedex 02 France, France

We consider the problem of describing mathematically the spontaneous activity of V1 by combining several important experimental observations including 1) the organization of the visual cortex into a spatially periodic network of hypercolumns structured around pinwheels, 2) the difference between short-range and long-range intra-cortical connections, the first ones being rather isotropic and producing naturally doubly-periodic patterns by Turing mechanisms, the second one being patchy and 3) the fact that the Turing patterns spontaneously produced by the short-range connections and the network of pinwheels have similar periods. By analyzing the Preferred Orientation (PO) map, we are able to classify all possible
singular points of the PO maps (the pinwheels) as having symmetries described by a small subset of the wallpaper groups. We then propose a description of the spontaneous activity of V1 using a classical voltage-based neural field model that features isotropic short-range connectivities modulated by non-isotropic long-range connectivities. A key observation is that, with only short-range connections and because the problem has full translational invariance in this case, a spontaneous doubly-periodic pattern generates a 2-torus in a suitable functional space which persists as a flow-invariant manifold under small perturbations, hence when turning on the long-range connections.

[a] Speaker

Mathematical tools for phase control in transient-states of spiking neurons

Gemma Huguet [a] [1], Oriol Castejón [1], Antoni Guillamon [1], Rafael De La Llave [2]

[1] Departament de Matemàtica Aplicada I - Universitat Politècnica de Catalunya (UPC) – Spain
[2] Georgia Institute of Technology (GATECH) – North Ave. Atlanta, Georgia 30332, United States

Phase resetting curves (PRCs) constitute a powerful resource in time-control problems in biological processes. They predict the effect of a perturbation on the phase of an oscillator, assuming that all the dynamics can be explained by the phase variable. However, factors like the rate of convergence to the oscillator, strong forcing or high stimulation frequency may invalidate the above assumption and raise the question of how is the phase variation away from an attractor. In this talk, I will present a numerical method to perform the effective computation of the phase advancement when we stimulate an oscillator which has not reached yet the asymptotic state (a limit cycle) using the concept of isochrons. To do so, we first perform a careful study of the theoretical grounds (the parameterization method for invariant manifolds), which allow us to describe the isochronous sections of the limit cycle. From it, we build up Phase Response Functions (PRF) and Amplitude Response Function (ARF) to control changes in the phase and the transversal variables, respectively. I will show some examples of the computations we have carried out for some well-known biological models. Finally, I will compare the predictions given by the PRC-approach (a 1D map) to those given by the PRF-ARF-approach (a 2D map), under pulse-train stimuli.

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[a] Speaker

Slow manifolds organising mixed-mode dynamics

Bernd Krauskopf [a] [1], Hinke Osinga [1], Jose Mujica [1]

[1] Department of Mathematics, The University of Auckland – New Zealand

This talk will discuss some mechanisms that give rise to mixed-mode oscillations in slow-fast systems with at least two slow variables. The focus will be on how attracting and repellling slow manifolds interact: locally to create small-amplitude oscillations and globally to give rise to large-amplitude excursions and reinjection.

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[a] Speaker

Delay-imposed structure on the network dynamics of oscillators

Spase Petkoski [a] [1], Viktor Jirsa [1]

[1] Institut de Neurosciences des Systèmes (INS) – Inserm : U1106, Université de la Méditerranée - Aix-Marseille II – Faculté de médecine Timone, 27 Bd Jean Moulin 13385 Marseille Cedex 5, France

The spatial structure of the brain is represented with tracts between different nodes. Due to the transmission speeds via these tracts, the overall dynamics of the brain is influenced by time-delays. Empirical data from the brain suggests that the tract lengths and thereafter the time delays follow a bimodal distribution. To study their influence on the dynamics, we introduce the Kuramoto model with link-dependent delays. We analyse three different network topologies with same distribution of the delays. In the first model, the distribution is completely random, while in the other two it imposes a structure in the network, dividing it into subpopulations. We observe that even when both subpopulations have identical distribution of natural frequencies and inter- and intra- population couplings, the subpopulations may still split in clusters that have different order parameters. In summary, large-scale brain networks are characterized by the functionally important presence of many signal transmission delays, which have systematic effects upon the global network synchronization behavior. We introduce spatio-temporal decomposition of the connectivity and demonstrate its capacity to explain various synchronization and clustering behaviors. These can be controlled by varying the spatial distribution for the discrete time delays. Our findings unveil the critical importance to the mean-field brain dynamics of its spatial dimension.

[a] Speaker

The role of homoclinic and heteroclinic cycles in neuronal models: individual and networked

Sergio Serrano [a] [1], Roberto Barrio, Marc Lefranc, Angeles Martinez, Marcos Rodriguez, Andrey Shilnikov

[1] University of Zaragoza – Spain

In this talk we present two applications of complete biparametric analysis of the bifurcations and different behaviours of single-neuron models, both giving a detailed information of the single-cell model as basic information to analyze small neuron networks (CPGs).
First, we characterize the systematic changes in the topological structure of chaotic attractors that occur as spike-adding and homoclinic bifurcations are encountered in the slow-fast dynamics of neuron models. This phenomenon is detailed in the Hindmarsh-Rose neuron model and in the inter-heart leech neuron model, where we show that the periodic orbits appearing after each spike-adding bifurcation have specific symbolic sequences in the canonical symbolic encoding of the dynamics of the system. This allows us to understand how these bifurcations modify the internal structure of the chaotic attractors.
Second, we reveal the existence of relevant bifurcations (heteroclinic cycles) in CPGs that creates slow switching oscillations that achieve the level of robustness and stability observed in nature. To study biologically plausible CPG models it is necessary to use specially adapted techniques to take into account that the equations are grouped by each neuron. But standard computational continuation techniques use the complete system without considering the internal structure of the network, while our techniques permits to detail the complete bifurcation process.

[a]Speaker

Spiking and Bursting in a biophysical model of excitable cell

Krasimira Tsaneva-Atanasova [a] [1]

[1] Department of Mathematics (CEMPS, University of Exeter) – Harrison Building, University of Exeter, Exeter EX4 4QF, U.K., United Kingdom

Spiking and bursting are core features of the functional physiology of all excitable cells. Only after firing a spike will a cell perform its task. Furthermore, the rate, timing and patterns of spike generation have important effects on the strength of downstream events such as neurotransmitter release. Thus understanding how these facets of action potential firing arise is crucial in helping us understand the functional physiology of these cells. Here we present bifurcation analysis of a detailed biophysical model that accounts for sppikng, bursting as well as mixed firing patterns observed in real cells. We study the dependence of these dynamics on key model parameters and discuss the underlying mathematical mechanisms.

[a]Speaker

Emergence of connectivity motifs via the interaction of long-term and sort-term plasticity.

Eleni Vasilaki [a] [1]

[1] Department of Computer Science, University of Sheffield – United Kingdom

The identification of synaptic mechanisms that underlie learning and memory is a key challenge for neuroscience. These mechanisms are currently assumed to be captured by persistent modifications to the synaptic connections among neurons. Synaptic connections in microcircuits and networks are not random; experimental observations indicate the existence of specific microscopicpatterns (or connectivity motifs), with non-random features. However, it is unclear how plasticity of individual synaptic connections contributes to the formation of the observed motifs. In particular, for cortical pyramidal neurons, the degree of bidirectional connectivity varies significantly between the visual and somatosensory cortex areas. Recent evidence in prefrontal cortex and in the olfactory bulb suggest that some other features of synaptic physiology, such as the short-term dynamical nature of the synapse, may be correlated to specific connectivity motifs. The causes for these structural differences are still unknown.
I will present a theory based on a phenomenological, long-term synaptic plasticity "learning rule", that is able to accurately reproduce a vast corpus of experimental data. The rule captures dependencies on both the timing and frequency of neuronal signals, providing a very simple mechanistic explanation for the emergence of connectivity motifs, while shedding light on the long debate about the nature of the neuronal code.

[a] Speaker

Coarse-grained analysis of patterned activity in a discrete-time neural network

Kyle Wedgwood [a] [1]

[1] School of Mathematical Sciences [Nottingham] (CMMB) – University Park, Nottingham, NG7 2RD, United Kingdom

At the single cell level, neurons typically exhibit all-or-nothing responses, dependent on the input currents they receive from the rest of the network. Due to far reaching processes, neurons can form connections with distant parts of the network, allowing for rapid communication across long distances.
Certain neural systems show computation through patterned activity; persistent localised activity, in the form of bumps has been linked to working memory whilst the propagation of activity in the form of waves has been associated with binocular rivalry tasks. The assumption of sufficiently slow synapses allows for the replacement of firing patterns with firing rates, resulting in a neural field model that is amenable to perturbative analysis. This description averages out fluctuations in both space and time ignoring these small scale effects. Our aim is to analyse a network that retains these small scale effects, but whose large scale effects can be predicted in an analogous way to neural field models.
We present analysis of a network of minimal three-state neurons whose transitions are probabilistic. By taking appropriate limits, we demonstrate the existence and compute stability of spatiotemporally patterned activity across the network. We then show how coarse-grained analysis can be used to construct bifurcation diagrams for the network when these limits are relaxed and illustrate how these can be used to reduce the complexity of the dynamics.

[a] Speaker

MS11 Mathematical Problems of General Relativity: S. Klainerman, J. Szeftel

Linear and non-linear wave equations on black hole backgrounds

Stefanos Aretakis [a] [1]

[1] Princeton University – United States

I will present recent results for linear and non-linear wave equations on black hole backgrounds. First I will discuss the characterisic gluing problem for the wave equation on general Lorentzian manifolds then present applications to extremal black holes including several non-decay and growth results. These instabilities results make the study of non-linear wave equations notoriously hard. I will then present a new physical space method for deriving very fast decay for general black holes (comparable to that of Price's law) and use this method to obtain several global well posedness results for non-linear wave equations on extremal black holes.

[a] Speaker

Escape coordinates outside black holes

Pieter Blue [a] [1]

[1] University of Edinburgh – School of Mathematics The University of Edinburgh James Clerk Maxwell Building The King's Buildings Peter Guthrie Tait Road Edinburgh EH9 3FD, United Kingdom

In proving estimates of decay of fields outside black holes, it is now common to use a foliation by spacelike hypersurfaces that cross the event horizon and null infinity. The traditional way that this has been done is to work with the standard radial Schwarzschild-like coordinates away from the horizon and infinity, to use something like Eddington-Finkelstein coordinates near the boundaries, and to apply a smooth cut-off function to transition from one set of coordinates to another.

In this brief talk, I will describe new set of coordinates with all the desirable properties and which can be written as analytic expressions in the standard Schwarzschild-like coordinates. Several of the vector fields that arise in the decay of fields have surprisingly simple expressions in these coordinates. For null geodesics, these vector fields generate energies that can be treated as escape functions.

[a] Speaker

The Hyperboloidal Foliation Method for Nonlinear Wave-Klein-Gordon Systems

Philippe G. Lefloch [a] [1]

[1] Laboratoire J-L Lions and CNRS (UPMC) – Université Pierre et Marie Curie (UPMC) - Paris VI – 4 place Jussieu - 75005 Paris, France

I will present a new method, based on a (3+1)-foliation of (the interior of a light cone in) Minkowski spacetime by hyperboloids and developed in collaboration with Yue Ma (UPMC) in *The hyperboloidal foliation method, World Scientific Press, 2014.* This method allows us to establish global-in-time existence results for small-amplitude solutions to nonlinear wave-Klein-Gordon systems posed on a curved spacetime. We use energy functionals that are invariant under Lorentz transformations, and are able to include wave equations and Klein-Gordon equations within a unified framework. The method provides energy bounds with almost optimal growth rates, while a natural application of this technique stands out: the nonlinear stability of Minkowski space for massive scalar fields.

Blog address: http://philippelefloch.org

[a]Speaker

Large data dispersive solutions to the Einstein-scalar field system

Jonathan Luk [a] [1]

[1] Cambridge University – United Kingdom

I will present recent work on the existence, stability and long time dynamics of large data dispersive solutions to the Einstein-scalar field system. This class of solutions, while having large initial data, are causally geodesically complete. This talk is based on joint works with Sung-Jin Oh and Shiwu Yang.

[a]Speaker

Unique Continuation in Asymptotically Anti-de Sitter Spacetimes

Arick Shao [a] [1], Gustav Holzegel [1]

[1] Imperial College London – United Kingdom

In this talk, we consider the problem of unique continuation from infinity for Anti-de Sitter (AdS) and asymptotically AdS spacetimes. We show, roughly, that given a solution ϕ of a linear (massive or massless) wave equation on AdS spacetime, if ϕ and its first derivative vanish to high enough order (depending on the mass) on a sufficiently large but finite portion of infinity, then ϕ must also necessarily vanish in a small neighborhood of infinity. In particular, this establishes a correspondence between data for ϕ at infinity and the value of ϕ in the interior. When available, we also connect our results to the well-posedness theory: we show that trivial Dirichet and Neumann data at (a large enough portion of) infinity along with sufficient regularity implies vanishing in the interior. Furthermore, all these results generalize to a large class of asymptotically AdS spacetimes, as well as to tensor-valued waves. These techniques are also viable for studying nonlinear wave equations; one application is to study corresponding uniqueness properties for the Einstein-vacuum equations with negative cosmological constant.

[a]Speaker

Stability and Instability of Scalar Fields on Kerr Spacetimes

Yakov Shlapentokh-Rothman [a] [1]

[1] Department of Mathematics - Princeton University – Department of Mathematics Fine Hall, Washington Road Princeton NJ 08544-1000 USA, United States

I will discuss some stability and instability results for wave and Klein-Gordon equations on sub-extremal Kerr exterior backgrounds. More specifically, for the wave equation we will see that general finite energy solutions have a uniformly bounded energy and satisfy an integrated local energy decay estimate. In contrast, for the Klein-Gordon equation we will see that there exist finite energy solutions which grow exponentially. We will also discuss the implications of these results for black hole stability. Some of this work is joint with Mihalis Dafermos and Igor Rodnianski.

[a]Speaker

A geometric approach for sharp Local well-posedness of quasilinear wave equations

Qian Wang [a] [1]

[1] Mathematical institute [Oxford] (OXPDE) – Andrew Wiles Building Radcliffe Observatory Quarter Woodstock Road Oxford OX2 6GG, United Kingdom

I will present my work on the problem of optimal well-posedness for general quasi-linear wave equations in $\mathbb{R}^{1+3}$. In general, equations of this type are ill-posed with H^s data for $s \leq 2$. The optimal result of the well-posedness with data in $H^s, s > 2$ was proved by Smith-Tataru by constructing parametrices using wave packets. I will present the proof by the vectorfield approach. This approach is initiated by Klainerman, developed by him and Rodnianski to achieve the result of $s > 2 + \frac{2-\sqrt{3}}{2}$ and then applied to the Einstein vacuum equations to achieve the result of $s > 2$. To achieve the optimal result for the general quasi-linear wave equations, one has to face the major hurdle caused by the Ricci tensor of the metric. This posed a question that if the geometric approach can provide the sharp result for the non-geometric equations. The optimal result is achieved based on geometric normalization and new observations on the regularity properties of the eikonal equation associated to the quasi-linear wave equations.

[a]Speaker

Vector field methods for transport equations with applications

Jacques Smulevici [a] [1]

[1] Laboratoire de Mathématiques d'Orsay (LM-Orsay) – CNRS : UMR8628, Université Paris XI - Paris Sud – France

I will present recent results concerning the development of a full vector field method for transport equations, in the spirit of the Klainerman vector field method for wave equations, including robust decay estimates of velocity averages for both the relativistic and the non-relativistic transport operators. In the second part of the talk, I will present several applications to the study of small data solutions of non-linear systems such as the Vlasov-Poisson or Vlasov-Nordstrom system. This is joint with David Fajman (Vienna) and Jérémie Joudioux (Vienna)

[a]Speaker

MS12 New Trends on Concentration Phenomena in Nonlinear Elliptic Equations: A. Pistoia, J.-C. Wei

Vortex Dynamics

Thomas Bartsch [a] [1]

[1] University of Giessen – Germany

The dynamics of point vortices in a two-dimensional domain is described by a singular, first-order Hamiltonian system. The Hamiltonian is the Kirchhoff-Routh or Kirchhoff-Onsager function which appears in several applications. We present recent results about the existence of equilibrium point vortex configurations in bounded planar domains and their desingularization to solutions of the Euler equations, and about periodic solutions.
This is joint work with Qianhui Dai and Angela Pistoia.

[a] Speaker

Multiple blowing-up solutions for the singular Liouville equation on closed surfaces

Teresa D'aprile [a] [1]

[1] University of Roma "Tor Vergata" – via della Ricerca Scientifica 1, 00133 Roma, Italy

Let (Σ, g) be a compact surface without boundary endowed with metric g. We are concerned with the existence of blowing-up solutions when the parameter ρ approaches the critical values $8\pi\mathbb{N}$ for the following singular Liouville equation:

$$-\Delta_g u = \rho\left(\frac{h(x)e^u}{\int_\Sigma h(x)e^u\,dV_g} - \frac{1}{|\Sigma|}\right) - 4\pi\sum_{i=1}^{\ell}\alpha_i\left(\delta_{p_i} - \frac{1}{|\Sigma|}\right),$$

where $\rho > 0$, $h : \Sigma \to \mathbb{R}$ is a smooth positive function, the points $p_i \in \Sigma$ are the singular sources with weights $\alpha_i > 0$. Here δ_p denotes the Dirac mass measure supported at p and $|\Sigma|$ is the area of Σ.
In particular, by employing a min-max scheme jointly with a finite dimensional reduction method, we construct solutions exhibiting a *blow-up* behavior near a finitely many number of points of Σ. We then discuss how new existence results may be deduced in a perturbative regime for the case of the sphere.
This is joint work with P. Esposito (Rome Tre University).

[a] Speaker

The singular Nirenberg problem

Francesca De Marchis [a] [1], Rafael Lopez Soriano [2]

[1] Università degli Studi di Roma Sapienza – Italy
[2] Universidad de Granada – Spain

I will consider the problem of prescribing the Gaussian curvature (under pointwise conformal change of the metric) on surfaces with conical singularities. This question has been first raised by Troyanov and it is a generalization of the Kazdan-Warner problem for regular surfaces, known as the Nirenberg problem on the sphere. Answer this question amounts to solve the following differential problem on a surface Σ

$$-\Delta_g u + 2K_g = 2Ke^u - 4\pi \sum_{j=1}^{m} \alpha_j \delta_{p_j}, \tag{1}$$

where K_g is the Gaussian curvature of the background metric g, K is the curvature we want to prescribe, p_j are the singular points and α_j the orders of the singularities of the new metric we are looking for: $\tilde{g} = e^u g$, verifying $K_{\tilde{g}} = K$. This equation has been studied first in the case $K > 0$. I will present some new results (obtained in collaboration with R. López-Soriano) in the case K sign-changing. When $\Sigma = S^2$, under some mild conditions on the nodal set of K we derived some sufficient conditions on K and on the singularities for the existence of solutions of (1). Even if we do not expect these conditions to be necessary, I will explain why they are somehow sharp.

[a]Speaker

Some results on entire solutions to a family of nonlinear elliptic systems

Alberto Farina [a] [1]

[1] Université de Picardie J. Verne (UPJV) – UPJV – France

We prove symmetry results and Liouville-type theorems for, possibly sign changing, entire distributional solutions to a family of nonlinear elliptic systems encompassing models arising in Bose-Einstein condensation and in nonlinear optics.

[a]Speaker

A bifurcation result for a general 2x2 Toda system

Francesca Gladiali [a] [1]

[1] University of Sassari – Italy

We consider a general 2×2 Toda system in $\mathbb{R}^N$ and we prove, using the bifurcation theory, that there exists a sequence of parameters at which a radial bifurcation occurs. The Legendre polynomials play a crucial role in the construction of the solutions.

[a]Speaker

A simplified approach to the regularising effect of nonlinear semigroups

Daniel Hauer [a] [1], Coulhon Thierry

[1] School of Mathematics and statistics - The University of Sydney – School of Mathematics and Statistics F07, University of Sydney, NSW 2006 Australia, Australia

Since the beginning of the 21st century, there appeared a huge flow of papers written on the regularising effect of nonlinear semigroups. Most authors of these papers follow the same approach: As a first step, a Log-Sobolev inequality is derived from a known Sobolev inequality. Then by using the Log-Sobolev inequality, one shows that the function $t \mapsto ||T_t||_{r(t)}$ satisfies a differential inequality which is strong enough to conclude an L^p-L^q-regularisation of the trajectories $t \mapsto T_t\varphi$ of the given semigroup $\{T_t\}$. In this talk, we present a simplified approach to this regularity effect and apply our method to varous examples.
The results presented here are obtained in joint work with Prof. Thierry Coulhon (Paris Sciences et Lettres, Département de Mathématiques et Applications, École Normale Supérieure, 62 bis rue Gay-Lussac, 75005 Paris, France)

[a] Speaker

Some Liouville type problems including a probability measure from 2D-turbulence

Tonia Ricciardi [a] [1]

[1] Dipartimento di Matematica e Applicazioni, Università di Napoli Federico II – Via Cintia, Monte S. Angelo I-80126 Napoli, Italy, Italy

We report some results concerning elliptic equations including an exponential type nonlinearity, defined on two-dimensional domains or manifolds. Such equations are motivated by the statistical mechanics description of 2D turbulence, as introduced by Onsager. We are particularly interested in the case where the variable vortex intensities and orientations are defined by a probability measure.
In particular, we study the blow-up behavior of solutions [1,4,5,6], the mass quantization and the existence of solutions by variational methods [4], the optimal Moser-Trudinger constants [3] and the existence of concentrating solutions by perturbation methods [2].

References: [1] H. Ohtsuka, T. Ricciardi, T. Suzuki, J. Differential Equations 249 n. 6 (2010), 1436-1465. [2] A. Pistoia, T. Ricciardi, arXiv: math.AP/1505.0530, submitted. [3] T. Ricciardi, T. Suzuki, J. of Eur. Math. Soc. (JEMS) 16 n.7 (2014), 1327-1348. [4] T. Ricciardi, G. Zecca, Differential and Integral Equations 25 n.3-4 (2012), 201-222. [5] T. Ricciardi, G. Zecca, arXiv:math.AP/1406-2925, submitted. [6] T. Ricciardi, G. Zecca, to appear on Advanced Nonlinear Studies.

[a] Speaker

Sharp asymptotic profiles for singular solutions to an elliptic equation with a sign-changing nonlinearity

Frédéric Robert [a] [1]

[1] Institut Élie Cartan de Lorraine (IECL) – Université de Lorraine, CNRS : UMR7502 – Boulevard des Aiguillettes BP 70239 54506 VANDOEUVRE LES NANCY CEDEX Ile du Saulcy - 57 045 METZ Cedex 01, France

We consider a positive solution to a nonlinear elliptic equation on a punctured ball. The linear part is the classical Laplacian. When the nonlinear part is positive and critical, this is similar to the classical problem studied by Caffarelli-Gidas-Spruck. When the nonlinear part is negative and a pure power, the problem is associated to a natural convex functional and the singularities are completely understood. In the present work, we mix the two nonlinearities. We show the existence of several potential behaviors. Two of them are natural extensions of the case of constant-sign nonlinearity. Two other behaviors are arising from the interaction of the two nonlinearity. In this talk, I will describe all the possible behaviors and I will show how the methods of apriori analysis in nonlinear elliptic problems are helping understanding this problem.

[a]Speaker

A rigidity result for overdetermined elliptic problems in the plane

David Ruiz [a] [1]

[1] University of Granada [Granada] – vda. del Hospicio, s/n C.P. 18071 Granada, Spain

A widely open problem is to classify the set of domains $\Omega \subset \mathbb{R}^n$ where there exists a bounded solution u to the overdetermined elliptic problem

$$\begin{cases} \Delta u + f(u) = 0 & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \\ \frac{\partial u}{\partial \vec{\nu}} = 1 & \text{on } \partial\Omega \end{cases} \tag{2}$$

for some Lipschitz function f. The case of a bounded domain was solved by J. Serrin in 1971: the ball is the unique such domain. Instead, the case of unbounded domains is not yet completely understood. In this talk we show that if $n = 2$ and $\partial\Omega$ is unbounded and connected, then Ω is a halfplane. This is joint work with Antonio Ros (U. Granada) and Pieralberto Sicbaldi (U. Aix Marseille).

[a]Speaker

Normalized solutions for coupled cubic Schrödinger equations

Nicola Soave [a] [1]

[1] Justus Liebig University of Giessen – Germany

We consider the system of coupled elliptic equations

$$\begin{cases} -\Delta u - \lambda_1 u = \mu_1 u^3 + \beta u v^2 \\ -\Delta v - \lambda_2 v = \mu_2 v^3 + \beta u^2 v \qquad \text{in } \mathbb{R}^3, \\ u, v > 0 \end{cases}$$

searching for solutions satisfying the additional condition

$$\int_{\mathbb{R}^3} u^2 = a_1^2 \quad \text{and} \quad \int_{\mathbb{R}^3} v^2 = a_2^2.$$

Assuming that a_1, a_2, μ_1, μ_2 are positive fixed quantities, we prove existence results for different ranges of the coupling parameter $\beta > 0$. The extension to systems with an arbitrary number of components is discussed, as well as the orbital stability of the solutions for the corresponding Schrödinger equations. This is a joint work with Thomas Bartsch and Louis Jeanjean.

[a] Speaker

Existence and symmetry of least energy nodal solutions for Hamiltonian elliptic systems

Hugo Tavares [a] [1], Miguel Ramos [2], Ederson Dos Santos [3], Denis Bonheure [4]

[1] Instituto Superior Tecnico [Lisboa] (IST) – Av. Rovisco Pais, 1 1049-001 Lisboa, Portugal
[2] Faculdade de Ciências [Lisboa] – Universidade de Lisboa, Campo Grande, Edificio CS, 7749-076 Lisboa, Portugal
[3] Universidade de São Paulo - São Carlos (USP) – Universidade de São Paulo - Campus de São Carlos Av. Trabalhador São-carlense, 400, Arnold Schimidt São Carlos - São Paulo CEP 13566-590, Brazil
[4] Département de Mathématique [Bruxelles] (ULB) – Avenue Franklin Roosevelt 50 - 1050 Bruxelles, Belgium

In this talk we will discuss recent existence results of least energy nodal solutions for the Hamiltonian elliptic system with Hénon–type weights

$$-\Delta u = |x|^{\beta}|v|^{q-1}v, \quad -\Delta v = |x|^{\alpha}|u|^{p-1}u \quad \text{in } \Omega, \qquad u = v = 0 \text{ on } \partial\Omega,$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$, $N \geq 1$, $\alpha, \beta \geq 0$ and the nonlinearities are superlinear and subcritical, namely

$$1 > \frac{1}{p+1} + \frac{1}{q+1} > \frac{N-2}{N}.$$

For the proof, we use a dual method applied to a perturbed system. Moreover, when Ω is either a ball or an annulus centred at the origin and $N \geq 2$, we show that these solutions display foliated Schwarz symmetry. It is natural to conjecture that these solutions are not radially symmetric. We provide such a symmetry breaking in a range of parameters where the solutions of the system behave like the solutions of a single equation.

[a] Speaker

Sign-changing solutions for the Brezis-Nirenberg problem

Giusi Vaira [a] [1]

[1] University of Roma, Sapienza (Dipartimento di Scienze di Base e Applicate per l'Ingegneria) – Via Antonio Scarpa, 14, Italy

We deal with the Brezis-Nirenberg problem in a bounded domain of R^N in a presence of a parameter λ which is positive and real, in the critical case, namely critical with respect to the Sobolev embedding. We show some recent existence result on sign-changing solutions for this problem in low dimensions and in higher dimensions.

[a] Speaker

MS13 Nonlinear Waves in Dispersive Equations: D. Pelinovsky, N. Visciglia

Cauchy problems for magnetic nonlinear Schrödinger equations

Nabile Boussaid [a] [1], Hichem Hajaiej, Slim Ibrahim, Laurent Michel

[1] LMB (LMB) – Université de Franche-Comté – France

This is a joint work with Hichem Hajaiej (NYU Shanghai), Slim Ibrahim (University of Victoria (BC)), and Laurent Michel (Université de Nice-Sophia Antipolis). We consider the well-posedness issue for nonlinear Schrödinger equations with non autonomous magnetic potentials. We show, without any gauge assumption, the well-posedness in the domain of the operator for L^2-subcritical nonlinearities. In the critical case, the blow-up threshold is the non-magnetic one. In the supercritical cases, for homogeneous fields, we also obtain blow-up and global well-posedness criterion, which relies both on the energy and angular momentum. We will show some results for nonlinear Schrödinger with non-autonomous external (electric) potentials.

[a]Speaker

Damped nonlinear Schrodinger equations

Remi Carles [a] [1]

[1] Institut Montpellierain Alexander Grothendieck – CNRS : UMR5149 – France

We describe the effect of the introduction of a nonlinear damping in the dynamics of the Schrödinger equation. A superlinear damping prevents finite time blow-up if the damping term grows faster than the focusing nonlinearity at infinity. Moreover, in the presence of confinement (compact manifold or harominc potential), the global solution converges to zero for large time. On the other hand, introducing a sublinear damping in the linear Schrödinger equation leads to finite time extinction, in the presence of confinement. This talk is based upon joint works with Paolo Antonelli and Christof Sparber, Clément Gallo, Tohru Ozawa.

[a]Speaker

Existence and multiplicity of solutions for a system of coupled NLS-KdV equations

Eduardo Colorado [a] [1,2]

[1] Departamento de Matemáticas, Universidad Carlos III de Madrid [Madrid] (UC3M) – Avda. Universidad 30, 28911-Leganés, Madrid, Spain
[2] Instituto de Ciencias Matemáticas (ICMAT) – C/Nicolás Cabrera 15, 28049 Madrid, Spain

Along this talk, will be focused the attention on the system of coupled NLS-KdV equations,

$$(S)\begin{cases} \mathrm{i}f_t + f_{xx} - \beta f g + |f|^2 f &= 0, \quad x\in\mathbb{R}, \quad t>0 \\ g_t + g_{xxx} + g g_x - \frac{1}{2}\beta(|f|^2)_x &= 0, \quad x\in\mathbb{R}, \quad t>0, \end{cases}$$

where i denotes the complex unit, $f(x,t)\in\mathbb{C}$, $g(x,t)\in R$, $\beta\in\mathbb{R}$ and $|f|$, $|g|\to 0$ as $|x|\to\infty$. This system appears in phenomena of interactions between short and long dispersive waves, arising in fluid mechanics, such as the interactions of capillary - gravity water waves. Indeed, f represents the short-wave, while g stands for the long-wave The main results that we will be shown deals with existence of positive bound and ground states for the corresponding stationary system when one looks for solitary-traveling wave solutions of (S). The results presented in this talk are contained in the following papers. **[1]** E. Colorado, *Existence of bound and ground states for a system of coupled nonlinear Schrödinger-KdV equations.* To appear in Comptes rendus - Mathématique, DOI:10.1016/j.crma.2015.03.011. **[2]** E. Colorado, *On the existence of bound and ground states for a system of coupled nonlinear Schrödinger-KdV equations.* Preprint ArXiv:1411.7283v3.

[a]Speaker

Improved time decay for magnetic Schrodinger evolutions

Luca Fanelli [a] [1]

[1] SAPIENZA Università di Roma (Roma 1) – P.le Aldo Moro 5, 00185 Roma, Italy

We will present some recent results concerning with electromagnetic Schrodinger equations, with scaling critical fields. While proving dispersive estimates, a quite explicit solution formula permit to reduce matters to the study of the boundedness of a suitable kernel. Moreover, we prove that, if the first eigenvalue of the angular hamiltonian is strictly positive, then the time decay rate is polynomially improved, by suitably weighting the topology.
The results are obtained in collaboration with V. Felli, M. Fontelos, G. Grillo, H. Kovarik, A. Primo.

[a]Speaker

Global existence and convergence of smooth solutions to Yang-Mills gradient flow over compact four-manifolds

Paul Feehan [a] [1]

[1] Department of Mathematics - Rutgers School of Arts and Sciences – Hill Center - Busch Campus 110 Frelinghuysen Road Piscataway, NJ 08854-8019, United States

We develop new results on global existence and convergence of solutions to the gradient flow equation for the Yang-Mills energy functional on a principal bundle, with compact Lie structure group, over a closed, four-dimensional, Riemannian, smooth manifold, including the following. If the initial connection is close enough to a local minimum of the Yang-Mills energy functional, in a norm or energy sense, then the Yang-Mills gradient flow exists for all time and converges to a Yang-Mills connection. If the initial connection is allowed to have arbitrary energy but we restrict to the setting of a Hermitian vector bundle over a compact, complex, Hermitian (but not necessarily Kaehler) surface and the initial connection has curvature of type (1,1), then the Yang-Mills gradient flow exists for all time, though bubble singularities may (and in certain cases must) occur in the limit as time tends to infinity. The Lojasiewicz-Simon gradient inequality plays a crucial role in our approach and we develop two versions of that inequality for the Yang-Mills energy functional.

[a]Speaker

Stability for solitons of the Landau-Lifshitz equation with an easy-plane anisotropy

Philippe Gravejat [a] [1]

[1] Centre de Mathématiques Laurent Schwartz – Ecole Polytechnique – France

We present two results in collaboration with A. de Laire (Université de Lille I), and by Y. Bahri (École polytechnique) concerning the orbital stability of sums of solitons, and asymptotic stability of a soliton of the Landau-Lifshitz equation with an easy-plane anisotropy.

[a]Speaker

The energy critical limit of ground state solitons

Stephen Gustafson [a] [1]

[1] The University of British Columbia [Vancouver] – 2329 West Mall Vancouver, B.C. V6T 1Z4, Canada

We consider the energy-critical limit of the ground state solitary waves of the pure power, focusing, energy sub-critical nonlinear Schroedinger equation in 3D. We show that after suitable (and precisely determined) rescaling, driven by the threshold resonance present in the energy critical linearized problem, the ground states converge to the the Aubin-Talenti function (with precise error estimate). We discuss implications for the sub-critical linearized NLS, and for the best constant in the Gagliardo-Nirenberg inequality. This is joint work with Tai-Peng Tsai and Ian Zwiers.

[a] Speaker

Transverse instability of periodic and generalized solitary waves for 5th order KP model

Mariana Haragus [a] [1], Erik Wahlén [2]

[1] Laboratoire de Mathématiques de Besançon (LMB) – CNRS : UMR6623, Université de Franche-Comté – Laboratoire de Mathématiques de Besançon, 16 route de Gray, 25 030 Besançon cedex, France
[2] Center for Mathematical Sciences (CMS) – Centre for Mathematical Sciences, Box 118, SE-22100, Lund, Sweden, Sweden

We consider a 5th order Kadomtsev-Petviashivili (KP) equation arising in the study of capillary-gravity water waves. This equation possesses a family of generalized solitary waves which are traveling solitary waves with periodic tails at infinity. We show that the periodic tails are transversely unstable and that this instability induces an essential transverse instability of the generalized solitary waves. We also discuss the question of extending these results to the Euler equations governing the water-wave problem.

[a] Speaker

Stability of Wave Trains in Whitham Equations

Mathew Johnson [a] [1], Vera Mikyoung Hur [2]

[1] University of Kansas, Department of Mathematics (KU) – Lawrence, Kansas, 66045, USA, United States
[2] University of Illinois at Urbana-Champaign, Department of Mathematics (UIUC) – Department of Mathematics 1409 W. Green Street Urbana, IL 61801, United States

In this talk, we will discuss recent advances in the existence and dynamics of periodic traveling waves in Whitham type equations for water waves. These nonlocal dispersive equations incorporate a canonical shallow water nonlinearity and the full dispersion relation for unidirectional surface water waves. Specifically, we will be interested in the impact of various physical effects (surface tension, constant vorticity, etc.) on the modulational instability of wave trains with sufficiently small amplitudes. Time permitting, we may also discuss results incorporating bidirectionality in the model.

[a] Speaker

The Nonlinear Schrödinger equation on a tadpole graph

Diego Noja [a] [1]

[1] Dipartimento di Matematica e Applicazioni Università di Milano Bicocca – Via Cozzi 55 20125 Milano, Italy

A tadpole graph is the structure given by a ring with a halfline attached at a point (vertex). A Nonlinear Schrödinger dynamics (NLS) is given on this carrier structure by imposing a boundary condition at the vertex which makes the dynamics hamiltonian. Here it is considered the so called Kirchhoff boundary condition, the most common in applications. The linear analogue is the simplest topologically nontrivial quantum graph and the nonlinear model is an example of interaction between a confined and an unbounded NLS dynamics.
The NLS on a tadpole graph has a surprisingly rich family of standing waves, and comprises a) branches of standing waves bifurcating from embedded linear eigenvalues; b) edge solitons, i.e. branches of standing waves bifurcating from the threshold of the essential spectrum c) family of standing waves without linear analogue undergoing saddle-node bifurcation. In this talk the above classification is described and the linearized and orbital stability of standing waves bifurcating from the essential spectrum is studied.

[a]Speaker

Blow up dynamics near the ground state for the focusing energy critical NLS

Galina Perelman [a] [1]

[1] Laboratoire d'Analyse et de Mathématiques Appliquées (LAMA) – Fédération de Recherche Bézout, CNRS : UMR8050, Université Paris-Est Créteil Val-de-Marne (UPEC) – Université de Paris-Est Créteil, Faculté des Sciences et Technologie, 61 av. du Général de Gaulle 94010 Créteil Cedex, France

In this talk I will review some recent results related to the construction of type II blow up solutions for the energy critical NLS arising from arbitrary small $\dot{H}^1$ perturbations of the ground state.

[a]Speaker

Quasineutral limit for the Vlasov-Poisson system

Frédéric Rousset [a] [1]

[1] Université Paris-Sud Orsay – université paris-sud Orsay – France

We shall study the quasineutral limit in Sobolev spaces for the Vlasov-Poisson system. As we shall see the validity of this limit is linked to linear stability properties of the profile in velocity of the initial data well-known as the Penrose stability criterion.(joint work with Daniel Han-Kwan)

[a]Speaker

Long time existence issues for some water waves systems

Jean-Claude Saut [a] [1]

[1] Université Paris-Sud – Université Paris-Sud – Laboratoire de Mathématiques 91405 Orsay, France, France

In order to fully justify asymptotic models for surface or internal waves one needs to provide long time (with respect to the inverse of a relevant small parameter) existence results for the solutions of the Cauchy problem, together with appropriate uniform bounds.
In this talk we will present new results and some open questions pertaining to this problematic.

[a]Speaker

Asymptotic Stability of the Toda m-soliton

C. Eugene Wayne [a] [1], Aaron Hoffman [2], G. Nicholas Benes [3]

[1] Boston University – United States
[2] Olin College of Engineering – United States
[3] Image Insight Incorporated – United States

We prove that multi-soliton solutions of the Toda lattice are both linearly and nonlinearly stable. Our proof uses neither the inverse spectral method nor the Lax pair of the model but instead studies the linearization of the Bäcklund transformation which links the $(m-1)$-soliton solution to the m-soliton solution. We use this to construct a conjugation between the Toda flow linearized about an m-solition solution and the Toda flow linearized about the zero solution, whose stability properties can be determined by explicit calculation. This is joint work with Nick Benes and Aaron Hoffman.

[a]Speaker

Collisions of vortex filaments

Evelyne Miot [a] [1]

[1] CNRS - Ecole Polytechnique – CNRS : UMR7640 – France

We study the issue of collisions for a model introduced by Klein, Majda and Damodaran for the evolution of vortex filaments. We focus on the cases of counter-rotating pairs of filaments on the one hand, and of an arbitray number of vortex filaments with polygonial symmetry on the other hand. We prove the existence of a solution such that the filaments collide at some point in finite time in a self-similar fashion. This is joint work with Valeria Banica and Erwan Faou.

[a]Speaker

Stable melting for the radial Stefan problem

Pierre Raphael [a] [1]

[1] Université de Nice-Sophia Antipolis and Institut Universitaire de France (UNS) – Université de Nice Sophia-Antipolis – France

I will consider the classical free boundary problem of melting of an ice ball and address the question of existence and stability of melting regimes. This is joint work with Mahir Hadzic (Kings college London).

[a]Speaker

MS14 Parabolic Equations in Life Sciences: Th. Lepoutre, A. Moussa

On Volume-Surface Reaction-Diffusion Systems and Applications in Cell-Biology

Klemens Fellner [a] [1]

[1] University of Graz (KFU) – Institute for Mathematics and Scientific Computing, Heinrichstr. 36, 8010 Graz, Austria

Volume-Surface Reaction-Diffusion (VSRD) systems appear naturally (besides many applications) in cell-biology when describing the dynamics of proteins exchanged between cytoplasm and cortex. In Drosophila SOP precursor stem-cells, the protein Lgl has been identified to play a key role in the asymmetric localisation of so-called cell-fate determinates during mitosis, i.e. during asymmetric stem-cell division. A mathematical VSRD models describing the kinetics between four conformations of Lgl is presented and mathematically analysed in terms of global existence, large-time behaviour and quasi-steady-state approximation. The rich behaviour of the model is illustrated via numerical simulations based on a suitable finite-element discretisation for such VSRD systems. A class of reduced, yet nonlinear models is likewise analysied and moreover shown to satisfy exponential convergence towards equilibrium via the entropy method. Finally, an entropy method without detailed balance condition is presented, which applies to complex balance and even more general first order reaction networks.

[a]Speaker

Asymptotic behavior of an inhomogeneous nonlocal equation

Salome Martinez [a] [1]

[1] Centro de Modelamiento Matemático (CMM) – Centro de Modelamiento Matemático Av. Blanco Encalada 2120 Piso 7 Santiago de Chile, Chile

In this talk we will study the asymptotic behavior of the solutions of

$$u_t(x,t) = \int_{\mathbb{R}} J\left(\frac{x-y}{g(y)}\right)\frac{u(y,t)}{g(y)}dy - u(x,t),$$

with $J : \mathbb{R} \to \mathbb{R}$ is a nonnegative even function with compact support such that $\int_{\mathbb{R}} J(y)dy = 1$. In this equation the dispersal is inhomogeneous in space since the step size $g(y)$ depends on the position y. In this talk we will study the asymptotic behavior of the initial value problem depending on the various behaviors of the step size function. Of special interest will be to prove the existence of L^1 steady state solutions, which characterize the asymptotic behavior of finite mass solutions.

[a]Speaker

Asymptotic spreading for general heterogeneous Fisher-KPP type equations

Grégoire Nadin [a] [1, 2], Henri Berestycki

[1] CNRS - UPMC – CNRS - UPMC – France
[2] EHESS – ehess – France

We will present some recent propagation results obtained with H. Berestycki for the solutions of multidimensional Fisher-KPP reaction-diffusion equations with general space-time heterogeneous coefficients. Namely, we will construct some spreading speeds through new notions of generalized principal eigenvalues and a game theory variational characterization.These estimates turn out to be optimal for almost periodic, asymptotically almost periodic and radially periodic equations or when the coefficients converge in radial segments. This last example may give rise to non-convex expansion sets.

[a]Speaker

An amazingly efficient L^2-estimate in reaction-diffusion systems

Michel Pierre [a] [1]

[1] ENS Rennes and IRMAR – Ecole normale supérieure de Rennes – France

We will recall how a simple L^2-estimate for a linear parabolic operator with discontinuous coefficients turns out to be a very powerful tool to solve several questions arising in reaction-diffusion systems. We will list relevant applications and discuss several extensions.

[a]Speaker

Extensions of Freidlin-Gartner's formula to general reaction terms

Luca Rossi [a] [1]

[1] Dipartimento di Matematica Pura e Applicata [Padova] – Via Trieste, 63 35121 Padova, Italy

The Freidlin-Gartner formula expresses the asymptotic speed of spreading for spatial-periodic Fisher-KPP equations in terms of the principal eigenvalues of a family of linear operators. One cannot expect the same formula to hold true for the other classes of reaction terms (monostable, combustion, bistable). However, these eigenvalues have been later related with the minimal speeds of pulsating travelling fronts, yielding a formula for the spreading speed which is not unreasonable to expect to hold for any reaction term. We will see that it is indeed the case. The method presented provides some partial results for equations whose terms depend arbitrarily on time and space, highlighting a general connection between the asymptotic speed of spreading and almost planar transition fronts.

[a]Speaker

From a microscopic model to a macroscopic model with cross-diffusion in Population Dynamics

Ariane Trescases [a] [1], Laurent Desvillettes [1], Thomas Lepoutre [2], Ayman Moussa [3]

[1] Centre de Mathématiques et de Leurs Applications - ENS Cachan (CMLA) – École normale supérieure [ENS] - Cachan, École normale supérieure (ENS) - Cachan – ENS Cachan CNRS, PRES UniverSud, 61 Avenue du Président Wilson 94230 Cachan, France

[2] Inria Rhone Alpes – INRIA – France

[3] Laboratoire Jacques-Louis Lions (LJLL) – INRIA, Université Paris VII - Paris Diderot, CNRS : UMR7598, Université Pierre et Marie Curie (UPMC) - Paris VI – B.C. 187 75252 Paris Cedex 05, France

We present new results of existence for a class of reaction-cross diffusion systems naturally arising in Population Dynamics. In the case where cross diffusion terms appear only in one of the two equations (triangular case), the solutions are obtained as the limit of the solutions of a microscopic model where only standard diffusions appear. The results use entropy and duality methods.

[a]Speaker

Analysis of degenerate cross-diffusion population models with volume filling

Nicola Zamponi [a] [1], Ansgar Jüngel[1]

[1] Technical University of Vienna (TU WIEN) – Austria

A class of parabolic cross-diffusion systems modeling the interaction of an arbitrary number of population species is analyzed in a bounded domain with no-flux boundary conditions. The equations are formally derived from a random-walk lattice model in the diffusion limit. Compared to previous results in the literature, the novelty is the combination of general degenerate diffusion and volume-filling effects. Conditions on the nonlinear diffusion coefficients are identified, which yield a formal gradient-flow or entropy structure. This structure allows for the proof of global-in-time existence of bounded weak solutions and the exponential convergence of the solutions to the constant steady state. The existence proof is based on an approximation argument, the entropy inequality, and new nonlinear Aubin-Lions compactness lemmas. The proof of the large-time behavior employs the entropy estimate and convex Sobolev inequalities. Moreover, under simplifiying assumptions on the nonlinearities, the uniqueness of weak solutions is shown by using the H^{-1} method, the E-monotonicity technique of Gajewski, and the subadditivity of the Fisher information.

[a]Speaker

Acceleration or not in some reaction diffusion equations

Matthieu Alfaro [a] [1]

[1] Institut de Mathématiques et de Modélisation de Montpellier (I3M) – CNRS : UMR5149, Université Montpellier II - Sciences et techniques – Case Courrier 051 Place Eugène Bataillon 34095 MONTPELLIER CEDEX 5, France

We consider reaction diffusion equations as population dynamics models. We aim at understanding if invasion occurs at a constant speed (via traveling waves) or by accelerating. We discuss recent developments on such an issue.

[a]Speaker

MS15 PDE Analysis of Three-dimensional Fluid Flows: J. Malek, E. Süli

On a class of dynamic outflow boundary conditions for incompressible Newtonian flows

Dieter Bothe [a] [1], Takahito Kashiwabara [2], Matthias Köhne [3]

[1] Technische Universität Darmstadt – Germany
[2] University of Tokyo – Japan
[3] Universität Düsseldorf – Germany

We derive physically meaningful boundary conditions at outflow boundaries which render the artificial boundary transparent in the sense that no unphysical dissipation is introduced into the system. The resulting class of dynamic ABCs is studied in the halfspace prototype case concerning local-in-time strong wellposedness.

[a]Speaker

Global existence of weak solutions for Navier-Stokes Equations with thermodynamically unstable pressure and anisotropic viscous stress

Didier Bresch [a] [1], Pierre-Emmanuel Jabin

[1] Laboratoire de Mathématiques de l'Université Savoie Mont-Blanc (LAMA UMR 5127 CNRS) – Université de Savoie, CNRS : UMR5127 – Université de Savoie, UFR SFA Domaine Universitaire, Bâtiment Le Chablais 73376 LE BOURGET DU LAC, France

We provide global existence of weak solutions for the compressible Navier-Stokes equations for more general stress tensor than those covered by P.-L. Lions and E. Feireisl's theory. More precisely we focus on more general pressure laws which are not thermodynamically stable and on anisotropy in the viscous stress tensor. Considering these two cases is important in practical situations for instance to describe solar events (virial pressure law), geophysical flows (eddy viscosity) or biological situations (anisotropy).

[a]Speaker

Large data analysis for the Kolmogorov two-equation model of turbulence

Miroslav Bulicek [a] [1]

[1] Mathematical Institute, Faculty of Mathematics and Physics, Charles University in Prague (MICUNI) – Sokolovska 83, 186 75 Prague, Czech Republic

A.N. Kolmogorov seems to have been the first to recognize that a two-equation model of tubulence might be used as the basis of turbulent flow prediction. Although his model has so far been almost unnoticed, it exhibits interesting features. First of all, its structure is similar to the Navier–Stokes(–Fourier) equations for incompressible fluid flow; the only difference is that the viscosity is not constant but depends on the fraction of two scalar quantities that measure the effect of turbulence: the average of the kinetic energy of velocity fluctuations and the measure related to the length scales of turbulence. The dependence is such that the material coefficients such as viscosity and turbulent diffusivities may degenerate, and thus the a priori control of the derivatives of the quantities involved is unclear. Furthermore, the system includes the dissipation of the energy, which is merely an L^1 quantity, appearing on the right-hand side of the equation for turbulent kinetic energy. We establish large data existence of a suitable weak solution to such a system completed by the initial and generalized Navier's slip and stick-slip boundary conditions.

[a] Speaker

Separation for the stationary Prandtl equation

Anne-Laure Dalibard [a] [1]

[1] Université Pierre et Marie Curie - Paris 6 (UPMC) – Université Pierre et Marie Curie [UPMC] - Paris VI, Université Pierre et Marie Curie (UPMC) - Paris VI – 4 place Jussieu - 75005 Paris, France

The Prandtl equation describes the motion of an incompressible flow with small viscosity near an obstacle. When the flow is stationary, and in the presence of an adverse pressure gradient, some separation phenomena have been observed in experiments: there exists a point on the boundary of the obstacle beyond which a back flow sets in along the wall, and the boundary layer separates from the surface. This behaviour has been explained by physicists thanks to auto-similar multiscale formal expansions near the point of separation.
The goal of this talk is to present a recent work with Nader Masmoudi in which we justify the separation and exhibit a separation profile and rate. Our proof relies on ideas developed by Merle and Raphaël in the context of singularity formation for the nonlinear Schrödinger equation.

[a] Speaker

Mathematical models of viscous, heat conducting fluids

Eduard Feireisl [a] [1]

[1] Institute of Mathematics of the Academy of Sciences of the Czech Republic (IM ASCR) – Zitna 25, 116 67 Praha 1, Czech Republic

We discuss several results concerning well-posedness as well as the possibility of numerical approximation of the complete Navier-Stokes-Fourier system describing the time evolution of a compressible viscous and/or heat conducting fluid. Several concepts of weak solutions are introduced along with the relevant existence theory based on proving convergence of an appropriate numerical scheme. Some counter examples to well-posedness, mostly in the context of invisicd fluids, are also discussed.

[a]Speaker

Effect of an irregular boundary on a rotating flow

David Gerard-Varet [a] [1]

[1] Institut de Mathématiques de Jussieu (IMJ) – CNRS : UMR7586, Université Paris VII - Paris Diderot – 2, place Jussieu 75251 Paris Cedex 05, France

We shall describe the effect of a rough boundary on a rotating vicous flow. When there is no roughness, a boundary layer called Ekman layer develops at the wall. When roughness is taken into acount (through a small amplitude/small wavelength oscillation), the dynamics of the boundary layer becomes nonlinear. We shall discuss the associated mathematical difficulties and qualitative changes of the flow (based on joint works with E. Dormy and AL Dalibard)

[a]Speaker

Recent advances concerning the three-dimensional primitive equations of atmospheric and oceanic dynamics

Edriss Titi [a] [1]

[1] Texas A & M University and Weizmann Institute of Science – Department of Mathematics, Texas A & M University, 4468 - TAMU, College Station, TX 77843, United States

In this talk I will survey some of the recent advances concerning the global regularity of the three-dimensional primitive equations of atmospheric and oceanic dynamics with various kinds of anisotropic viscosity and diffusion. Moreover, I will also show the finite-time blowup for certain class of solutions of the non-viscous primitive equations.

[a]Speaker

Error estimates for the compressible Navier-Stokes equations

Antonin Novotny [a] [1]

[1] Institut Mathématiques de Toulon, University of Toulon – Insttut Mathématiques de Toulon, University of Toulon – France

We present a general method based on the investigation of the relative energy of the system, that provides an unconditional error estimate for the approximate solution of the barotropic Navier–Stokes equations obtained by time and space discretization. We use this methodology to derive an error estimate for a specific DG/finite element scheme for which the convergence was proved in [1]. The talk contains material from two recent papers [2] and [3]. [1] T.K. Karper. A convergent FEM-DG method for the compressible Navier-Stokes equations. Numer. Math. 125(3) :441–510, 2013. [2] T. Gallouet, R.Herbin, D.Maltese and A. Novotny. Error estimate for a numerical approximation to the compressible barotropic Navier-Stokes equations. IMA Journal of Numerical Analysis, to appear [3] E. Feireisl, R. Hosek, D. Maltese and A. Novotny. Error estimates for a numerical method for the compressible Navier-Stokes system on suficiently smooth domains, Preprint

[a] Speaker

MS16 PDE-constrained Bayesian Inverse Problems and Data Assimilation: D. Crisan, M. A. Iglesias, K. Law

On the probabilistic interpretation of variational data assimilation

Jochen Broecker [a] [1]

[1] School of Mathematical and Physical Sciences, University of Reading (UOR) – Whiteknights, PO Box 217, READING, Berkshire, RG6 6AX, United Kingdom

Data assimilation is a term mainly used in the geosciences; it refers to matching trajectories of a dynamic model with observations. In variational data assimilation, this is usually accomplished by minimising an error functional which penalises both deviations of the solution from the observations as well as deviations from being a solution of the dynamical model. Motivated by heuristic considerations and the case of discrete time with gaussian perturbations, a quadratic error functional is often used which (in discrete time) can be interpreted as the likelihood of a trajectory. In continuous time, this interpretation breaks down though. Possible alternatives for the error functional will be discussed along with their statistical interpretation. Finally, the problem of existence of solutions for the emerging variational problems will be touched upon.

[a] Speaker

Probability Measures on Numerical Solutions of ODEs and PDEs for Uncertainty Quantification and Inference

Patrick Conrad [a] [1], Mark Girolami [1], Simo Sarkka [2], Andrew Stuart [1], Konstantinos Zygalakis [3]

[1] University of Warwick – United Kingdom
[2] Aalto University – Finland
[3] University of Southampton – United Kingdom

Deterministic ODE and PDE solvers are widely used, but characterizing the error in numerical solutions within a coherent statistical framework is challenging. We successfully address this problem by constructing a probability measure over functions consistent with the solution that provably contracts to a Dirac measure on the unique solution at rates determined by an underlying deterministic solver. The measure straightforwardly derives from important classes of numerical solvers and is illustrated on uncertainty quantification and inverse problems.

[a] Speaker

Inverse problems for stochastic transport equations

Dan Crisan [a] [1], Yoshiki Otobe [2], Szymon Peszat [3]

[1] Department of Mathematics - Imperial College London – Department of Mathematics South Kensington Campus Imperial College London LONDON SW7 2AZ United Kingdom, United Kingdom
[2] Department of Mathematical Sciences, Shinshu University – 3-1-1 Asahi, Matsumoto 390-8621, Japan
[3] Institut of Mathematics - Polish Academy of Sciences – Sw. Tomasza 30/7, 31 027 Krakow, Poland, Poland

I will discuss inverse problems for a a class of stochastic linear transport equations driven by a temporal or spatial white noise. The equations depend on an unknown potential and have either additive noise or multiplicative noise. The main result of the paper is that one can approximate the potential with arbitrary small error when the solution of the stochastic linear transport equation is observed over time at some fixed point in the state space.

[a]Speaker

Multilevel ensemble Kalman filtering

Hoel Hakon [a] [1], Kody Law [2], Raul Tempone [2]

[1] Department of Mathematics, University of Oslo (UiO) – University of Oslo, P.O. Box 1053, Blindern, Norway, Norway
[2] King Abdullah University of Science and Technology (KAUST) – King Abdullah University of Science and Technology P.O.Box 55455 Jeddah 21534, Saudi Arabia, Saudi Arabia

The ensemble Kalman filter (EnKF) is a sequential filtering method that uses an ensemble of particle paths to estimate the means and covariances of the Kalman filter by the use of sample moments, i.e., the Monte Carlo method. EnKF is often both robust and efficient, but its performance may suffer in settings where the computational cost of accurate simulations of particles is high. The multilevel Monte Carlo method (MLMC) is an extension of classical Monte Carlo methods which by sampling stochastic realizations on a hierarchy of resolutions may reduce the computational cost of moment approximations by orders of magnitude. In this work we have combined the ideas of MLMC and EnKF to construct the multilevel ensemble Kalman filter (MLEnKF). The main ideas of this method is to compute particle paths on a hierarchy of resolutions and to apply multilevel estimators on the ensemble hierarchy of particles to compute Kalman filter means and covariances. Theoretical results and a numerical study of the performance gains of MLEnKF over EnKF will be presented.

[a]Speaker

Diffusion Forecast: A nonparametric modeling

John Harlim [a] [1], Tyrus Berry [1], Dimitris Giannakis [2]

[1] The Pennsylvania State University – United States
[2] Courant Institute of Mathematical Science (CIMS) – United States

I will discuss a nonparametric modeling approach for forecasting stochastic dynamical systems on low-dimensional manifolds. The key idea is to represent the discrete shift maps on a smooth basis which can be obtained by the diffusion maps algorithm. In the limit of large data, this approach converges to a Galerkin projection of the semigroup solution of the backward Kolmogorov equation of the underlying dynamics on a basis adapted to the invariant measure. This approach allows one to quantify evolve the probability distribution of non-trivial dynamical systems with equation-free modeling.

[a] Speaker

A Bayesian level-set approach for geometric inverse problems

Matthew Dunlop [1], Marco Iglesias [a] [2], Yulong Lu [1], Andrew Stuart [1]

[1] University of Warwick – United Kingdom
[2] University of Nottingham – United Kingdom

We discuss a computational Bayesian framework for the solution of PDE-constrained inverse problems where the unknown is a geometric feature of the underlying PDE forward model. In this talk we introduce a level-set approach to infer and quantify the uncertainty of unknown geometric features within an infinite dimensional Bayesian framework. Numerical experiments are displayed to show the capabilities of the level-set approach for (i) identifying geologic structures in groundwater flow and (ii) estimating electric conductivity in the complete electrode model for EIT. In these applications, the unknown parameters have sharp interfaces characterised by the level-set function which is inferred, within the Bayesian framework, from observational data that arises from the PDE (forward) model under consideration. By means of state-of-the-art MCMC methods we describe numerical experiments which explore the posterior distribution that arises from the Bayesian level-set formulation.

[a] Speaker

Sequential Monte Carlo Methods for High-Dimensional Inverse Problems

Nikolas Kantas [a] [1]

[1] Department of Mathematics - Imperial College London – Department of Mathematics South Kensington Campus Imperial College London LONDON SW7 2AZ United Kingdom, United Kingdom

We consider the inverse problem of estimating the initial condition of a partial differential equation, which is only observed through noisy measurements at discrete time intervals. In particular, we focus on the case where Eulerian measurements are obtained from the time and space evolving vector field, whose evolution obeys the two-dimensional Navier-Stokes equations defined on a torus. We will adopt a Bayesian formulation resulting from a particular regularisation that ensures the problem is well posed. In the context of Monte Carlo based inference, it is a challenging task to obtain samples from the resulting high dimensional posterior on the initial condition. In this work we will propose a generic adaptive Sequential Monte Carlo (SMC) sampling approach for high dimensional inverse problems that overcomes some of these difficulties. The method builds upon appropriate Markov chain Monte Carlo (MCMC) techniques, which are currently considered as benchmarks for evaluating data assimilation algorithms used in practice. In our numerical examples, the proposed SMC approach achieves the same accuracy as MCMC but in a much more efficient manner. If time permits we will discuss some extensions of these ideas for high dimensional non-linear filtering problems. The talk is based on joint work with Alexandros Beskos (UCL), Ajay Jasra (NUS), Alexandre Thiery (NUS) and Dan Crisan (Imperial)

[a] Speaker

Stability of the Ensemble Kalman Filter

David Kelly [a] [1]

[1] Courant Institute of Mathematical Science (CIMS) – United States

The Ensemble Kalman Filter (EnKF) owes its great succcess in geophysical data assimilation to its ability to provide filtering skill for high dimensional turbulent forecast models at a relatively low computational cost. Despite its practical importance and intuitive derivation, very little is known theoretically about EnKF, particularly in the regime of finite ensemble size. In this talk we will present results concerning the stability properties of EnKF in this regime, namely well-posedness (uniform boundedness in time) and robustness of the filter to initialization errors (geometric ergodicity). The results are obtained under a very natural assumption on the forecast-observation model, which we call the 'observable energy criterion'. We also show that, when this criterion is violated, innocuous looking forecast models can lead to drastic malfunction and vast instability of EnKF. This is joint work with Andy Majda and Xin Tong (NYU).

[a] Speaker

Multilevel Sequential Monte Carlo Samplers

Kody Law [a] [1,2], Ajay Jasra [3], Yan Zhou [3], Alex Beskos [4], Raul Tempone [5]

[1] King Abdullah University of Science and Technology (KAUST) – Saudi Arabia
[2] Oak Ridge National Laboratory (ORNL) – United States
[3] National University of Singapore (NUS) – Singapore
[4] University College London (UCL) – United Kingdom
[5] King Abdullah University of Science and Technology (KAUST) – Saudi Arabia

Consider the approximation of expectations w.r.t. probability distributions associated to the solution of partial differential equations (PDEs); this scenario appears routinely in Bayesian inverse problems. In practice, one often has to solve the associated PDE numerically, using, for instance finite element methods and leading to a discretisation bias, with the step-size level h_L. In addition, the expectation cannot be computed analytically and one often resorts to Monte Carlo methods. In the context of this problem, it is known that a multilevel Monte Carlo (MLMC) approach can reduce the amount of computational effort to estimate expectations, for a given level of error. This is achieved via a telescoping identity associated to a Monte Carlo approximation of a sequence of probability distributions with discretisation levels $\infty > h_0 > h_1 \cdots > h_L$. In many practical problems of interest, one cannot achieve an i.i.d. sampling of the associated sequence of probability distributions. A sequential Monte Carlo (SMC) version of the MLMC method is introduced to deal with this problem. It is shown that under appropriate assumptions, the attractive property of a reduction of the amount of computational effort to estimate expectations, for a given level of error, can be maintained within the SMC context. The approach is numerically illustrated on a Bayesian inverse problem.

[a]Speaker

A parameter identification problem for random heterogeneous materials

Frederic Legoll [a] [1,2]

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This work is concerned by transport phenomena in porous media, modeled by the Darcy equation. We consider heterogeneous materials, the properties of which are modeled by a random stationary permeability. Based on experimental results, the conductivity, at the microscopic scale, is supposed to be distributed according to a Weibull probability law, the parameters of which are yet unknown. In this work, we discuss the identification of the parameters of this microscopic model on the basis of some observed quantities at the macroscopic scale, including the effective (homogenized) permeability.
This is joint work with W. Minvielle, A. Obliger and M. Simon.
Related publication: F. Legoll, W. Minvielle, A. Obliger and M. Simon, ESAIM Proceedings 2015.

[a]Speaker

Analysis of the Ensemble Kalman Filter for Inverse Problems

Claudia Schillings [a] [1], Andrew Stuart [1]

[1] Mathematics Institute, University of Warwick – United Kingdom

The ideas from the Ensemble Kalman Filter (EnKF) introduced by Evensen in 1994 can be adapted to inverse problems by introducing artifical dynamics. In this talk, we will discuss an analysis of the EnKF based on the continuous time scaling limits, which allows to derive estimates on the long-time behavior of the EnKF and, hence, provides insights into the convergence properties of the algorithm. Results from various numerical experiments supporting the theoretical findings will be presented.

[a]Speaker

High Dimensional Non-Gaussian Bayesian Inference with Transport Maps

Alessio Spantini [a] [1], Youssef Marzouk [1]

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Characterizing the high dimensional posterior distributions that arise from nonlinear inverse problems in the Bayesian framework is a challenging task. A recent approach to this problem seeks to construct a deterministic transport map from a reference distribution to the posterior. Posterior samples can then easily be obtained by pushing forward reference samples through the map. In this talk, we address the computation of the transport map in high dimensions.

Formally, the transport map can be obtained as the solution of an optimization problem over a function space. We show that the first variation of this optimization objective can be analyzed and evaluated in closed form, and that it reveals low-dimensional structure that may be present in the inference problem. We then propose an algorithm that composes a sequence of transport maps, interleaved with rotations, that act on progressively fewer dimensions and that adapt to the structure of the target/posterior distribution. We demonstrate the algorithm on high-dimensional inference problems arising in spatial statistics and PDEs.

[a]Speaker

MS17 Singularities in the Calculus of Variations and PDE: S. Alama, L. Bronsard

Nematic colloids and defects

Stan Alama, Lia Bronsard, Xavier Lamy [a] [1]

[1] Institut Camille Jordan (ICJ) – Université Claude Bernard - Lyon I (UCBL), CNRS : UMR5208 – Bât. Jean Braconnier, 43 Bd du 11 novembre 1918, 69622 VILLEURBANNE CEDEX, France

In nematic liquid crystals, molecules tend to align in a common direction. This long-range orientational order is very sensitive to inclusion of foreign particles, which create topological defects – inducing fascinating self-assembly phenomena. The size of particles, and the anchoring at their surface, have a crucial influence on the type of colloidal interactions thus enforced. In a joint work with S. Alama and L. Bronsard we use Landau-de Gennes theory to give an accurate description of the defects created by a single spherical particle, depending on size and anchoring.

[a]Speaker

Caractérisation cinétique des vortex

Pierre Bochard [a] [1]

[1] Laboratoire de Mathématiques d'Orsay (LM-Orsay) – CNRS : UMR8628, Université Paris XI - Paris Sud – France

Les champs de rotationnel nul à valeur dans la sphère apparaissent naturellement comme limite de configurations minimisant certaines énergies dépendant d'un paramètre tendant vers 0, comme par exemple l'énergie d'Aviles-Giga et des énergies issues du micromagnétisme.
Dans cet exposé d'un travail en collaboration avec R. Ignat, nous montrerons comment une formulation cinétique jusque là étudiée en dimension 2 dans le cadre du micromagnétisme permet en dimension supérieure de caractériser les champs de la forme $\pm\frac{x-P}{|x-P|}$.
Nous donnerons une interprétation géométrique de ce résultat en terme de surface totalement ombilicale, c'est-à-dire dont la seconde forme fondamentale est un multiple de l'identité.

[a]Speaker

Critical points of the Cahn-Hilliard Energy in a critical regime

Michael Gelantalis [a] [1], Alfred Wagner [1], Maria Westdickenberg [1]

[1] Rheinische-Westfälische Technische Hochschule (RWTH) – RWTH Aachen Templergraben 55 52056 Aachen, Germany

The Cahn-Hilliard energy landscape on the torus is explored in the critical regime of large system size and mean value close to -1. Existence and properties of a "droplet-shaped" local energy minimizer and approximately-mountain-pass-type critical point are established. The proofs employ the Γ-limit (identified in a previous work), quantitative isoperimetric inequalities, and variational arguments. This is joint work with Alfred Wagner and Maria Westdickenberg

[a]Speaker

Sawtooth profile in smectic A liquid crystals

Tiziana Giorgi [a] [1]

[1] New Mexico State University (NMSU) – United States

We analyze the Chen-Lubensky free energy functional to understand the zigzag pattern formed in a smectic A liquid crystal in the presence of an applied magnetic field. We use Gamma-convergence to show that in a suitable regime of fields a sawtooth profile is favored. We also carry out numerical simulations illustrating the zigzag structure. Our mathematical and numerical results are consistent with the experimental picture presented in the physics literature. This is joint work with Carlos J. Garcia-Cervera and Sookyung Joo.

[a]Speaker

Dimension reduction for the Landau-de Gennes model in nematic thin films.

Dmitry Golovaty [a] [1]

[1] The University of Akron – United States

We use the method of Γ-convergence to study the behavior of the Landau-de Gennes model for a nematic liquid crystalline film in the limit of vanishing thickness. In this asymptotic regime, surface energy plays a greater role and we take particular care in understanding its influence on the structure of the minimizers of the derived two-dimensional energy. We assume general weak anchoring conditions on the top and the bottom surfaces of the film. The constants in the weak anchoring conditions are chosen so as to enforce that a surface-energy-minimizing nematic Q-tensor has the normal to the film as one of its eigenvectors. We establish a general convergence result and then discuss the limiting problem in several parameter regimes. This is joint work with Alberto Montero and Peter Sternberg.

[a]Speaker

Topological solitons in chiral magnetism

Christof Melcher [a] [1]

[1] RWTH Aachen University – Pontdriesch 14- 16, 52056 Aachen, Germany

Chiral symmetry breaking described by Lifshitz invariants gives rise to a class of topological solitons, the so-called chiral skyrmions. This form of chirality occurs in various condensed matter systems including ferromagnets and liquid crystals. We shall discuss the occurence of modulated phases in appropriate parameter regimes and the stabilization of isolated chiral skyrmions as a new excited state.

[a]Speaker

The fractional Allen-Cahn equation and nonlocal minimal surfaces

Vincent Millot [a] [1]

[1] Laboratoire Jacques-Louis Lions (LJLL) – INRIA, Université Paris VII - Paris Diderot, CNRS : UMR7598 – B.C. 187 75252 Paris Cedex 05, France

In this talk, I will present a convergence result for a fractional version of the Allen-Cahn equation where the Laplacian is replaced by the fractional Laplacian with exponent s between 0 and 1/2. In a singular limit, (arbitrary) solutions of this equation tend to stationary nonlocal minimal surfaces, i.e., critical point of the fractional 2s-perimeter introduced recently by Caffarelli-Roquejoffre-Savin. This generalizes to the fractional setting a result due to Hutchinson and Tonegawa. My talk is based on a joint work with Yannick Sire and Kelei Wang.

[a]Speaker

Detecting phase defaults of circle-valued maps

Petru Mironescu [a] [1]

[1] Institut Camille Jordan – Université de Lyon – France

Life in Sobolev spaces of circle-valued maps u would be easy if it were possible to write every such u as $u = e^{\imath\varphi}$ with φ as smooth as u and controlled by u. Unfortunately this is not the case. I will present two techniques allowing to control the defaults. The one is the factorization method, inspired by null Lagrangian techniques and the theory of weighted Sobolev spaces. The other one is a PDEs flavor method. We will present applications to variational problems.

[a]Speaker

Stationary harmonic functions whose Laplacian is a Radon measure

Rémy Rodiac [a] [1]

[1] Laboratoire d'Analyse et de Mathématiques Appliquées (LAMA) – Fédération de Recherche Bézout, CNRS : UMR8050, Université Paris-Est Créteil Val-de-Marne (UPEC) – Université de Paris-Est - Créteil, 61 avenue du général de Gaulle, 94010 Créteil cedex, France

In this talk I will study regularity properties of limiting vorticity measures associated to the Ginzburg-Landau equations without magnetic field. E.Sandier and S.Serfaty proved that, in some cases, such a limiting vorticity measure can be written as the Laplacian of a stationary harmonic function h in H^1. Assuming that h is a H^1 stationary harmonic function such that Δh is a measure, we can prove that, locally, near almost every point of the domain, h can be written as $h = |H|$ for some smooth harmonic function H. In particular we deduce that the measure Δh is concentrated on lines, which are sets of zeros of harmonic functions. This problem is also related to vorticity measure of the time independant Euler system in fluid mechanics and to limiting vorticity measures of system of point vortices.

[a]Speaker

Asymptotic Behavior of Critical Points of Energy Involving ‘Circular Well’ Potential

Itai Shafrir [a] [1], Petru Mironescu [2]

[1] Technion - I.I.T. – Israel
[2] Institut Camille Jordan – Université Claude Bernard Lyon 1 – France

We study the singular limit of critical points of an energy with a penalization term depending on a small parameter. The energy involves a potential which is a nonnegative function on the plane, vanishing on a closed curve. We generalize to this setting the results obtained by Bethuel, Brezis, and Helein for the Ginzburg–Landau energy. This is a joint work with Petru Mironescu (Lyon I).

[a]Speaker

On the leapfrogging phenomenon in fluid mechanics

Didier Smets [a] [1]

[1] Laboratoire Jacques-Louis Lions (LJLL) – Université Pierre et Marie Curie (UPMC) - Paris VI – B.C. 187 75252 Paris Cedex 05, France

The mathematical analysis of the time evolution of arbitrary vortex tubes in fluid mechanics remains widely open. The particular case of vortex rings, somewhat simpler due to the reduction to cylindrical symmetry, was tackled by Helmholtz and Kelvin in the second half of the nineteenth century, and the subject has largely developed since then. Helmholtz did already described, without being able to observe it, an interaction phenomenon between rings which is now referred to as "leapfrogging", and which has been observed in real fluids since then.

In the talk, I'll describe the problem in its historical perspective and I'll next report on recent works with R.L. Jerrard (for quantum fluids) and E. Miot and P. Gravejat (for classical fluids) which aim at a rigorous mathematical justification of the leapfrogging phenomenon.

[a]Speaker

Optimal convergence rates via distance, energy, and dissipation

Maria G. Westdickenberg [a] [1], Felix Otto [2]

[1] RWTH Aachen University – Germany
[2] Max Planck Institute for Mathematics in the Sciences (MPI) – Germany

A result of Brezis delivers optimal decay rates of energy and dissipation for a gradient flow with respect to a convex energy. In joint work with Felix Otto we seek corresponding results in the nonconvex case. The first idea is to use a perturbed version of the natural algebraic and differential relationships in the mildly nonconvex setting. We apply this framework to derive decay rates for the 1-d Cahn Hilliard equation on the line. The second idea is to do even better by observing that the ODE argument of Brezis does not require positive definiteness of the full Hessian, but only positive definiteness in suitable directions. We apply this idea in the setting of the 2-d Mullins Sekerka evolution.

[a]Speaker

MS18 Spectral Properties of Hyperbolic Dynamical Systems: C. Guillarmou, G. Rivière

Exponential decay of correlations for Sinai billiard flows

Viviane Baladi [a 1], Mark Demers [2], Carlangelo Liverani [3]

[1] Département de Mathématiques et Applications (DMA) – CNRS : UMR8553, École normale supérieure [ENS] - Paris – France
[2] Fairfield University – Fairfield, CT 06824, United States
[3] Dipartimento di Matematica [Roma II] (DIPMAT) – Via della Ricerca Scientifica, 00133 Roma, Italie, Italy

We prove exponential decay of correlations for two-dimensional Sinai billiard flows with finite horizon.This is achieved by studying the spectrum (resonances) of a transfer operator on a suitable Banach space of anisotropic distributions, combining techniques of Dolgopyat and Liverani (as adapted in a previous work of Baladi-Liverani on piecewise hyperbolic contact flows) and methods of Demers-Liverani (as adapted in a previous work of Demers and Zhang giving a new proof of Young's result of exponential decay of correlations for Sinai billiard maps).

[a]Speaker

Where are the resonances of negatively curved cusp surfaces ?

Yannick Bonthonneau [a 1,2]

[1] Département de Mathématiques et Applications (DMA) – CNRS : UMR8553, École normale supérieure [ENS] - Paris – France
[2] Université Paris Sud – Université Paris XI - Paris Sud – France

We call cusp surface a complete surface whose ends are exact real hyperbolic cusps. On such a manifold the laplacian Δ has continuous spectrum (as well as eigenvalues). The resonances appear as the poles of the analytic continuation of the resolvant $R(s) = (\Delta - s(1-s))^{-1}$. When the curvature is negative, I will explain how one can show that most resonances lay in a vertical strip near the spectrum (at least for generic metrics).

[a]Speaker

Discrete band spectrum and zeta functions for geodesic flow on negative curvature manifold

Frederic Faure [a 1]

[1] Institut Fourier (IF) – CNRS : UMR5582, Université Joseph Fourier - Grenoble I – France

The geodesic flow on a negative curvature manifold is a model of chaotic dynamics. The vector field has discrete spectrum in specific anisotropic Sobolev spaces called Ruelle-Pollicott spectrum. It governs decay of correlations. We will explain that this spectrum is structured in bands separated by gaps. Work with Masato Tsujii.

[a]Speaker

Distorted plane waves in chaotic scattering

Maxime Ingremeau [a] [1]

[1] Institut de Physique Théorique (IPhT) – CEA – France

Distorted plane waves, sometimes called Eisenstein functions, are a family of eigenfunctions of a Schrödinger operator that are not square integrable. More precisely, they can be written as the sum of a plane wave and a purely outgoing wave. We shall study distorted plane waves in the semiclassical limit, on manifolds that are Euclidean near infinity, under the hypothesis that the classical dynamics is hyperbolic close to the trapped set, and that some topological pressure is negative.

[a]Speaker

Ruelle spectrum of Analytic circle maps

Frederic Naud [a] [1]

[1] Laboratoire de mathématiques d'Avignon (LMA) – Université d'Avignon et des Pays de Vaucluse – Université d'Avignon et des Pays de Vaucluse Faculté des sciences 33 Rue Louis Pasteur 84000 Avignon, France

Expanding circle maps are the simplest toy models of (uniformly) hyperbolic dynamical systems. The Ruelle Spectrum (or correlation spectrum) describes the asymptotics of correlations functions for the SRB measure. Despite the fact that this theory dates back to the 70's, almost nothing is known on this correlation spectra from a quantitative viewpoint. In this talk we will show how to prove optimal lower bounds on the density of Ruelle eigenvalues using complex deformations of Blaschke products and potential theoretic tools. This is a joint work with Oscar Bandtlow.

[a]Speaker

Spreading and Equidistribution of propagated coherent states on hyperbolic surfaces.

Roman Schubert [a] [1]

[1] University of Bristol – United Kingdom

Coherent states are strongly localised wave packets which play an important role in quantum mechanics and many of its applications. One reason why they are so useful is that their time evolution can be described in the semiclassical limit by a single trajectory of the classical Hamiltonian dynamics together with the linearised flow around that trajectory. But if the trajectory is hyperbolic this approximation breaks down at some fraction of the so called Ehrenfest time, due to the spreading of the wave packet induced by the hyperbolic dynamics. In a recent paper with R. Vallejos and F. Toscano we developed an approach which allows to describe what happens to the coherent state at the Ehrenfest time, namely it becomes an isotropic state which is localised on the unstable manifold of the central trajectory. In this talk we apply this approach to the propagation of coherent states on surfaces of constant negative curvature. The classical dynamics is given by the geodesic flow, which is a classical example of a uniformly hyperbolic system, and the construction describing the propagation of coherent states and their transition to isotropic states at the Ehrenfest time can be made very explicit. We furthermore show that mixing of the geodesic flow implies equidistribution of propagated coherent states beyond the Ehrenfest time and up to at least twice the Ehrenfest time.

[a]Speaker

Counting in infinite measure

Pierre Vidotto [a] [1]

[1] Laboratoire de Mathématiques Jean Leray (LMJL) – Université de Nantes, CNRS : UMR6629, Ecole Centrale de Nantes – France

Let X be a Hadamard manifold of dimension $\geqslant 2$ with sectionnal curvature pinched between two negative constants. We consider the action of some discrete group Γ of positive isometries of X, torsion free and acting properly and discontinuously on X. These exotic groups enable us to work in the context of manifolds X/Γ with infinite measure. We will investigate two problems: the first concerns the asymptotic behavior of the orbital function N_Γ of Γ. The second is to find an asymptotic to the number $\mathrm{N}(R)$ of primitive closed geodesics on X/Γ with length smaller than R. After an introduction of main notations, we will present a coding of the unit tangent bundle $\mathrm{T}^1\mathrm{X}/\Gamma$ which allows us to write the previous quantities N_Γ and $\mathrm{N}(R)$ as sums of iterates of some perturbation of a transfert operator. Finally we will explain how to deduce the results using some spectral properties of this operator.

[a]Speaker

Weighted zeta functions for Anosov flows

Tobias Weich [a] [1]

[1] Universität Paderborn – Germany

Recently Giulietti-Liverani-Pollicott, Faure-Tsujii and Dyatlov-Zworski introduced zeta functions for Anosov flows. Using their techniques we construct a weighted version of dynamical zeta functions which allows to obtain more details on the spectral properties of the transfer operator (joint work with C.Guillarmou and J. Hilgert).

[a] Speaker

MS19 Stability of Nonlinear Waves: M. Beck, K. Zumbrun

Numerical stability analysis for thin film flow

Blake Barker [a] [1]

[1] Division of Applied Mathematics (DAM) – 182 George Street Providence, RI 02912, United States

We discuss various aspects of numerical stability analysis of periodic roll wave solutions arising in equations of inclined thin film flow, with an eye toward the development of guaranteed error bounds. In particular, we rigorously verify stability of a family of periodic wave solutions arising in a generalized Kuramoto-Sivashinsky equation in the Korteweg-de Vries limit, that is, in the limit viscosity goes to zero.

[a]Speaker

Stability of Viscous Roll Waves

Blake Barker [1], Mathew Johnson [a] [2], Pascal Noble, Luis Miguel Rodrigues [3,4], Kevin Zumbrun

[1] Division of Applied Mathematics (DAM) – 182 George Street Providence, RI 02912, United States
[2] University of Kansas, Department of Mathematics (KU) – Lawrence, Kansas, 66045, USA, United States
[3] KALiFFE (UCBL / INRIA Grenoble Rhône-Alpes / INSMI) – INSMI (CNRS), INRIA, Université Claude Bernard - Lyon I (UCBL) – Institut Camille Jordan, Université de Lyon Université Claude Bernard, 43, boulevard du 11 Novembre 1918, F-69622, Villeurbanne cedex, France
[4] Institut Camille Jordan (ICJ) – Institut National des Sciences Appliquées [INSA], Ecole Centrale de Lyon, Université Claude Bernard - Lyon I (UCBL), CNRS : UMR5208, Université Jean Monnet - Saint-Etienne – Bât. Jean Braconnier n° 101 43 Bd du 11 novembre 1918 69622 VILLEURBANNE CEDEX, France

Roll-waves are a well observed hydrodynamic instability occurring in inclined thin film flow, mathematically described as periodic traveling wave solutions of the St. Venant system. In this talk, I will discuss recent progress concerning the spectral and nonlinear stability of viscous roll-waves in a variety of asymptotic regimes, including near the onset of hydrodynamic instability, large-Froude number analysis, and (possibly) in the inviscid limit.

[a]Speaker

Dynamics near the subcritical transition of the 3D Couette flow

Jacob Bedrossian [a] [1], Nader Masmoudi [2], Pierre Germain [2]

[1] University of Maryland, College Park (UMD-CP) – United States
[2] Courant Institute, New York University – United States

We discuss the dynamics of small perturbations of the plane, periodic Couette flow in the 3D incompressible Navier-Stokes equations at high Reynolds number. For sufficiently regular initial data, we determine the stability threshold and show that all solutions near the threshold rapidly converge to a class of global "2.5 dimensional" slowly-evolving solutions referred to as "streaks". The primary stability mechanisms are an anisotropic enhanced dissipation effect and an inviscid damping effect of the velocity component normal to the shear, both a result of the mixing caused by the large mean shear. Joint work with Pierre Germain and Nader Masmoudi.

[a]Speaker

Creating a spectral gap through inverse spectral theory

Martina Chirilus-Bruckner [a] [1]

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A means of simplifying the analysis of dynamical systems is given by center manifold theory, which – under certain conditions, such as the existence of a spectral gap – allows to reduce the dimension of the problem. Especially in the context of partial differential equations, which can be viewed as infinite-dimensional dynamical systems, such a reduction is of particular interest, but often inhibited due to the absence of a spectral gap. We explain how center manifold theory can be applied to non-autonomous problems in which, at first sight, such an endeavor seems hopeless, but a reduction becomes possible after solving an inverse spectral problem. In other words, we demonstrate that the infinitely many parameters inherent in non-autonomous coefficients can be used to overcome challenges that are out of reach in the autonomous case. We illustrate the successful use of this strategy along a range of problems whose common theme is the search of time-periodic solutions in the setting of dispersive equations with spatially periodic coefficients. This is joint work with C.E. Wayne.

[a]Speaker

Anomalous spreading in a system of coupled Fisher-KPP equations

Matt Holzer [a] [1]

[1] George Mason University – United States

Anomalous spreading refers to a scenario wherein the coupling of two equations leads to faster spreading speeds in one of the components. The existence of these spreading speeds can be predicted from the linearization about the unstable state. Whether this linear speed is observed in the nonlinear regime depends on the particular parameter values being considered. We discuss mechanisms leading to these faster speeds and prove that for some parameters initial data consisting of compactly supported perturbations of Heaviside step functions spreads asymptotically with the anomalous speed. The proof makes use of a comparison principle and the explicit construction of sub and super solutions.

[a] Speaker

A Geometric Approach to Counting Eigenvalues

Robert Marangell [a] [1]

[1] School of Mathematics and statistics [Sydney] – School of Mathematics and Statistics F07 University of Sydney NSW 2006 Australia, Australia

In this talk I will demonstrate a geometrically inspired technique for computing temporal eigenvalues of linearised operators about travelling waves. Using the examples of the Fisher-KPP equation and a Keller-Segel model of bacterial chemotaxis, I will show how to produce an Evans function which is computable through several orders of magnitude in the spectral parameter and show how such a function can naturally be extended into the continuous spectrum. In both examples, I will use this function to numerically verify the absence of eigenvalues in a large region of the right half of the spectral plane.

[a] Speaker

Transverse steady bifurcation of viscous shock solutions of a system of parabolic conservation laws

Rafael Monteiro Da Silva [a] [1]

[1] Department of mathematics [Bloomington] (IU - Bloomington) – 831 E. Third St. — Bloomington, IN 47405, United States

I will describe how a recent proof of instability of inviscid slow shocks in magneto hydrodynamics (MHD) motivated an analytical study of the existence of nonplanar viscous shocks and also an extensive numerical study (in joint work with Kevin Zumbrun and Blake Barker) of transitions to instability for viscous multi-D MHD.
The talk will focus on nonlinear, steady, bifurcations for a class of strict parabolic models that features O(2) symmetry in a strip . The applications in mind are "cellular instabilities" occuring in detonation and MHD. Curiously, a similar phenomena in MHD was observed by astrophysicists in late 90's through numerical studies of slow shocks in white dwarf stars.
Time permitting, I will also show further results on instabilities in MHD that involve both numerical and analytical techniques.

[a] Speaker

Concatenated traveling waves

Stephen Schecter [a] [1], Xiao-Biao Lin [1]

[1] North Carolina State University (NC State) – Raleigh, North Carolina 27695, United States

We consider concatenated traveling wave solutions of reaction-diffusion systems. These are solutions that look like a sequence of traveling waves with increasing velocity, with the right state of each wave equal to the left state of the next. I will present an approach to the stability theory of such solutions that does not rely on treating them as a sum of traveling waves. It is based instead on exponential dichotomies and Laplace transform.

[a] Speaker

MS20 Statistical Properties of Dynamical Systems: V. Baladi, I. Melbourne

Open Sets of Axiom A Flows with Exponentially Mixing Attractors

Oliver Butterley [a] [1]

[1] University of Vienna (UNIVIE) – Oskar-Morgenstern-Platz 1, 1090 Wien, Austria

I will discuss how, for any dimension $d \geq 3$, we may construct $\mathcal{C}^1$-open subsets of the space of $\mathcal{C}^3$ vector fields such that the flow associated to each vector field is Axiom A and exhibits a non-trivial attractor which mixes exponentially with respect to the unique SRB measure. This is joint work with V. Araújo & P. Varandas.

[a]Speaker

Local Central Limit Theorem in fast-slow partially hyperbolic dynamical systems

Jacopo De Simoi [a] [1]

[1] Department of Mathematics - University of Toronto (UofT) – Department of Mathematics - University of Toronto - Bahen Centre - 40 St. George St. - Toronto, Ontario - CANADA - M5S 2E4, Canada

We prove a Local Central Limit Theorem for an open class of fast-slow partially hyperbolic dynamical systems. The proof relies on a blend of Standard Pairs techniques and spectral theory. This is joint work with C. Liverani

[a]Speaker

Stability index, uncertainty exponent and thermodynamic formalism for intermingled basins of chaotic attractors

Gerhard Keller [a] [1]

[1] Department Mathematik, Universität Erlangen-Nürnberg – Cauerstraße 11, 91058 Erlangen, Germany

I will discuss simple discrete time dynamical systems with two chaotic attractors that have intermingled basins of attraction. The "degree of intermingledness" of the basins can be described by (at least) two local scaling quantities: the uncertainty exponent introduced by the Maryland group already in 1985, and the stability index introduced by Podvigina and Ashwin in 2011. Using random walk and diffusion approximation arguments approximate formulas for the uncertainty exponent were derived by Ott et al. in 1993. Here we present precise formulae for both exponents, derived from thermodynamic formalism, for a class of two-dimensional model systems including in particular the maps studied by Bonifant and Milnor in 2008. The key words for the proofs are telescoping, pressure function, and large deviations.

[a]Speaker

Almost sure invariance principles for sequential and non-stationary dynamical systems

Matthew Nicol [a] [1], Nicolai Haydn[2], Andrew Torok[1], Sandro Vaienti[3]

[1] University of Houston – United States
[2] University of Southern California – United States
[3] CPT, Luminy – Centre National de la Recherche Scientifique - CNRS – France

We establish almost sure invariance principles, a strong form of approximation by Brownian motion, for non-stationary time-series arising as observations on dynamical systems. Our examples include observations on sequential expanding maps, perturbed dynamical systems, non-stationary sequences of functions on hyperbolic systems as well as applications to the shrinking target problem in expanding systems.

[a] Speaker

Potential kernel, hitting probabilities and limit theorems for Abelian extensions of dynamical systems

Damien Thomine [a] [1]

[1] Département de Mathématiques-Université de Paris XI – Université Paris XI - Paris Sud – France

For well-behaved recurrent random walks, there is a general relation between the potential kernel and the probability that an excursion starting from the origin hits a given point before going back to the origin. I will present a way to recover this relation for recurrent, ergodic Abelian extensions of a well-behaved family (ergodic Gibbs-Markov maps) of hyperbolic dynamical systems.
We compute the limit distribution of the Birkhoff sums of well-chosen observables with two different methods: the method of moments, and a method involving the excursions from the origin. The equality of the limits yields the aforementionned relation, among other results of interest. Applications include for instance the geodesic flow on Abelian covers of compact hyperbolic manifolds.
This is a work in progress, joint with Françoise Pène (Université de Brest).

[a] Speaker

Hitting time laws in dynamical systems

Mike Todd [a] [1]

[1] University of St Andrews – Mathematical Institute, University of St Andrews, North Haugh, St Andrews, Fife, KY16 9SS, United Kingdom

Laws of recurrence are fundamental to the understanding of the statistics of a dynamical system. In this talk I'll give a general form of hitting time law which covers recurrence laws for systems with holes (open systems), asymptotic hitting time laws to shrinking targets as well as other related hitting time laws. This has applications for both non-uniformly hyperbolic and uniformly hyperbolic systems, yielding different types of behaviour depending on the speed of mixing.

[a]Speaker

Shadowing and stochastic stability

Dmitry Todorov [a] [1,2]

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[2] Saint Petersburg State University (SPbSU) – 7-9, Universitetskaya nab., St.Petersburg, 199034, Russia

There is a strong and long-lasting interest in chaotic dynamical systems as mathematical models of various processes in different areas of science. Like for any other mathematical models for chaotic systems to be useful it is desirable that they have stability properties.
There are exist different stability properties. In particular, there exist two notions of stability with respect to small per-iteration perturbations – shadowing property and stochastic stability. System is said to have shadowing property if every (pseudo)trajectory with small errors can be uniformly approximated by a trajectory without errors. System is stochastically stable if the noise perturbing the system is considered to be random and invariant measures for the stochastic process corresponding to the perturbed system converge to an invariant measure of the unperturbed system.
Although conceptually these properties are somewhat similar and it is known that some chaotic systems have both properties, no direct relations between shadowing and stochastic stability were established so far.
I will discuss some of these relations both for qualitative and quantitative versions of shadowing and stochastic stability.

[a]Speaker

Conformally symplectic systems

Rafael De La Llave [a] [1]

[1] Georgia Institute of Technology (GATECH) – North Ave. Atlanta, Georgia 30332, United States

Conformally symplectic systems are systems which transform a symplectic form into a multiple of itself.
They appear naturally in mechacnical systems with friction proportional to the velocity, thermostated systems or as Euler Lagrange equations of discounted functionals in echonomics. We present a KAM theory for these systems.
The results is formulated in an a-posteriory format which says than an approximate solution of an invariance equation
which is not degenerate leads to a true solution.Note that the formulation does not make any reference to a quasi-integrable system. The method also leads to a very efficient algorithm that is guaranteed to
converge to the maximal domain. This algorithm has been implemented.
The method also allows to study the domain of analyticity of the solutions in singular perturbation theory.
This is joint work with A. Celletti and R. Calleja

[a] Speaker

Non trivial limit distributions for systems with holes

Dalia Terhesiu [a] [1]

[1] Faculty of Mathematics – Oskar-Morgenstern-Platz 1 1090 Wien, Austria, Austria

Recent work by M. Demers and B. Fernandez shows that the pushforward measure of open interval maps with indifferent fixed points at the origin converges to the point mass supported at the origin; in this context, an 'open' interval intermitent map is a map with a 'non-small' hole (roughly, a cylinder) that does not contain any neighborhood of the origin. In work in progress with R. Zweimueller, we study the existence of a non trivial limit distribution under a different normalization of the pushforward transfer operator (or some type of average) that could be eventually used in the study of statistical properties of the open system.

[a] Speaker

MS21 Stochastic Dynamics: H. Crauel, A. J. Homburg

On FitzHugh-Nagumo SDEs and SPDEs

Nils Berglund [a] [1], Christian Kuehn [2], Damien Landon

[1] Université d'Orléans (MAPMO) – CNRS : UMR7349 – Université d'Orléans, UFR Sciences Bâtiment de mathématiques - Route de Chartres B.P. 6759 - 45067 Orléans cedex 2, France
[2] Vienna University of Technology – Austria

We will discuss two results on stochastic FitzHugh-Nagumo equations arising in neuroscience. The first result concerns the interspike interval distribution in FitzHugh-Nagumo SDEs for a single neuron in the excitable regime. We prove that in a large parameter range, this distribution is asymptotically exponential, with a parameter which can be expressed in terms of the principal eigenvalue of a substochastic Markov chain. The second result is on local existence of solutions for FitzHugh-Nagumo SPDEs in two and three space dimensions for a large population of neurons. The proof relies on Martin Hairer's theory of regularity structures, and requires the equation to be renormalised.

References:

Nils Berglund and Damien Landon, Mixed-mode oscillations and interspike interval statistics in the stochastic FitzHugh-Nagumo model, Nonlinearity 25:2303-2335 (2012)
Nils Berglund and Christian Kuehn, Regularity structures and renormalisation of FitzHugh-Nagumo SPDEs in three space dimensions, preprint arXiv:1504.02953 (2015)

[a] Speaker

Approximate Slow Manifolds and Interface Motion for Stochastic PDEs

Dirk Blömker [a] [1]

[1] Universität Augsburg [Augsburg] – Universitätsstr. 12, 86159 Augsburg, Germany

We consider stochastic partial differential equations like the Cahn-Hilliard or Allen-Cahn equation perturbed by additive noise, and study the dynamics of interfaces or droplets for theses stochastic models. For small noise the dynamics of the stochastic infinite dimensional system is given by the motion along a deterministic finite dimensional deterministic slow manifold M parametrized by the interface or droplet positions. The manifold describes the motion well for very long time-scales until an interface breaks down or the droplets merge. Main results presented include stochastic stability and attractivity for the manifold M and the derivation of an effective equation for the positions of interfaces or droplets.

[1] *D. Antonopoulou, D. Blömker, G. Karali*: Front motion in the one-dimensional stochastic Cahn-Hilliard equation. SIAM J. Math. Anal. 44(5), 3242–3280, (2012).

[2] *D. Antonopoulou, P. Bates, D. Blömker, G. Karali*: Motion of a droplet for the mass-conserving stochastic Allen-Cahn equation. Preprint, 2015.

[a] Speaker

Random dynamical systems generated by stochastic lattices

Tomás Caraballo [a] [1]

[1] University of Seville [Seville] – C/ S. Fernando, 4, C.P. 41004-Sevilla, Spain

The aims of this talk is to report on recent advances in the topic of random dynamical systems generated by stochastic lattice differential systems. We will focus on problems containing additive and multiplicative noise and will emphasize the differences when considering a finite number of noisy terms at each node (essentially the same noise in each node) or a different noisy perturbation at each one. We will show how these systems generate a random dynamical system possessing a random attractor.

[a]Speaker

Brownian Motion in a Multiscale Potential

Andrew Duncan [a] [1], Serafim Kalliadasis [2], Marc Pradas [3], Grigorios Pavliotis [1]

[1] Department of Mathematics [Imperial College London] – Department of Mathematics South Kensington Campus Imperial College London SW7 2AZ London UK, United Kingdom

[2] Imperial College London – Department of Chemical Engineering, Imperial College London, London SW7 2AZ, UK, United Kingdom

[3] The Open University (OU) – Department of Mathematics and Statistics, The Open University, Milton Keynes, MK7 6AA. UK., United Kingdom

In this talk we consider the problem of Brownian motion in a rough potential, modelled as a slowly-varying potential perturbed by periodic multiscale fluctuations. We show that the effective behaviour of this model can be described by an overdamped Langevin equation possessing multiplicative noise, for which detailed balance with respect to a coarse grained invariant measure will hold. We demonstrate how the small scale fluctuations in the potential can give rise to dynamical behaviour which is qualitatively different from that of the original, unperturbed model, and through numerical examples and analysis we will explore this behaviour in a number of regimes, particularly in the limit of increasingly many length scales. This is joint work with Serafim Kalliadasis (Imperial College), Marc Pradas (Open University) and Grigorios Pavliotis (Imperial College).

[a]Speaker

Synchronization by noise for order-preserving random dynamical systems

Benjamin Gess [a] [1]

[1] University Bielefeld – Germany

We present recent results on weak synchronization by noise for order-preserving random dynamical systems on Polish spaces. That is, under these conditions we prove the existence of a weak point attractor consisting of a single random point. This generalizes previous results in two directions: First, we do not restrict to Banach spaces and second, we do not require the partial order to be admissible nor normal. As a second main result and application we present weak synchronization by noise for stochastic porous media equations with additive noise. This is joint work with Franco Flandoli and Michael Scheutzow.

[a]Speaker

Mean-square random dynamical systems and mean-square dichotomies

Peter Kloeden [a] [1]

[1] Huazhong University of Science and Technology (HUST) – Wuhan, China

Mean-square random dynamical systems are formulated as nonautonomous dynamical systems on the time-variable state spaces X_t $L_2[\Omega, \mathcal{A}_t, \mathbb{R}^d]$ of nonanticipiative mean-square radom variables. They are generated by meanfield stochastic differential equations. Many concepts from nonautonomous dynamical systems carry over, e.g.g, a mean-square attractor is a pulback attractor. However, lack of compactness criteria for the spaces $L_2[\Omega, \mathcal{A}_t, \mathbb{R}^d]$make it difficult to apply the theory of nonautonomous dynamical systems. Mean-square dichotomies are defined for lienar meanfield SDE and the corresonding spectrum found and is used to illustrate the d bifurcation ofa zero solution to a nontrivial mean-square attractor

[a]Speaker

Large deviations

Davit Martirosyan [a] [1]

[1] Laboratoire d'Analyse, Géométrie et Modélisation (AGM) – CNRS : UMR8088, Université de Cergy Pontoise – France

The talk is devoted to the derivation of large deviations principle for the family of stationary measures of the Markov process generated by the flow of the damped wave equation with vanishing smooth white noise. The main novelty is that we do not assume that the limiting equation possesses a unique equilibrium and that we do not impose roughness on the noise. Our proof is based on a development of the approach introduced by Freidlin and Wentzell for the study of large deviations for stationary measures of stochastic ODEs on a compact manifold, and some ideas introduced by Sowers. A key ingredient of the proof relies on a new result according to which any discrete-time Markov chain possesses a positive finitely additive stationary measure.

[a] Speaker

A random and set-valued dynamical systems perspective on critical transitions

Martin Rasmussen [a] [1]

[1] Department of Mathematics - Imperial College London – Department of Mathematics South Kensington Campus Imperial College London LONDON SW7 2AZ United Kingdom, United Kingdom

Critical transitions describe the occurrence of sudden changes of behaviour in the dynamics of complex systems in nature, technology and society. We identify certain critical transitions with bifurcations in both random dynamical systems and set-valued dynamical systems and describe fundamental aspects of both approaches in detail. We also address the question of predicting critical transitions.
This talk is based on three papers, with co-authors J.S.W. Lamb, G. Malavolta (Imperial College London), Doan Thai Son (Vietnam Academy of Science and Technology), C. Kuehn (Vienna University of Technology) and C.S. Rodrigues (MPI Leipzig).

[a] Speaker

MS22 Stochastic PDEs: A. Debussche, F. Flandoli

Regularization by noise for the stochastic transport equation

Lisa Beck [a] [1]

[1] University of Augsburg – Germany

We discuss several aspects of weak (L^∞-) solutions to the stochastic transport equation

$$du = b \cdot Du\, dt + \sigma Du \circ dW_t$$

with Stratonovich multiplicative noise. Here, b is a time dependent vector field (the drift), u is the unknown, σ a real number, and W_t a Brownian motion.
For the deterministic equation ($\sigma = 0$) it is well-known that multiple solutions may exist and that solutions may blow up from smooth initial data in finite time provided that the drift is not regular enough (basically less than Lipschitz in space). For the stochastic equation ($\sigma \neq 0$) instead, a suitable integrability condition on the drift is sufficient to prevent the formation of non-uniqueness and of singularities.
In my talk I will explain some ideas of a joint project with F. Flandoli, M. Gubinelli and M. Maurelli, which concern this phenomenon of regularization by noise. While a similar result was achieved via stochastic characteristics, we have obtained the conservation of Sobolev regularity of the initial data relying on PDE techniques.

[a] Speaker

Stochastic Navier-Stokes Equations for Compressible Fluids

Dominic Breit [1], Martina Hofmanova [a] [2]

[1] Heriot-Watt University – Riccarton Edinburgh EH14 4AS, UK, United Kingdom
[2] Technical University Berlin (TU Berlin) – Straße des 17.Juni 136 10623 Berlin, Germany

We study the Navier-Stokes equations governing the motion of isentropic compressible fluid in three dimensions driven by a multiplicative stochastic forcing. In particular, we consider a stochastic perturbation of the system as a function of momentum and density and establish existence of the so-called finite energy weak martingale solution under the condition that the adiabatic constant satisfies $\gamma > 3/2$. The proof is based on a four layer approximation scheme together with a refined stochastic compactness method and a careful identification of the limit procedure.

[a] Speaker

Renormalisation Group in Regularity Structures

Yvain Bruned [a] [1]

[1] LPMA (Paris 6) – UPMC – France

The recent theory by Martin Hairer on Regularity Structures allows us to treat singular SPDEs such as the generalised KPZ. We will present a new description of the renormalisation group which is a crucial step in the resolution of such SPDEs.

[a]Speaker

SPDEs on narrow domains and on graphs: an asymptotic approach

Sandra Cerrai [a] [1]

[1] University of Maryland – United States

We introduce here a class of stochastic partial differential equations defined on a graph and we show how they are obtained as the limit of suitable stochastic partial equations defined in a narrow channel, as the width of the channel goes to zero. This is a joint work with M. Freidlin.

[a]Speaker

Stochastic scalar conservation laws

Benjamin Gess [a] [1]

[1] University Bielefeld – Germany

In this talk we will review recent results for stochastic scalar conservation laws with random flux. In the first part we will focus on a well-posedness theory for the case of spatially inhomogeneous, random fluxes as they appear in mean field games. In the second part we will investigate the long time behavior and regularity of solutions to stochastic scalar conservation laws on the torus. In particular, we will observe a certain regularizing effect due to the noise. This is joint work with Panagiotis E. Souganidis.

[a]Speaker

Weak universality of the parabolic Anderson model

Nicolas Perkowski [a] [1]

[1] Humboldt Universität (HU Berlin) – Germany

We study a nonlinear version of the two-dimensional lattice parabolic Anderson model (PAM) with small potential and see that its rescaled solution converges to the linear continuum PAM, universally for all centered i.i.d. potentials with sufficiently many moments and for all nonlinearities vanishing at the origin. The proof is based on paracontrolled distributions. Joint work with Khalil Chouk and Jan Gairing.

[a]Speaker

Densities for the Navier-Stokes equations with noise

Marco Romito [a] [1]

[1] Dipartimento di Matematica, Università di Pisa – Largo Bruno Pontecorvo 5, 56127 Pisa,, Italy

We present a proof of existence of the density with respect to the Lebesgue measure, as well as of regularity in Besov spaces, for the solution of stochastic differential equations with "non-smooth" data.
As an application we show existence of densities for the finite dimensional marginal distributions of the law of solutions of the 3D Navier-Stokes equations forced by Gaussian noise. Classical methods, such as the Malliavin calculus, do not work in this setting for reasons that are strongly related to the three dimensional case.
The same method provides also regularity in time of the densities, as well as absolute continuity of the laws of some quantities (energy and dissipation rate, for instance) that depend on a infinite number of components.
A stronger result, namely Hölder continuity of the densities, is available through a suitable conditioned Fokker-Planck equation.
When the random forcing has no full support in Fourier space the problem is open. In this case we can prove the mere existence of a density, without any regularity property, by using the backward local smoothness of trajectories, and weak-strong uniqueness.

[a]Speaker

Some recent results on stochastic kinetic equations

Julien Vovelle [a] [2,1]

[2] Institut Camille Jordan (ICJ) – Université Claude Bernard - Lyon I (UCBL), CNRS : UMR5208 – Bât. Jean Braconnier n° 101 43 Bd du 11 novembre 1918 69622 VILLEURBANNE CEDEX, France
[1] CNRS – CNRS : UMR5208 – France

I will present various results and problematics on stochastic kinetic equations : diffusion limits, regularisation by noise, long-time behaviour, case of space-time white noise (joint works with Debussche, De Moor, Fedrizzi, Flandoli, Priola, Rodrigues).

[a]Speaker

MS23 Stochastic Homogenization: S. Armstrong, A. Gloria

Quenched local CLT for symmetric diffusions in a degenerate random environment

Jean-Dominique Deuschel [a] [1], Alberto Chiarini [2]

[1] Institut für Mathematik [Berlin] – Sekr. MA 7-2 Straße des 17.Juni 136 10623 Berlin, Germany
[2] Technische Universitaet Berlin – Germany

We study a symmetric diffusion X on $\mathbb{R}^d$ in divergence form in a stationary and ergodic environment, with measurable unbounded and degenerate coefficients. The diffusion is formally associated with $L^\omega u = \nabla\cdot(a^\omega \nabla u)$. We prove for X a quenched local central limit theorem, under some moment conditions on the environment; the key tool is a parabolic Harnack's inequality obtained using the celebrated Moser's iteration technique.

[a] Speaker

The Clausius-Mossotti formulas and beyond

Mitia Duerinckx [a] [1]

[1] Universite Libre de Bruxelles – Belgium

We study the behavior of the homogenized coefficients associated with some random stationary ergodic medium under a Bernoulli perturbation. More precisely, a stationary family of possible inclusions is considered, and each of them is chosen independently according to a Bernoulli process, thus yielding perturbed inclusions in a reference stationary medium. Introducing a notion of derivative with respect to the perturbation of the medium at each possible inclusion – very reminiscent of the vertical derivatives used in stochastic homogenization, or of the randomized derivatives used e.g. in the context of Stein's method –, we prove for the perturbed homogenized coefficients a proxy for a Taylor expansion in terms of these derivatives. This justifies the so-called cluster expansions formally used by physicists, and proves the analyticity of the homogenized coefficients with respect to the Bernoulli parameter. Our approach holds under the minimal assumptions of stationarity and ergodicity, both in the scalar and vector cases. In particular, the first-order term yields the celebrated (electric and elastic) Clausius-Mossotti formulas for isotropic spherical random inclusions in an isotropic reference medium. This work constitutes the first rigorous proof of these formulas in the case of random inclusions.

[a] Speaker

A pointwise two-scale expansion for equations with random coefficients

Yu Gu [a] [1], Jean-Christophe Mourrat [2]

[1] Stanford University – United States
[2] École normale supérieure de Lyon (ENS LYON) – École Normale Supérieure (ENS) - Lyon – 15 parvis René Descartes - BP 7000 69342 Lyon Cedex 07, France

Quantitative stochastic homogenization of operators in divergence form has witnessed important progress recently. Our goal is to go beyond the error bound to analyze statistical fluctuations around the homogenized limit. We prove a pointwise two-scale expansion for equations in divergence form. The approach is probabilistic. The main ingredients include the Kipnis-Varadhan method applied to reversible diffusion in random environment and a quantitative martingale central limit theorem.

[a]Speaker

On the Stochastic Homogenization of Parabolic Equations

Jessica Lin [a] [1], Charles Smart[2]

[1] University of Wisconsin-Madison [Madison] – 480 Lincoln Dr. Madison, WI 53706, United States
[2] Cornell University – 91709 – Cornell University – Ithaca, New York 14853 US, United States

We will present an overview of the study of stochastic homogenization of fully nonlinear uniformly parabolic equations in spatio-temporal media. This model includes the stochastic homogenization of linear nondivergence form equations with coefficients varying in space and time. We will discuss the qualitative theory, the associated regularity theory for parabolic equations, and error estimates for this problem. In particular, we obtain quenched error estimates which are algebraic in the rate of decay.

[a]Speaker

Individual invariance principles for degenerate diffusions in a periodic potential.

Pierre Mathieu [a] [1]

[1] Université d'Aix-Marseille (AMU) – Ministère de l'Enseignement Supérieur et de la Recherche Scientifique – Aix-Marseille Université Jardin du Pharo 58, bd Charles Livon -13284 Marseille Cedex 07, France

I will explain pointwise (individual) homogenization results (invariance principle) for a diffusion operator of the form $\Delta - \nabla V \nabla$, where V is measurable, periodic and such that exp(-V) and exp(V) are integrable. The key ingredient of the proof is a weighted Sobolev-type inequality. This is joint work with M. Ba that is to appear in Siam Journal on Mathematical Analysis.

[a]Speaker

A selection approach in stochastic homogenization: Special Quasirandom Structures

William Minvielle [a] [2,1]

[2] MATHERIALS (INRIA Paris - Rocquencourt) – INRIA, École des Ponts ParisTech (ENPC) – France
[1] Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS) – Université Paris Est (UPE), École des Ponts ParisTech (ENPC) – 6 et 8 avenue Blaise Pascal Cité Descartes - Champs sur Marne 77455 Marne la Vallée Cedex 2, France

In this work, we introduce a selection approach for the homogenization of a random, linear elliptic second order partial differential equation set on a bounded domain in $\mathbb{R}^d$. The random diffusion coefficient matrix field is assumed to be uniformly elliptic, bounded and stationary ("periodic in law"). In the limit when $\varepsilon \to 0$, the solution of the equation converges to that of a homogenized problem of the same form, the coefficient field of which is a deterministic and constant matrix $A^\star$ given by an average involving the so-called corrector function that solves a random auxiliary problem set on the entire space.
In practice, the corrector problem is approximated on a bounded domain Q_N as large as possible. A by-product of this truncation procedure is that the deterministic matrix $A^\star$ is approximated by a random, apparent homogenized matrix $A^\star_N(\omega)$. We select only random realizations that satisfy special conditions (e.g. in a bi-composite material with equal probability for each phase, we enforce that each phase is present with equal volume in the finite supercell).
We prove, under mild hypothesis (symmetry of A), that our approach is convergent. We demonstrate in special cases that it is efficient. A significant part is devoted to the introduction of the conditioning we used, since the efficiency is related to the conditioning. The method is illustrated with numerical results.

[a]Speaker

Describing the fluctuations in stochastic homogenisation

Jean-Christophe Mourrat [a] [1], Yu Gu, James Nolen, Felix Otto

[1] École normale supérieure de Lyon (ENS LYON) – École Normale Supérieure (ENS) - Lyon – 15 parvis René Descartes - BP 7000 69342 Lyon Cedex 07, France

Consider the solution of a divergence-form problem with random coefficients. Under suitable assumptions on the law of the coefficients, homogenisation theory ensures that as the correlation length of the random coefficients is sent to 0, this solution converges to the solution of a similar problem with constant, "homogenised" coefficients. The problem of quantifying the error in this convergence has witnessed tremendous progress recently. The goal of this talk is to explain how one can go beyond error bounds, and describe precisely the statistics of the fluctuations in terms of a finite number of new effective parameters. (Joint works with Yu Gu, James Nolen and Felix Otto.)

[a]Speaker

A regularity theory for elliptic systems with random coefficients

Stefan Neukamm [a] [1], Antoine Gloria, Felix Otto

[1] Technische Universität Dresden (TU Dresden) – TU Dresden 01062 Dresden, Germany

We study the regularity of elliptic systems with stationary and ergodic, random coefficients and consider the associated stochastic homogenization problem from a quantitative perspective. Following the philosophy of Avellaneda and Lin, developed in the context of deterministic, periodic homogenization, we lift the regularity theory of the homogenized limit to the heterogeneous situation and obtain $C^{1,\alpha}$ estimates on large scales, where "large" is quantified in terms of a minimal radius that measures the (sublinear) growth of the generalized corrector. In a purely deterministic part, we show that on length scales larger than the minimal radius, the equation features the same regularity properties as the constant-coefficient equation. In a second part, based on a quantification of ergodicity via a coarsened logarithmic Sobolev inequality (which allows for arbitrarily slow-decaying correlations), we derive stretched exponential moment bounds for the minimal radius. Finally, we apply the theory to stochastic homogenization and obtain various quantitative results, e.g. on the spatial growth of the corrector and the error of the two-scale expansion. The talk is based on a joint work with Antoine Gloria and Felix Otto.

[a] Speaker

MS24 Swarming: A. Bernoff, Ch. Topaz

A blob method for the aggregation equation and Gamma-convergence of regularized nonlocal interaction energies

Katy Craig [a] [1], Andrea Bertozzi, Ihsan Topaloglu

[1] Department of Mathematics [UCLA] (UCLA) – UCLA Mathematics Department Box 951555 Los Angeles, CA 90095-1555, United States

The aggregation equation models the motion of particles moving to minimize a nonlocal interaction energy. Often, the interaction between particles is chosen to scale according to a power law potential, leading to aggregation or repulsion, depending on the sign of the potential. In general, the corresponding interaction energies are neither convex nor differentiable, placing them outside the scope of most existing results on energy minimization and gradient flow In this talk, I will present joint work with Andrea Bertozzi on a new numerical method for the aggregation equation, inspired by vortex blob methods for the Euler equations. I will present quantitative results on the convergence of this regularized particle method, along with numerical examples exploring its qualitative behavior. I will then present recent work with Ihsan Topaloglu, in which we examine the effect of regularization on the nonlocal interaction energies corresponding to the aggregation equation and prove Gamma-convergence results showing that minimizers converge to minimizers and gradient flows converge to gradient flows.

[a] Speaker

Three-dimensional swarming states induced by hydrodynamic interactions

Maria D'Orsogna [a] [1]

[1] CSUN (CSUN) – 18111 Nordhoff Avenue Northridge CA, United States

Swarming patterns arising from self-propelled particles have been extensively studied, particularly in two-dimensions and in the absence of an embedding medium. We consider the dynamics of more realistic three dimensional self-propelled particles interacting in a fluid medium. The fluid interaction terms generated by direct short-ranged pairwise interactions may impart much longer-ranged hydrodynamic forces, effectively amplifying the coupling between individuals. We consider two limiting cases of fluid interactions, a "clear fluid" where particles have direct knowledge of their own velocity, that of others and of the fluid, and an "opaque fluid" where particles are able to determine their velocity only in relation to the surrounding fluid flow. We discuss emergent patterns that are unstable in fluid-free environments and that become stabilized by opaque fluid couplings such as rotating mills.

[a] Speaker

Anisotropic interactions in a first-order aggregation model

Razvan Fetecau [a] [1], Joep Evers [2], Lenya Ryzhik [3]

[1] Simon Fraser University – Canada
[2] Eindhoven University of Technology – Netherlands
[3] Stanford University – United States

We consider the following anisotropic extension of a well-studied first-order ODE model for collective behaviour:

$$\frac{dx_i}{dt} = v_i,$$
$$v_i = -\frac{1}{N} \sum_{j \neq i} \nabla_{x_i} K(|x_i - x_j|) w_{ij},$$

where x_i ($i = 1, \ldots, N$) denote the positions of N particles (individuals) in $\mathbb{R}^d$, K is an aggregation potential which incorporates inter-individual social interactions, and w_{ij} represent weights that model limitations in sensorial perception (e.g., limited field of vision). We assume that the weights w_{ij} depend on the direction of motion of individuals (i.e., on $v_i/|v_i|$) and consequently, the equations that determine the velocities v_i become *implicit*. This fact brings major new analytical issues, such as non-uniqueness and jump discontinuities in velocities. We study the well-posedness of the anisotropic model and discuss its modes of breakdown. To extend solutions beyond breakdown we propose a relaxation system containing a small parameter ε, which can be interpreted as a small amount of inertia or response time. We show that the limit $\varepsilon \to 0$ can be used as a jump criterion to select the physically correct velocities. We illustrate the results with numerical simulations in two dimensions.

[a]Speaker

Exact and asymptotic steady states of aggregation equations with power-law potentials

Yanghong Huang [a] [1], Carrillo Jose [1]

[1] Imperial College London – United Kingdom

Despite the rich steady patterns observed in nonlocal interaction models, there are very few steady solutions with explicit closed expressions. In this talk, several exact solutions are presented for the aggregation equation with power-law potential, using explicit inverse of some singular integral operators. The asymptotic behaviours near the boundary are also discussed.

[a]Speaker

The dynamics of synchronous human clapping: from experiment to model

Ryan Lukeman [a] [1]

[1] St. Francis Xavier University (CANADA) – Canada

The emergence of collective synchrony among interacting units such as people, cells, or animals has been a well-studied phenomenon. Coupled oscillator models are often used to study how synchrony emerges in these instances, yet interesting dynamics can develop in real systems after synchrony is established. In this work, we study a specific example of post-synchrony dynamics via groups of people clapping in unison. By collecting audio data on individuals and groups (including groups of over 100 people) attempting to maintain a collective rhythm, the evolution of group frequency, period, and level of synchrony are presented. These metrics form the blueprint for construction of an integrate-and-fire type model to capture the essential dynamics of the experimental observations, to pinpoint what aural interactions might govern such collective behaviour.

[a]Speaker

Driving and Response in Insect Swarms

Nicholas Ouellette [a] [1]

[1] Department of Mechanical Engineering and Materials Science, Yale University – United States

Aggregations of social animals, such as flocks of birds, schools of fish, or swarms of insects, are beautiful, natural examples of self-organized behavior far from equilibrium. Because so many of these systems display large-scale ordered patterns and because quantitative data for real animals is still relatively uncommon, it has become the norm in modeling animal aggregations to focus on this order. Large-scale patterns alone, however, are not sufficient information to characterize an animal aggregation fully, and do not provide stringent enough conditions to benchmark models. We have therefore developed methods to drive a particular example of collective behavior—laboratory mating swarms of the non-biting midge Chironomus riparius—and measure the response at both the individual and the collective level. Our results allow us to begin to characterize the swarms in term of state variables and response functions, and point towards a more specific way to describe the collective behavior than simply the overall pattern.

[a]Speaker

Contagion Shocks in One Dimension

Martin Short [a] [1], Li Wang [2], Jesus Rosado [3], Andrea Bertozzi [2]

[1] Georgia Institute of Technology – United States
[2] UCLA – United States
[3] Universidad de Buenos Aires – Argentina

We consider an agent-based model of emotional contagion coupled with motion in one dimension that has recently been studied in the computer science community. The model involves movement with speed proportional to a "fear' variable that undergoes a temporal consensus averaging with other nearby agents. We study the effect of Riemann initial data for this problem, leading to shock dynamics that are studied both within the agent-based model as ODEs as well as in a continuum limit as PDEs. We examine the model under distinguished limits as the characteristic contagion interaction distance and the interaction timescale both approach zero. Here, we observe a threshold for the interaction distance vs. interaction timescale that produces qualitatively different behavior for the system - in one case particle paths do not cross and there is a natural Eulerian limit involving nonlocal interactions and in the other case particle paths can cross and one may consider only a kinetic model in the continuum limit. Time permitting, we will also discuss recent extensions of the model to two dimensions.

[a]Speaker

Topological data analysis of biological aggregation models

Chad Topaz [a] [1], Lori Ziegelmeier [1], Tom Halverson [1]

[1] Macalester College – Saint Paul, MN 55105, United States

We use topological data analysis to characterize global dynamics of the biological aggregation models of Vicsek et al. (1995) and D'Orsogna et al. (2006). Numerical position and velocity data from simulations are construed as point clouds varying over time. Using methods from persistent homology, we measure topological features such as connected components and circles that persist over multiple spatial scales. The topological analysis detects dynamical events that classical order parameters do not.

[a]Speaker

MS25 Topological Methods in Differential Equations: L. Malaguti, V. Obukhovskii

On non compact fractional order differential inclusions with generalized boundary conditions and impulses

Irene Benedetti [a] [1], Valeri Obukhovskii [2], Valentina Taddei [3]

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Fractional calculus deals with generalizations of ordinary integral and differential operators. Fractional derivative and integrals, as non-integer derivatives and integrals, give more flexibility helping to model real-life problems. In particular, they all are very useful at describing the anomalous phenomena, providing an excellent tool for the description of memory and hereditary properties of various materials and processes. On the other hand, impulses take into account parameters subjected to short-term perturbations in time, as is the case in various processes in physics, population dynamics, biotechnology and economics.

In this talk I will show existence results for a fractional differential inclusion with non local conditions and impulses in a reflexive Banach space. I will apply a technique based on weak topology to avoid any kind of compactness assumptions on the nonlinear term. These arguments are motivated by an application to a parabolic differential equation with the nonlinearity depending on an integral term.

This is a joint work with Valeri Obukhovskii and Valentina Taddei.

[a]Speaker

Dirichlet problem in billiard spaces

Grzegorz Gabor [a] [1]

[1] Nicolaus Copernicus University, Faculty of Mathematics and Computer Science (UMK) – Poland

An elementary observation is that the dynamical system of a billiard type with a uniform motion can be modeled by the simple impulsive second-order system

$$\begin{cases} \ddot{x}(t) = 0, & \text{for } t \geq 0, \\ \dot{x}(s+) = \dot{x}(s) + I(x(s), \dot{x}(s)), & \text{if } x(s) \in \partial K, \end{cases} \tag{3}$$

where $K = \overline{intK} \subset \mathbb{R}^n$ is a compact subset, and I is an impulse function describing the impact law. If we have a crocked table the acceleration becomes nontrivial and the problem becomes nonlinear and complicated.

We shall discuss the constrained Dirichlet boundary value problem

$$\begin{cases} \ddot{x}(t) = f(t, x(t)), & \text{for } t \in [0,T], x(t) \in int\, K, \\ \dot{x}(s+) = \dot{x}(s) + I(x(s), \dot{x}(s)), & \text{if } x(s) \in \partial K, \\ x(0) = x(T) = 0, \end{cases} \tag{4}$$

where I describes the impact law of the equality of the angle of incidence and angle of reflection and the equality of a length of the velocity vector before and after the impact. The results presented in the talk develop the research on impulsive Dirichlet problems with state-dependent impulses. The main part will be devoted to both the existence and multiplicity results in one dimensional billiards ($K = [-r, r]$). Several observations concerning the multidimensional case and important open problems will also be given.

[a] Speaker

Geometric inverse problem for non-stationary Stokes system

Rakia Malek [a] [1], Maatoug Hassine [1]

[1] Faculté des Sciences de Monastir (FSM) – Université de Monastir Rue Salem BCHIR Skanes 5000 Monastir Tunisie, Tunisia

In this work we focus on an inverse problem related to the non-stationary Stokes system. It comes from the detection of an object immersed in a fuid from boundary measurements. We propose an alternative approach based on the Kohn-Vogelius formulation and the topological gradient method. The inverse problem is formulated as a topology optimization one. The topological sensitivity analysis method gives the variation of a criterion with respect to the creation of a small hole in the domain. In the numerical part, we propose a one-shot reconstruction algorithm and we present some numerical results, showing the efficiency and accuracy of our approach.

[a] Speaker

On Some New Applications of the Guiding Functions Method

Valeri Obukhovskii [a] [1]

[1] Voronezh State Pedagogical University (VSPU) – (394 068) Voronezh, Russia

In this talk, we consider applications of the guiding functions method (see, e.g., [1]) to the study of qualitative behavior of solutions of differential equations and inclusions and trajectories of nonlinear control systems. In particular, the notion of an A-bifurcation for a system of differential equations in a Hilbert space is described and the existence and uniqueness of an A-bifurcation point is presented. The abstract result is applied to the study of the global structure of the solution set of a feedback control system governed by integro-differential equations. A multiparameter global bifurcation problem for periodic solutions of operator-differential inclusions whose multivalued parts are not necessarily convex-valued is described. Some results concerning the asymptotic behavior of solutions of differential and functional differential inclusions are considered. References. [1] V. Obukhovskii, P. Zecca, N.V. Loi, and S. Kornev, Method of Guiding Functions in Problems of Nonlinear Analysis, Lecture Notes in Math. 2076, Springer, Berlin – Heidelberg, 2013.

[a] Speaker

Nonlocal problem in Hilbert spaces

Valentina Taddei [a] [1]

[1] Department of Physics, Informatics and Mathematics, University of Modena and Reggio Emilia – I-41125, Italy

We consider nonlinear differential inclusions both of first and second order in a separable Hilbert space compactly embedded in a Banach space. A wide family of nonlocal boundary value problems is treated, including periodic, anti-periodic, mean value and multipoint conditions. We give existence results based on an approximation solvability method and on a Scorza Dragoni-type result for multivalued maps. We make use of a continuation principle in a finite dimensional setting, embedding the problem into a family of linearized problems depending on a parameter. The transversality condition is strictly localized on the boundary of a suitable open set. In the second order case we assume suitable Nagumo conditions. We conclude the talk showing some applications to integro-differential equations arising from a nonlocal dispersal model or from the telegraph equation. The work is in collaboration with I. Benedetti (Perugia), N.V. Loi (Petrovietnam) and L. Malaguti (Modena and Reggio Emilia).

[a] Speaker

Delay evolution equations with measures and nonlocal initial conditions

Ioan Vrabie [a] [1], Irene Benedetti [2], Luisa Malaguti [3], Valentina Taddei [4]

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We consider the semilinear delay evolution equation with measures and nonlocal initial data of the form

$$\begin{cases} du(t) = \{Au(t) + f(t, u_t)\}dt + dg(t), & t \in \mathbb{R}_+, \\ u(t) = h(u)(t), & t \in [\,-\tau, 0\,], \end{cases}$$

where $\tau \geq 0$, $A : D(A) \subseteq X \to X$ is the infinitesimal generator of a C_0-semigroup, $f : \mathbb{R}_+ \times \mathcal{R}([\,-\tau, 0\,]; X) \to X$ is continuous, $g \in BV_{\mathrm{loc}}(\mathbb{R}_+; X)$ and $h : \mathcal{R}_b(\mathbb{R}_+; X) \to \mathcal{R}([\,-\tau, 0\,]; X)$ is nonexpansive, and we prove an existence result for $\mathcal{L}^\infty$-solutions. Here $\mathcal{R}_b(\mathbb{R}_+; X)$ stands for the space of bounded and piecewise continuous functions from $\mathbb{R}_+$ to X and $\mathcal{R}([\,-\tau, 0\,]; X)$ denotes the space of piecewise continuous functions from $\mathbb{R}_+$ to X, both endowed with the sup-norm.

[a] Speaker

Stability of semilinear parabolic systems in L_2 and $W^{1,2}$

Martin Väth [a] [1], Pavel Gurevich [2]

[1] Institute of Mathematics of the Academy of Sciences of the Czech Republic – Žitná 25 115 67 Prague 1, Czech Republic
[2] Freie University Berlin – Kaiserswerther Straße 16-18, 14195 Berlin, Allemagne, Germany

Starting from the "naive" question how to prove stability of semilinear parabolic systems in the spaces L_2 and $W^{1,2}$ by means of linearization, one is led to two different "natural" approaches. The aim of the talk is to compare both approaches and to reveal some connections, e.g. with Kato's square root problem.

[a] Speaker

The generalized version of the Miranda theorem with applications

Kryszewski Wojciech [a] [1]

[1] Nicolaus Copernicus University – Poland

In the talk I will give a survey on some recent developments of the celebrated Miranda theorem and its applications in the theory of differential equations and inclusions. We shall discuss global as well as local aspects of this result, along with some topological tools (such as the fixed point index and the topological degree) allowing to study maps defined on closed subsets of a Banach space with values in the space and detect their zeros or fixed points. The existence of stationary solutions to some elliptic problems and periodic solutions of the corresponding parobolic problems under the presence of state constraints will be presented.

[a]Speaker

Nonlocal solutions of hyperbolic type equations

Kentarou Yoshii [a] [1]

[1] Tokyo University of Science (TUS) – 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan

In this talk we consider existence and uniqueness of (classical) solutions to abstract nonlocal Cauchy problems for nonlinear evolution equations of the form

$$\begin{cases} \dfrac{d}{dt}u(t) + A(t)u(t) = f(t) + C(t, K(t)u)g(t) & \text{for } t \in I := [0, T], \\ u(0) = u_0 + Mu. \end{cases} \tag{P}$$

Here $\{A(t); t \in I\}$ is a family of closed linear operators in the complex Hilbert space X, $K(t) : C(I; Y) \to \mathbb{C}$ is linear and bounded for all $t \in I$, $K(\cdot)$ is continuous on I and $M : C(I; Y) \to Y$ is a bounded linear operator, where Y is a subspace of X. By [OY], we can show that $A(t)$ has a unique evolution operator. And hence, by Schauder Tychonoff's fixed point theorem, we are able to solve problem (P). The result will appear in [MY].Typical examples of M and $K(t)$ and applications to Schrödinger equations will be discussed.

[MY] L. Malaguti and K. Yoshii, *Nonlocal solutions of hyperbolic type equations and their controllability*, in preparation.

[OY] N. Okazawa and K. Yoshii, *Linear Schrödinger evolution equations with moving Coulomb singularities*, J. Differential Equations **254 (2013), 2964–2999.**

[a]Speaker

MS26 Water Waves: D. Lannes, E. Wahlén

Control of water waves

Thomas Alazard [a] [1]

[1] Département de Mathématiques et Applications (DMA) – CNRS : UMR8553, École normale supérieure [ENS] - Paris – France

Water waves are disturbances of the free surface of a liquid. They are, in general, produced by the immersion of a solid body or by impulsive pressures applied on the free surface. The question we discuss in this talk is the following: which waves can be generated by blowing on a localized portion of the free surface. Our main result asserts that one can generate any small amplitude, periodic in x, two-dimensional, gravity-capillary water waves. This is a result from control theory. More precisely, we prove the local exact controllability of the incompressible Euler equation with free surface. This is a joint work with Pietro Baldi and Daniel Han-Kwan.

[a] Speaker

Splash singularities for the free boundary Navier-Stokes equations

Angel Castro [a] [1]

[1] Universidad Autonoma de Madrid [Madrid] (UAM) – Madrid, Spain

In this talk we will present our recent work on establishing the existence of splash singularities for the free boundary Navier-Stokes equations. The existence of that kind of singularities have been proved for the water waves problem previously. We will show that the viscosity can not prevent the formation of a splash. This is a joint work with D. Córdoba, C. Fefferman, F. Gancedo and J. Gómez-Serrano.

[a] Speaker

The small density contrast limit for multilayer shallow water systems

Vincent Duchene [a] [1]

[1] Institut de Recherche Mathématique de Rennes (IRMAR) – Universite de Rennes 1, CNRS : UMR6625 – France

We will discuss the behaviour of the inviscid multilayer Saint-Venant (or shallow water) system in the limit of small density contrast. We will see that, under reasonable hyperbolicity conditions on the flow, the system is well-posed on a time interval independent of the small density contrast parameter, and that the solutions converge towards solutions of the rigid-lid system under the Boussinesq approximation. The asymptotic behaviour is similar to that of the incompressible limit for Euler equations, in the sense that there exists a small initial layer in time for ill-prepared initial data, accounting for rapidly propagating “acoustic” waves (here, the so-called barotropic mode) which interact only weakly with the “incompressible” component (here, baroclinic).

[a] Speaker

Validity of the KdV and the NLS approximation for the water wave problem

Wolf-Patrick Duell [a] [1]

[1] Universitaet Stuttgart – Germany

Many mathematical models for hydrodynamic problems are so complicated that a qualitative understanding of the solutions to the full problems usable for practical applications does not seem within reach for the near future, neither analytically nor numerically. Therefore, it is reasonable to approximate these models in various parameter regimes by appropriate reduced models whose qualitative properties are more easily accessible. To understand to which extent these reduced models yield correct predictions of the behavior of the original problems it is important to justify the validity of these approximations by estimates of the approximation errors in the typical length and time scales.
In this talk, we discuss mathematically rigorous justifications of the approximations of the water wave equations by the Korteweg-de Vries equation and the Nonlinear Schrödinger equation.

[a] Speaker

On Whitham's conjecture of a highest cusped wave for a nonlocal shallow water wave equation

Mats Ehrnstrom [a] [1]

[1] Norwegian University of Science and Technology (NTNU) – Alfred Getz vei 1 NO-7491 Trondheim NORWAY, Norway

We consider the Whitham equation $u_t + 2uu_x + Lu_x = 0$, where L is the non-local (Fourier multiplier) operator given by the symbol $m(\xi) = (\frac{\tanh(\xi)}{\xi})^{1/2}$. G. B. Whitham conjectured that for this equation there would be a highest, cusped, travelling-wave solution. We find this wave as a limiting case at the end of the main bifurcation curve of P-periodic solutions, and prove that it belongs to the Hölder space C^α for all $\alpha < 1/2$, but to no Hölder space C^α with $\alpha > 1/2$. Further properties of the wave, and of traveling-wave solutions of the Whitham equation in general, are given. An essential part of the proof consists in a precise analysis of the integral kernel corresponding to the symbol m.

[a] Speaker

Spatial dynamics methods for axisymmetric solitary waves on a ferrofluid jet

Mark Groves [a] [1]

[1] Universität des Saarlandes – Universität des Saarlandes Campus D-66123 Saarbrücken, Germany

We consider the irrotational flow of an incompressible, inviscid ferrofluid of constant density surrounding a metal wire in a vacuum, and examine whether the magnetic force in the ferrofluid induced by a constant current flowing in the wire can, together with surface tension, support axisymmetric solitary waves on its free surface. The hydrodynamic problem is formulated as an infinite-dimensional Hamiltonian system in which the longitudinal spatial direction is the time-like variable. A centre-manifold reduction technique is employed to reduce the system to a locally equivalent Hamiltonian system with a finite number of degrees of freedom. Homoclinic solutions to the reduced system, which correspond to axisymmetric solitary waves, are detected by a variety of dynamical systems methods.

[a]Speaker

Incompressible limit for the free surface Navier-Stokes equations

Frédéric Rousset [a] [1]

[1] Université Paris-Sud Orsay – université paris-sud Orsay – France

We shall discuss the incompressible limit of the free surface Navier Stokes equations. in particular, we will explain how to get estimates of the solution in strong norms uniform with respect to the Mach number. (Joint with Nader Masmoudi)

[a]Speaker

Global bifurcation of steady gravity water waves with critical layers

Eugen Varvaruca [a] [1]

[1] University of Reading – United Kingdom

I will present some recent results on the problem of two-dimensional travelling water waves propagating under the influence of gravity in a flow of constant vorticity over a flat bed. By means of a conformal mapping and an application of Riemann-Hilbert theory, the free-boundary problem is equivalently reformulated as a one-dimensional pseudodifferential equation which involves a modified Hilbert transform and, moreover, has a variational structure. Using the new formulation, existence is established, by means of real-analytic global bifurcation theory, of a family of solutions which includes waves of large amplitude, even in the presence of critical layers in the flow. This is joint work with Adrian Constantin and Walter Strauss.

[a]Speaker

Contributed talks

Galoisian Analysis of Schrödinger Equations with Generalized Lennard-Jones Potentials

Primitivo Acosta-Humanez [a] [1]

[1] Universidad del Atlantico (Uniatlantico) – KM 7 via Puerto Colombia, Barranquilla, Colombia

In this talk it will be presented a proof of non-integrability, through Liouvillian functions, of Schrödinger equations with Lennard-Potential $6 - 12$ using differential Galois theory. A generalization of Lennard-Jones potentials, given by

$$V(r) = -\frac{A}{r^m} + \frac{B}{r^{2m-2}},$$

is analysed with differential Galois theory to obtain integrability of the Schrödinger equations with these potentials. In particular, we obtain Liouvillian solutions whenever $m = 6$, which is very closed to the classical Lennard-Jones potentials.

[a] Speaker

Elliptic regularity theory applied to time harmonic Maxwell's equations

Giovanni S. Alberti [a] [1], Yves Capdeboscq [2]

[1] École normale supérieure – Département de mathématiques et applications – France
[2] University of Oxford – United Kingdom

In this talk I will show how the L^p theory for elliptic equations can be applied to study the regularity of solutions to time harmonic Maxwell's equations with anisotropic complex coefficients. In particular, the solutions are Hölder continuous provided that the coefficients are $W^{1,p}$ for some $p > 3$. This improves existing regularity estimates, where the minimum assumption was the Lipschitz continuity of the coefficients. Moreover, I shall show that this approach can be easily extended to the case with bi-anisotropic materials.

[a] Speaker

Existence of solitary-wave solutions to nonlocal equations

Mathias Nikolai Arnesen [a] [1]

[1] Department of Mathematical Sciences, Norwegian University of Science and Technology (NTNU) – 7491 Trondheim, Norway

We prove existence and conditional energetic stability of solitary wave solutions for the two classes of pseudodifferential equations $u_t + (f(u))_x - (Lu)_x = 0$ and $u_t + (f(u))_x + (Lu)_t = 0$, where f is a nonlinear term, typically of the form $c|u|^p$ or $cu|u|^{p-1}$, and L is a Fourier multiplier operator of positive order. The former class includes for instance the Whitham equation with capillary effects and the generalized Korteweg–de Vries equation, and the latter the Benjamin–Bona–Mahony equation. Existence and conditional energetic stability results have earlier been established using the method of concentration–compactness for a class of operators with symbol of order $s \geq 1$. We extend these results to symbols of order $0 < s < 1$, thereby improving upon the results for general operators with symbol of order $s \geq 1$ by enlarging both the class of linear operators and nonlinearities admitting existence of solitary waves. Instead of using abstract operator theory, the new results are obtained by direct calculations involving the nonlocal operator L, something that gives us the bounds and estimates needed for the method of concentration–compactness.

[a] Speaker

Rough flows and homogenization in fast-slow dynamics

Ismael Bailleul [a] [1], Sebastian Riedel [2], Rémi Catellier [3]

[1] IRMAR – université Rennes 1 – France
[2] TU Berlin – Germany
[3] Université Rennes 1 (IRMAR) – université Rennes 1 – France

A general machinery for constructing flows from approximate flows has recently been introduced in the work "Flows driven by rough paths", where it was used to give a simple approach to the core results on rough differential equations. This framework is wider though, and allows to extend the standard theory of stochastic flows beyond the setting of Itô-Stratonovich integration theory. This requires to introduce a setting in which (random) velocity fields are lifted to enriched objects, in the same spirit as rough paths theory. The integration theory of these objects is simple in the above-mentioned setting of approximate flows and shares the same benefits as Lyons' theory : the continuity of the solution map. This continuity result is the key step to get a quick access to interesting homognization results for fast-slow dynamics, by lifting these systems into the setting of rough flows, and by combining deterministic and probabilistic arguments.

[a] Speaker

Harmonic perturbations with delay of periodic separated variables differential equations

Luca Bisconti [a] [1], Marco Spadini [1]

[1] Dipartimento di Matematica e Informatica "U. Dini", Università degli Studi di Firenze – Viale Morgagni, 67/a - 50134 - Firenze, Italy

In this talk we present recent results jointly obtained with Marco Spadini (University of Florence). We investigated the set of harmonic solutions of T-periodic perturbations of T-periodic separated variables ODEs on manifolds, allowing the perturbing term to contain a finite delay. Namely, given $T > 0$ and a boundaryless smooth manifold $N \subseteq \mathbb{R}^d$, we consider T-periodic solutions of equations of the form

$$\dot{\zeta}(t) = a(t)\Phi(\zeta(t)) + \lambda\Xi\big(t, \zeta(t), \zeta(t-r)\big), \quad \lambda \geq 0, \tag{5}$$

where $r > 0$ is a finite time lag, $a : \mathbb{R} \to \mathbb{R}$ is continuous and T-periodic, $\Phi : N \to \mathbb{R}^d$ and $\Xi : \mathbb{R} \times N \times N \to \mathbb{R}^d$ are given continuous *tangent vector fields* on N, in the sense that $\Phi(\xi)$ belongs to the tangent space $T_\xi N$, for any $\xi \in N$, and Ξ is T-periodic in the first variable and tangent to N in the second one, i.e. $\Xi(t, \xi, \eta) = \Xi(t + T, \xi, \eta) \in T_\xi N$, $(t, \xi, \eta) \in \mathbb{R} \times N \times N$. We assume that the average $\bar{a}$ of a on $[0, T]$ is nonzero, that is: $\bar{a} := \frac{1}{T}\int_0^T a(t)dt \neq 0$.
By applying degree-theoretic methods we obtain a global continuation result for the T-periodic solutions of (5) and we provide sufficient conditions for the existence of branches of T-periodic solutions.

[a]Speaker

Convergence of a Spectral Method for Improved Boussinesq Equation

Handan Borluk [a] [1], Gulcin Muslu [2]

[1] Istanbul Kemerburgaz University – Turkey
[2] Istanbul Technical University – Turkey

In [1], Bogolubsky showed that the bad Boussinesq equation is unstable under short wave perturbation and then he proposed that the bad Boussinesq equation could be approximated by the improved Boussinesq (IBq) equation. The IBq equation has been widely studied over the last couple of decades both numerically and analytically. In this study, we proved the convergence of the scheme which is a combination of a spectral and finite difference methods.
This work has been supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under the project MFAG-113F114. [1] I. L. Bogolubsky, "Some examples of inelastic soliton interaction", Comput. Phys. Commun. 13, (1977), 149-155.

[a]Speaker

Bistable reaction-diffusion equations and propagation properties in unbounded domains

Juliette Bouhours [a] [1], Henri Berestycki[2], Guillemette Chapuisat[3]

[1] Mathematical and statistical sciences, University of Alberta, Canada – 632 CAB University of Alberta Edmonton, Alberta T6G 2G1 CANADA, Canada
[2] Centre d'analyse et de mathématique sociale (CAMS) – CNRS : UMR8557, Ecole des Hautes Etudes en Sciences Sociales (EHESS) – 54 boulevard Raspail 75006 Paris, France
[3] Centre de Mathématiques et Informatique [Marseille] (CMI) – Université de Provence - Aix-Marseille I – 39 rue Frédéric Joliot-Curie 13453 Marseille cedex, France

We consider a bistable reaction-diffusion equation in unbounded domains and we investigate the existence of propagation phenomena, possibly partial, in some direction or, on the contrary, of blocking phenomena. This problem is motivated by the question of propagation mechanisms in several biological systems.
We will start by describing the biological motivations behind our mathematical model. Then we will present our main results on propagation and blocking, illustrating them with biological interpretations and numerical simulations.

We start by proving the well-posedness of the problem. Then we prove that when the domain has a decreasing cross section with respect to the direction of propagation there is complete propagation. Further, we prove that a wave can be blocked as it comes to an abrupt geometry change. Finally we discuss various general geometrical properties that ensure either partial or complete invasion by 1. In particular, we show that in a domain that is "star-shaped" with respect to an axis, there is complete invasion by 1.

This a work in collaboration with Henri Berestycki and Guillemette Chapuisat.

[a]Speaker

Non-Newtonian fluids under random influences

Dominic Breit [a] [1]

[1] Heriot-Watt University (UNITED KINGDOM) – Riccarton Edinburgh EH14 4AS UK, United Kingdom

We present new results for generalized Navier-Stokes equations which are perturbed by a Bronian motion. First we aderss existence of martingale weak solutions to the SPDE based on a stochastic pressure decomposition and L^∞-truncation.
For slow flows (i.e. if the convective term is neglected) we establish regularity results and study space-time discretizations.
We show that the approximation has convergence rate one w.r.t. the space discretization and 1/2 w.r.t. to the discretization in time.

[a]Speaker

Optimal regularity and long-time behavior of solutions for the Blackstock-Crighton equation

Rainer Brunnhuber [a] [1], Stefan Meyer [2]

[1] Universität Klagenfurt – Universitätsstraße 65-67, 9020 Klagenfurt, Austria
[2] Martin Luther Universität Halle-Wittenberg – Theodor-Lieser-Straße 5, 06120 Halle (Saale), Germany

We are going to consider the non-homogeneous Dirichlet boundary value problem

$$\begin{cases} (a\Delta - \partial_t)(u_{tt} - b\Delta u_t - c^2\Delta u) = (k(u_t)^2 + |\nabla u|^2)_{tt} & \text{in } \mathbb{R}_+ \times \Omega, \\ (u, u_t, u_{tt}) = (u_0, u_1, u_2) & \text{on } \{t = 0\} \times \Omega, \\ (u, \Delta u) = (g, h) & \text{on } \mathbb{R}_+ \times \Gamma, \end{cases}$$

which is motivated from the Blackstock–Crighton equation modeling the propagation of finite-amplitude sound in thermoviscous fluids. The spatial domain $\Omega \subset \mathbb{R}^n$ is assumed to have smooth boundary Γ. Moreover, $u_0, u_1, u_2 \colon \Omega \to \mathbb{R}$ and $g, h \colon J \times \Gamma \to \mathbb{R}$ are given, $u = u(t, x)$ is the unknown, and a, b, c, k are positive constants.
We show that, for small initial and boundary data, there exists a unique global solution with optimal L_p-regularity which converges to zero at an exponential rate as time tends to infinity. Our techniques are based on maximal L_p-regularity for parabolic problems and the implicit function theorem.
Additionally, we will provide a short outlook on how to treat the case of non-homogeneous Neumann boundary conditions which are relevant for applications of high intensity focused ultrasound in a medical context.

[a]Speaker

SDEs with distributional drift and Polymer measure

Giuseppe Cannizzaro [a] [1]

[1] Technische Universität Berlin [Berlin] (TUB) – Straße des 17. Juni 135 10623 Berlin, Germany

We study existence and uniqueness of solution for stochastic differential equations with distributional drift by giving a meaning to the Strook-Varadhan martingale problem associated to such equations. The approach we exploit is the one of paracontrolled distributions recently introduced by Gubinelli, Imkeller and Perkowski in their by now celebrated paper "Paracontrolled Distributions and Singular PDEs". As a result, we make sense of the two and three dimensional polymer measure. Joint work with Khalil Chouk

[a]Speaker

Pulses with oscillatory tails in the FitzHugh-Nagumo system

Paul Carter [a] [1], Bjorn Sandstede [1]

[1] Brown University – United States

Numerical studies indicate that the FitzHugh-Nagumo system exhibits stable traveling pulse solutions with oscillatory tails. We discuss an analytical result regarding the existence of such pulses using geometric blow up techniques and singular perturbation theory. We also describe numerical results regarding the stability of the pulses, and we propose a mechanism that explains the transition from single to double pulses that was observed in earlier numerical studies.

[a]Speaker

Weak stability for the 2D plasma-vacuum interface problem

Davide Catania [a] [1,2]

[1] eCampus University – Italy
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We consider the free boundary problem for the plasma-vacuum interface in 2D ideal compressible magnetohydrodynamics (MHD). In the plasma region, the flow is governed by the usual compressible MHD equations, while in the vacuum region we consider the Maxwell system for the electric and the magnetic fields, in order to investigate the linear stability of the problem, in particular in relation with the electric field in vacuum. At the free interface, driven by the plasma velocity, the total pressure is continuous and the magnetic field on both sides is tangent to the boundary.

Under suitable conditions satisfied at each point of the plasma-vacuum interface, we prove an energy estimate for the linearized boundary value problem with a loss of derivatives, due to the failure of the uniform Kreiss–Lopatinskii condition. The proof follows by an analysis of the Lopatinskii determinant and the construction of a suitable symmetrizer.

These results have been obtained in a joint work with Marcello d'Abbicco and Paolo Secchi.

[a]Speaker

Fractional operators with singular drift: Smoothing properties and Morrey-Campanato spaces

Diego Chamorro [a] [1], Stéphane Menozzi [1]

[1] Laboratoire de Mathématiques et Modélisation d'Evry UMR CNRS 8071 (LaMME) – Université d'Evry-Val d'Essonne – 23 Boulevard de France 91037 Evry Cedex, France

We investigate some smoothness properties for the following equation involving a class of Lévy type operators with singular divergence-free drift:

$$\begin{cases} \partial_t\theta(t,x) + \nabla\cdot(v\,\theta)(t,x) + \mathcal{L}\theta(t,x) = 0, \\ \theta(0,x) = \theta_0(x), \qquad \text{for } x\in\mathbb{R}^n,\ n\geq 2, \\ \text{with } \nabla\cdot(v) = 0 \text{ and } t\in[0,T]. \end{cases}$$

The operator $\mathcal{L}$ is given by the expression $\mathcal{L}(f)(x) = \text{v.p.}\int_{\mathbb{R}^n}\big[f(x)-f(x-y)\big]\pi(y)dy$, where $\pi(y)dy$ is a Lévy measure.
The underlying motivation of this framework is given by equations from fluid dynamics. Our argument is based on a duality method using the molecular decomposition of Hardy spaces through which we derive some Hölder continuity. This property will be fulfilled as far as the drift v belongs to a suitable Morrey-Campanato space $M^{q,a}$ for which the regularizing properties of the Lévy operator suffice to obtain global Hölder continuity.

[a]Speaker

The Weyl Criterion for the Spectrum

Nelia Charalambous [a] [1]

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The essential spectrum of the Laplacian on functions over complete noncompact manifolds has been extensively studied. It is known that on hyperbolic space a spectral gap appears, whereas is has been conjectured that on manifolds with uniformly subexponential volume growth and Ricci curvature bounded below the essential spectrum is the nonnegative real line.
In our work with Zhiqin Lu we prove a generalization of Weyl's criterion for the spectrum of a self-adjoint and nonnegative operator on a Hilbert space. We then apply this generalized criterion to expand the set of manifolds over which the essential spectrum of the Laplacian on functions is the nonnegative real line. We also use our criterion to compute the essential spectrum of complete shrinking Ricci solitons and weighted manifolds. Finally, we apply our criterion to study the spectrum of the Laplacian on k-forms under a continuous deformation of the metric. The results that we obtain allow us to compute the essential spectrum of the Laplacian on k-forms for an asymptotically flat manifold.

[a]Speaker

Validated Function Calculus for the Rigorous Solution of Differential Equations

Pieter Collins [a] [1]

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A *validated function calculus* is a numerical method for manipulating functions as first-class objects in a rigorous way. It provides a powerful extension of interval methods (for rigorous computation with numbers) allowing computed solutions of the *flow* $\phi(x_0, t)$ of a differential equation to be used in further analysis, such as reachability tests or computation of periodic orbits. A validated function calculus based on *Taylor models* is used in Berz & Makino's tool COSY Infinity (1998), and an abstract, extensible function calculus is implemented in the tool Ariadne (C. et al, 2005–) for rigorous numerics and verification of hybrid systems.
In this talk I will briefly explain the basic ideas of validated function calculus, giving examples of function calculi based on polynomials and Fourier series, and demonstrate their use in the study of ordinary differential equations, including approaches based on the Picard operator and Taylor series. I will demonstrate the solution of delay differential equations, and composite systems via modular decomposition, and outline a method for the analysis of reduced-order systems, which requires the solution of differential inclusions (Zivanovic & C., 2010). Finally, I will describe challenges for further research in the rigorous solution of differential equations, including stiff systems with multiple time-scales, and the simplification problem for enclosure sets.

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[a]Speaker

On strong cosmic censorship with a cosmological constant.

João L. Costa [a] [1]

[1] Lisbon University Institute - ISCTE / CAMGSD - ULisboa (ISCTE/CAMGSD) – Portugal

Motivated by the Strong Cosmic Censorship Conjecture (SCCC) we consider the problem of global uniqueness for the Einstein-Maxwell-scalar field system with a cosmological constant, for spherically symmetry characteristic initial data.
First we consider the situation where the outgoing data is stationary (i.e., prescribed by a subextremal Reissner Nordstroem black hole event horizon) and the remaining data is otherwise free. In that case, one can find an open set of free data for which it is possible to construct regular extensions of the maximal (globally hyperbolic) development. This provides indirect evidence for the failure of the SCCC in the case of a positive cosmological constant.
To go from indirect evidence to results applying unequivocally to the conjecture at hand we present some preliminary results concerning the case where the outgoing data, instead of stationary, satisfies Price's law.
This is joint work with: P. Girão, J. Natário and J. D. Silva.

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[a]Speaker

Some weighted estimates for elliptic operators with singular data

Roberta D' Ambrosio [a] [1], Loredana Caso [1]

[1] Department of Mathematics, University of Salerno – Italy

Let Ω be an open subset of $\mathbb{R}^n$, $n \geq 2$, not necessarily bounded or regular. Consider in Ω the elliptic operator

$$L = \sum_{i,j=1}^{n} a_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{n} a_i \frac{\partial}{\partial x_i} + a \,,$$

where the coefficients a_{ij} are locally VMO while the data a_i, a belong to a weighted Sobolev space whose weight σ is a function of distance type from a nonempty subset of $\partial\Omega$. We prove that if $s \in \mathbb{R}$, $p \in]n/2, +\infty[$ and u is a solution of the problem

$$\begin{cases} u \in W^{2,p}_{loc}(\Omega) \\ Lu \geq f \quad f \in L^p_{loc}(\Omega) \\ \limsup\limits_{x \to x_o} \sigma^s(x)\, u(x) \leq 0 \qquad \forall\, x_o \in \partial\Omega \\ \limsup\limits_{|x| \to +\infty} \sigma^s(x)\, u(x) \leq 0 \quad \text{if } \Omega \text{ is unbounded,} \end{cases}$$

then u verifies the following Aleksandrov type estimate

$$\sup_{x \in \Omega} \sigma^s(x) u(x) \leq c \Big(\frac{1}{|B|} \int_B |\sigma^{s+2} f^-|^p \, dx \Big)^{\frac{1}{p}} \,,$$

where $B \subset\subset \Omega$ is an open ball, f^- is the negative part of f and $c \in \mathbb{R}_+$ is independent of u. As a consequence, some uniqueness results for singular elliptic problems are obtained.

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[a] Speaker

Hypocoercivity for a linearized multi-species Boltzmann system

Esther S. Daus [a] [1]

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In this joint work with A. Juengel, C. Mouhot and N. Zamponi a new coercivity estimate on the spectral gap of the linearized Boltzmann collision operator for multiple species is proved. The assumptions on the collision kernels include hard and Maxwellian potentials under Grad's angular cut-off condition. Two proofs are given: a non-constructive one, based on the decomposition of the collision operator into a compact and a coercive part, and a constructive one, which exploits the "cross-effects" coming from collisions between different species and which yields explicit constants.
Furthermore, the essential spectra of the linearized collision operator and the linearized Boltzmann operator are calculated. Based on the spectral-gap estimate, the exponential convergence towards global equilibrium with explicit rate is shown for solutions to the linearized multi-species Boltzmann system on the torus. The convergence is achieved by the interplay between the dissipative collision operator and the conservative transport operator and is proved by using the hypocoercivity method of Mouhot and Neumann.

[a]Speaker

Slow-fast factorization of the Evans function via the Riccati transformation

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[2] Department of Mathematics, University of Bremen – Germany

In the spectral stability analysis of pattern solutions, the presence of a small parameter can reduce the complexity of the linear stability problem. The spectrum of the linearization about certain type of patterns is given by the zero set of an analytic function, the so-called Evans function. Our reduction method yields a factorization of the Evans function in accordance with the scale separation induced by the small parameter. For some specific equations this product structure has yet been established by geometric arguments. Our analytic method formalizes and generalizes the factorization procedure. The main tool for the reduction is the Riccati transformation. We employ our techniques to study the spectral stability of stationary, spatially periodic pulse patterns to singularly perturbed reaction-diffusion systems. The asymptotic structure of the factors of the Evans function can be determined explicitly in this setting.

[a]Speaker

Stability of delay differential systems

Alexander Domoshnitsky [a] [1]

[1] Ariel University (AU) – Ariel University, Qiryat University, 40700 Ariel, Israel

In this talk we present new explicit tests of the exponential stability of delay differential systems. We demonstrate, for example, that although the second order ordinary differential equation x"(t)+p(t)x(t)=0 can be oscillatory and unstable, the delay differential equation x"(t)+a(t)x(h(t))-b(t)x(g(t))=0, where a(t)-b(t)=p(t), can be nonoscillatory and exponentially stable. Exponential stability of second order delay equations can be achieved without damping term. Results of this sort were considered impossible. New stability results for delay differential systems are obtained on the basis of new results about nonoscillation of scalar delay equations. For the study of nonoscillation, the technique of differential inequalities is developed. Applications to stabilization of various models are discussed.

[a]Speaker

Numerical Approximation of Solitary waves in the Benjamin-Ono and the Intermediate Long Wave systems

Angel Duran [a] [1], Dimitrios Mitsotakis [2]

[1] University of Valladolid (UVA) – Spain
[2] Victoria University of Wellington – New Zealand

We consider the Benjamin-Ono and the Intermediate Long Wave systems derived in [1] that model two-way propagation of long internal waves of small amplitude along the interface of two fluid layers under the effects of gravity. After reviewing some theoretical properties of the models at hand, we present numerical evidence of the existence of solitary waves. Some properties of the waves, suggested by the numerical experiments, are discussed including the speed-amplitude relation and their asymptotic decay rate. We also present some numerical studies concerning the dynamics of the waves which involve experiments about their interactions, stability properties along with comparisons with their unidirectional counterparts.

[1] J.L. Bona, D. Lannes and J.-C. Saut, Asymptotic models for internal waves, *J. Math. Pures Appl.*, **89** (2008), pp. 538-566.

[a]Speaker

Some Qualitative Properties of a Fractional-type Camassa-Holm Equation

Nilay Duruk Mutlubas [a] [1], Husnu Ata Erbay [2], Albert Erkip [3]

[1] Istanbul Kemerburgaz University – Turkey
[2] Ozyegin University – Turkey
[3] Sabanci University – Turkey

The equation of motion of one-dimensional nonlocal nonlinear elasticity is:

$$u_{tt} = (\beta * (u + g(u)))_{xx}$$

where $*$ denotes the spatial convolution operator and $\beta(x)$ is the kernel function. In a recent work [1], using asymptotic expansion method, a fractional generalization of the Camassa-Holm equation have been derived by showing that, when the Fourier transform of the kernel function $\beta(x)$ has fractional powers, the unidirectional propagation of small-but-finite amplitude waves are governed by the fractional Camassa-Holm equation:

$$u_t + \frac{6}{5}u_x + 3uu_x + (-D_x^2)^\nu u_t = -\frac{3}{5}[2(-D_x^2)^\nu (uu_x) + u(-D_x^2)^\nu u_x] \tag{6}$$

with a constant $\nu \geq 1$ being not necessarily an integer and $(-D_x^2)^\nu \omega = \mathcal{F}^{-1}(|\xi|^{2\nu}\mathcal{F}\omega)$ where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse, respectively. In this talk, we discuss local well-posedness of solutions to the Cauchy initial value problem for (6) and wave breaking phenomenon.

[1] H. A. Erbay, S. Erbay, A. Erkip *Derivation of the Camassa-Holm equations for elastic waves*, Physics Letters A, **379**, 2015, 956-961.

[a] Speaker

Reduced models for domain walls in soft ferromagnetic films

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Domain walls are transition layers that separate domains of constant magnetization in a ferromagnet. In weakly anisotropic films of moderate thickness, a type of domain wall (the "asymmetric Néel wall") may be favored that exhibits a two-scale structure, combining a narrow, divergence-free wall core with long-range tails that complete the rotation between neighboring domains.

In this talk, we present a reduced model for the internal structure of an isolated domain wall in an infinitely extended ferromagnetic film (joint work with R. Ignat and F. Otto), as well as its generalization to periodic systems of parallel walls with potentially interacting tails. Both results have been obtained by Gamma-convergence, starting from the Landau-Lifshitz energy. The reduced models describe the amount of rotation in the wall core as a function of the strength of an external magnetic field and the (suitably non-dimensionalized) film thickness. In the case of interacting walls, also a prediction of the average magnetization in the direction of the field in the domains is available. This prediction additionally depends on the domain width and, for CoFeB films, agrees well with experimental data, provided the film thickness is not too large (joint work with C. Hengst, F. Otto and R. Schäfer).

[a]Speaker

Validity of the Camassa-Holm equation for elastic waves

Saadet Erbay [a] [1], Husnu Ata Erbay [1], Albert Erkip [2]

[1] Ozyegin University – Turkey
[2] Sabanci University – Turkey

In a recent study [1], using a double-asymptotic expansion, the Camassa-Holm equation has been derived to model unidirectional propagation of waves in a nonlocal and nonlinear elastic medium. In the present work, we discuss rigorously the validity of the Camassa-Holm equation for elastic waves and give uniform estimates for the error terms in suitable norms with respect to the small parameters measuring the nonlinear and dispersive effects.

[1] H.A. Erbay, S. Erbay, A. Erkip, "Derivation of the Camassa-Holm equations for elastic waves", Physics Letters A 379, 956-961 (2015).

[a]Speaker

Regularization by noise for transport and kinetic equations

Ennio Fedrizzi [a] [1], Franco Flandoli [2], Enrico Priola [3], Julien Vovelle [4,5]

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[4] CNRS – CNRS : UMR5208 – France
[5] Institut Camille Jordan (ICJ) – Université Claude Bernard - Lyon I (UCBL), CNRS : UMR5208 – Bât. Jean Braconnier n° 101 43 Bd du 11 novembre 1918 69622 VILLEURBANNE CEDEX, France

For some differential equations the addition of a carefully chosen, random noise term can produce a regularizing effect (e.g. solutions are more regular, or restored uniqueness). I will first consider a few easy examples (ODEs) to introduce some of these regularizing mechanisms, then detail two cases where we have regularization for a PDE: the (stochastic) linear transport equation

$$\partial_t u(t,x) + b(t,x)\cdot\nabla u(t,x) \quad \Big(+ \nabla u(t,x) \circ \frac{dW_t}{dt} \Big) = 0$$

and a (stochastic) kinetic equation with force term

$$\partial_t f(t,x,v) + v\cdot\nabla_x f(t,x,v) + F(x)\cdot\nabla_v f(t,x,v) \quad \Big(+ \nabla_v f(t,x,v) \circ \frac{dW_t}{dt} \Big) = 0\,.$$

I will present some classical results for these two equations, related to well-posedness and regularity of solutions, that can be obtained under weaker hypothesis in the stochastic setting.
This work is partially supported by LABEX MILYON / ANR-10-LABX-0070.

[a] Speaker

A comparison result for solutions of anisotropic elliptic problems via symmetrization

Filomena Feo [a] [1], Angela Alberico,
Giuseppina Di Blasio

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In the last years, there has been an increasing interest in the study of anisotropic problems in calculus of variations and in
partial differential equations motivated by their applications to mathematical models in various branches of applied science.
We treat the following class of anisotropic problems

$$\begin{cases} -div\left(a\left(x,u,\nabla u\right)\right)=f\left(x\right)-div \quad g\left(x\right) & \text{in } \Omega \\ \\ u=0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded open subset of $\mathbb{R}^N$
$N \geq 2$, $a : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^N$
is a Carathéodory function such that, for a.e. $x \in \Omega$,

$$a(x,\eta,\xi)\cdot\xi \geq \Phi\left(\xi\right) \quad \text{for } (\eta,\xi) \in \ \mathbb{R}\times\mathbb{R}^N$$

with Φ an $N-$dimensional Young function, and the data f and g are measurable functions fulfilling a suitable summability condition.
Our aim is to obtain a comparison result for solutions to this class of problems by means of anisotropic symmetrization. As consequence we deduce a priori estimates for norms of the relevant solutions.

[a]Speaker

Breaking of Ergodicity in Expanding Systems of Globally Coupled Piecewise Affine Circle Maps

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To identify and to explain coupling-induced phase transitions in Coupled Map Lattices (CML) has been a lingering enigma for about two decades. In numerical simulations, this phenomenon has always been observed preceded by a lowering of the Lyapunov dimension, suggesting that the transition might require changes of linear stability. Yet, recent proofs of co-existence of several phases in specially designed models work in the expanding regime where all Lyapunov exponents remain positive.

In this talk, I will consider a family of CML composed by piecewise expanding individual maps, global interaction and finite number N of sites, in the weak coupling regime where the CML is uniformly expanding.

I will show, mathematically for N=3 and numerically for N¿3, that a transition in the asymptotic dynamics occurs as the coupling strength increases. The transition breaks the (Milnor) attractor into several chaotic pieces of positive Lebesgue measure, with distinct empiric averages. It goes along with various symmetry breaking, quantified by means of magnetization-type characteristics.

Despite that it only addresses finite-dimensional systems, to some extent, this result reconciles the previous ones as it shows that loss of ergodicity/symmetry breaking can occur in basic CML, independently of any decay in the Lyapunov dimension.

[a] Speaker

A higher-order large-scale regularity theory for random elliptic operators

Julian Fischer [a] [1], Felix Otto [1]

[1] Max-Planck-Institute for Mathematics in the Sciences, Leipzig – Germany

We develop a large-scale regularity theory of higher order for divergence-form elliptic equations with heterogeneous coefficient fields a in the context of stochastic homogenization. Under the assumptions of stationarity and slightly quantified ergodicity of the ensemble, we derive a $C^{2,\alpha}$-"excess decay" estimate on large scales and a $C^{2,\alpha}$-Liouville principle: For a given a-harmonic function u on a ball B_R, we show that its energy distance to the space of a-harmonic "corrected quadratic polynomials" on some ball B_r has the natural decay in the radius r above some minimal (random) radius r_0. Our Liouville principle states that the space of a-harmonic functions growing at most quadratically has (almost surely) the same dimension as in the constant-coefficient case. The existence of a-harmonic "corrected quadratic polynomials" – and therefore our regularity theory – relies on the existence of second-order correctors for the homogenization problem. By an iterative construction, we are able to establish existence of subquadratically growing second-order correctors.

[a] Speaker

A trajectory-free framework for analysing multiscale systems

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[1] The University of New South Wales (AUSTRALIA) (UNSW Australia) – School of Mathematics and Statistics UNSW Australia, UNSW 2052 Australia, Australia
[2] University of Sydney – Australia
[3] Monash University – Australia

This paper introduces coordinate-independent methods for analyzing multiscale dynamical systems using numerical techniques based on the transfer operator and its adjoint. In particular, we present a method for testing whether an arbitrary dynamical system exhibits multiscale behavior and for estimating the time-scale separation. For systems with such behavior, we establish techniques for analyzing the fast dynamics in isolation, extracting slow variables for the system, and accurately simulating these slow variables at a large time step. By avoiding trajectory integration, the developed techniques are highly computationally efficient. We illustrate our method with numerical examples and show how the reduced slow dynamics faithfully represents statistical features of the full dynamics.

[a]Speaker

Preventing blow-up in the 2D chemotaxis system by decaying signal-dependent sensitivity

Kentarou Fujie [a] [1], Takasi Senba [2]

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[2] Kyushu Institute of Technology – Fukuoka, 804-8550, Japan

We will report latest development in a Keller-Segel system with decaying signal-dependent sensitivity. More precisely, we establish absence of blow-up in the Keller-Segel system

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla \chi(v)) & \text{in } \Omega \times (0, \infty), \\ \tau v_t = \Delta v - v + u & \text{in } \Omega \times (0, \infty), \end{cases}$$

under the Neumann boundary condition in a smoothly bounded domain in 2D with general sensitivity function $\chi(v)$ satisfying $\chi' > 0$ and decaying property: $\chi'(s) \to 0$ as $s \to \infty$. As to the case of the linear sensitivity function $\chi(v) = v$, it is well known that the large initial data induces blow-up solutions. In contrast, the logarithmic case $\chi(v) = \chi_0 \log v \, (\chi_0 > 0)$ enables us to see an absolutely different picture from linear one. Namely, independently the size of initial data, global existence and boundedness in the system were proved under some smallness condition on χ_0 by Nagai-Senba (1998), Biler (1999), Winkler (2011) etc. In this context, removing the smallness condition on χ_0 in 2D was conjectured by Biler and Velázquez. We give an answer to this conjecture in a more general setting. Moreover, we note that unlike the previous results, our method does not depend on any particular structure of $\chi(v)$. The main idea is focusing on concentration of mass around each point of $\overline{\Omega}$ locally.

[a]Speaker

Multiplicity results for sign changing bound state solutions of a semilinear equation

Carmen Cortázar [1], Marta García-Huidobro [a 2], Pilar Herreros [1]

[1] Pontificia Universidad Católica de Chile (PUC) – Chile
[2] Pontificia Universidad Católica de Chile (PUC) – Facultad de Matemáticas, PUC Vicuña Mackenna 4860, Chile

In this talk we give conditions on the nonlineary f so that the problem

$$\Delta u + f(u) = 0, \quad x \in \mathbb{R}^N, N \geq 2, \quad \lim_{|x|\to\infty} u(x) = 0,$$

has at least two solutions having a prescribed number of nodal regions and for which $u(0) > 0$. Any nonconstant solution to (7) is called a bound state solution. Bound state solutions such that $u(x) > 0$ for all $x \in \mathbb{R}^N$, are referred to as a first bound state solution, or a ground state solution. The existence of ground states for (7) has been established by many authors under different regularity and growth assumptions on the nonlinearity f, both for the Laplacian operator and the degenerate Laplacian operator. The main assumptions on the nonlinearity f are

(f_1) f is a continuous function defined in $\mathbb{R}$, and f is locally Lipschitz in $\mathbb{R} \setminus \{0\}$.

(f_2) There exists $\delta > 0$ such that if we set $F(s) = \int_0^s f(t)dt$, it holds that $F(s) < 0$ for all $0 < |s| < \delta$, and $\lim_{s\to-\infty} F(s) = \lim_{s\to\infty} F(s)$, $F(s) < \lim_{s\to\infty} F(s)$ for all $s \in \mathbb{R}$.

(f_3) F has a local maximum at some $\gamma \in (\delta, \infty)$ and $F(\gamma) > 0$.

(f_4) there exists $\theta \in (0, 1)$ such that

$$\lim_{s\to\infty}\Big(\inf_{s_1,s_2\in[\theta s,s]} Q(s_2)\Big(\frac{s}{f(s_1)}\Big)^{N/2}\Big) = \infty, \tag{7}$$

where $Q(s) := 2NF(s) - (N-2)sf(s)$.

[a]Speaker

On the wave length of smooth periodic traveling waves of the Camassa-Holm equation

Anna Geyer [a] [1], Jordi Villadelprat

[1] Universitat Autònoma de Barcelona (UAB) – 08193 Bellaterra (Barcelona) SPAIN, Spain

This paper is concerned with the wave length λ of smooth periodic traveling wave solutions of the Camassa-Holm equation. The set of these solutions can be parametrized using the wave height a (or "peak-to-peak amplitude"). Our main result establishes monotonicity properties of the map $a \longmapsto \lambda(a)$, i.e., the wave length as a function of the wave height. We obtain the explicit bifurcation values, in terms of the parameters associated to the equation, which distinguish between the two possible qualitative behaviours of $\lambda(a)$, namely monotonicity and unimodality. The key point is to relate $\lambda(a)$ to the period function of a planar differential system with a quadratic-like first integral, and to apply a criterion which bounds the number of critical periods for this type of systems.

[a]Speaker

Skew products of interval maps over sub-shifts

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We treat step skew products over transitive subshifts of finite type with interval fibers. The fiber maps are diffeomorphisms on the interval; we assume that the end points of the interval are fixed under the fiber maps. We further assume positive Lyapunov exponents at the interval end points. Our work thus extends work by V. Kleptsyn and D. Volk who treated step skew products where the fiber maps map the interval strictly inside itself.

We clarify the dynamics for an open and dense subset of such skew products. In particular we prove existence of a finite collection of disjoint attracting invariant graphs. These graphs are contained in disjoint areas in the phase space called trapping strips. Trapping strips are either disjoint from the end points of the interval (internal trapping strips) or they are bounded by an end point (border trapping strips). The attracting graphs in these different trapping strips have different properties.

[a]Speaker

Stability analysis for combustion fronts traveling in hydraulically resistant porous media

Anna Ghazaryan [a] [1]

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We study front solutions of a system that models combustion in highly hydraulically resistant porous media. The spectral stability of the fronts is tackled by a combination of energy estimates and numerical Evans function computations. Our results suggest that there is a parameter regime for which there are no unstable eigenvalues. We use recent works about partially parabolic systems to prove that in the absence of unstable eigenvalues the fronts are convectively unstable. This is a joint work with S. Lafortune and P. McLarnan.

[a]Speaker

On bifurcations of area-preserving maps with quadratic homoclinic tangencies on non-orientable surfaces

Marina Gonchenko [a] [1], Amadeu Delshams [2], Sergey Gonchenko [3]

[1] Uppsala University – Sweden
[2] Universittat Politecnica de Catalunya – Spain
[3] University of Nizhny Novgorod – Russia

We study bifurcations of non-orientable area-preserving maps with quadratic homoclinic tangencies. We study the case when the maps are given on non-orientable two-dimensional surfaces. We consider one and two parameter general unfoldings and establish the results related to the emergence of elliptic periodic orbits.

[a]Speaker

On some new properties of the solution to an initial-boundary-value problem for the time-fractional diffusion equation with $\alpha \in (0, 2)$

Demian Nahuel Goos [a] [1,2], Gabriela Reyero[1]

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[2] Consejo Nacional de Investigaciones Científicas y Técnicas - Rosario (CONICET) – Argentina

The time-fractional diffusion equation is a fractional partial differential equation which interpolates the heat equation and the wave equation via fractional derivatives in Caputo's sence and which provides a mathematical model for the intermediate behaviour of linear viscoelastic media subject to stress pulses.
In this work we study the following initial-boundary-value problem in the upper half-plane for the one-dimensional time fractional diffusion equation with $\alpha \in (0, 2)$:

$$\begin{cases} \frac{\partial^\alpha}{\partial t^\alpha} u(x,t) = \lambda \frac{\partial^2}{\partial x^2} u(x,t) & \text{if } x \in \mathbb{R}; \quad t \in \mathbb{R}_0^+ \\ u(x,0) = \varphi(x) & \text{if } x \in \mathbb{R} \\ u \text{ bounded} \end{cases}$$

A solution to this problem, identical to the one given by other authors, is obtained; as well as the first proof to this result. Different new properties of the solution to this problem, which are strongly connected to the physical context by viscoelastic media, are presented. Among these properties, the asymtotic behaviour and the limiting behaviour $\alpha \to 1$ and $\alpha \to 2^-$ stand out. Finally, some new advances considering the generalization of the order of derivation $\alpha > 0$ are presented.

[a] Speaker

On two nonlocal equations showing chaotic behaviour

Rafael Granero Belinchon [a] [1]

[1] University of California, Davis (UC Davis) – United States

In this talk we will introduce our results concerning a pair of nonlocal equations. The first equation is analogous to the Kuramoto-Sivashinsky equation, in which short waves are stabilized by a possibly fractional diffusion of order less than or equal to two, and long waves are destabilized by a backward fractional diffusion of lower order. The second equation is a chemotaxis model that arises related to the three-component urokinase plasminogen invasion model.

Numerical simulations show that both equations have chaotic solutions. For the nonlocal Kuramoto-Sivashinsky, the spatial structure of the solution consists of interacting traveling waves resembling viscous shock profiles. In the nonlocal model of tumor growth, the solution develop a number of peaks that emerge and, eventually, mix with other peaks.

We will present the global existence, uniqueness, and analyticity of solutions for both models. We will also show the existence of a compact attractor and an analytical bound on the number of oscillations that the solutions may develop.

[a]Speaker

On the two-dimensional stationary Navier-Stokes equations

Julien Guillod [a] [1]

[1] Department of Theoretical Physics – University of Geneva – Switzerland

The stationary Navier-Stokes equations in two-dimensional unbounded domains are still not completely understood mathematically. In 1933, Leray proved the existence of weak solutions for this problem, but their behavior at infinity is still an open question. If the prescribed velocity at infinity is a nonzero constant vector field, some results were obtained by different methods. If the velocity at infinity is zero, almost nothing is known, unless particular symmetries or boundary conditions are assumed. In this talk, we discuss the main reasons which make this problem highly nontrivial: the well-known Stokes paradox but also the decay the non-linearity, which is critical or even supercritical in some cases. We will present asymptotic expansions and numerical simulations, which show that the velocity field decays like $|\boldsymbol{x}|^{-1/3}$ and has a wake-structure.

[a]Speaker

Convergence analysis of the Navier-Stokes-α equations on bounded domains

Juan V. Gutiérrez-Santacreu [a] [1], Marko A. Rojas-Medar [2]

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The Navier-Stokes-αequations were developed as a LES (Large Eddy Simultion) model for governing turbulent fluid flows using physical principles such as Lagrangian averaging and asymptotic expansions in Hamilton's principle to the turbulence in the flow being statistically homogeneous and isotropic. These equations can also be seen as a regularitization *à la Leray* of the Navier-Stokes equations associated with the parameter α being the smaller scale being capture for the model. In this talk we will present some convergence results between the approximate solutions being computed by using eigenfunctions of the Stokes operator of the two-dimensional Navier-Stokes-α equations and a solution of the two-dimensional Navier-Stokes equations with respect to the parameter α and the eigenvalues. In particular, we show two different convergence results (a) local-in-time error estimates and (b) global-in-time error estimates. The latter will establish by using the concept of stability for solutions of the Navier-Stokes equations in terms of the L^2 norm.

[a] Speaker

Dynamical bifurcation and final patterns for fourth order phase transition equations

Jongmin Han [a] [1]

[1] Kyung Hee University – Department of Mathematics, 26, Kyungheedae-ro, Dongdaemun-gu, Seoul, 130-701, South Korea

In this talk, we consider bifurcation and stability of the fourth order phase transition equations including the Swift-Hohenrberg equation and the damped Kuramoto-Sivashinsky equation. We show that the equations bifurcates from the trivial solutions to an attractor as a bifurcation parameter passes a critical number. This attractor is reponsible for the final patterns of solutions and we analyze it via a center manifold analysis.

[a] Speaker

Critical oscillation constants for half-linear differential and difference equations

Petr Hasil [a] [1]

[1] Department of Mathematics and Statistics, Faculty of Science, Masaryk University (MUNI) – Kotlarska 2, 611 37 Brno, Czech Republic

The main subject of this talk is to present the results concerning the conditional oscillation of second order half-linear differential equations. We show that the existence of the mean values of coefficients is sufficient for Euler-type equations to be conditionally oscillatory (i.e., that there exists a border value given by their coefficients which separates oscillatory equations from non-oscillatory ones). We explicitly find oscillation constants even for the considered equations whose coefficients can change sign. These results cover known results concerning equations with periodic and almost periodic positive coefficients and extend them to larger classes of equations. Then, we aim to the conditional oscillation of half-linear difference equations and point out the similarities and differences in comparison with the continuous case.

[a]Speaker

New formulation of the compressible Navier-Stokes system with degenerate viscosity coefficient and the highly compressible limit

Boris Haspot [a] [1]

[1] Université Paris Dauphine, Ceremade – Ceremade – Place du Maréchal de Lattre de Tassigny, 75016 Paris., France

In this talk, we will study the highly compressible regime for global weak solution of Navier-Stokes equations with degenerate viscosity coefficient (it means when the Mach Number ϵ goes to $+\infty$). To do this we will propose a new formulation of compressible Navier-Stokes equations by introducing a suitable effective velocity v such that the system becomes parabolic on the density ρ and curlv and hyperbolic on divv. It allows us in particular to prove the existence of global strong solution with large initial data in one dimension for the viscous shallow water system. In a second time, we will show that for particular choice on the viscosity coefficients the solution of compressible Navier-Stokes equation converges to solution of the pressureless system when ϵ goes to $+\infty$. This limit solution is related to the fast diffusion, heat or porous medium equation, indeed the limit density ρ verifies these equations following the choice of the viscosity coefficients. We will particularly focus on the case of initial density with compact support, indeed "the speed of propagation" is finite for porous medium equation. Roughly speaking we will observe that the main part of the mass corresponding to solution ρ_ϵ is located in the support of the solution of the porous medium equation (which is compact). The mass outside of this support tends to be small in terms of $1/\epsilon$.

[a]Speaker

Regularization of planar piecewise smooth two-folds

Kristian Kristiansen[1], John Hogan [a] [2]

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[2] Department of Engineering Mathematics, University of Bristol (EMUOB) – Merchant Venturer's Building, Woodland Road, Bristol BS8 1UB, United Kingdom

This talk will demonstrate the use of the blow up method of Krupa and Szmolyan to study the Sotomayor-Teixeira regularization of two-folds in planar piecewise smooth systems. We will describe the perturbation of associated limit cycles, pseudo-equilibria, and canard-like orbits connecting stable sliding with unstable sliding. We will also present results on how the regularization function can introduce additional bifurcation.

[a]Speaker

Topological sensitivity applied to cavity reconstruction from elasticity linear surface measurements and Kohn-Vogelius formulation

Mourad Hrizi [a] [1], Maatoug Hassine

[1] Faculty of Science of Monastir – Tunisia

In this work, we focus on the geometric inverse problems in linear elasticity related to the detection, topology and geometry of an cavity in bounded domain Ω via a part of the exterior boundary measurement on $\partial\Omega$. We propose a one-shot geometric algorithm based on the asymptotic expansion for the Kohn-Vogelius formulation using the topological gradient method. The inverse problem is formulated as a topology optimization one. A topological sensitivity analysis is derived form an energy function. Then, we present a numerical method for the geometric reconstruction of the unknown cavity using a level curve of the topological gradient. Finally, we illustrate the efficiency and accuracy of the proposed algorithm by some numerical results.

[a]Speaker

Global existence in a 2D semilinear chemotaxis-Navier-Stokes system with position sensitivity under the small initial data

Sachiko Ishida [a] [1], Xinru Cao [2], Yulan Wang [3]

[1] Tokyo University of Science – 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan
[2] Dalian University of Technology – China
[3] Xihua University – China

We will prove that a coupled semilinar chemotaxis-fluid model with position dependent sensitivity has a global solution under suitable smallness conditions on the initial data.

We deal with the system

$$\begin{cases} n_t = \Delta n - \nabla \cdot (S(n,c,x) n \nabla c) - u \cdot \nabla n, \\ c_t = \Delta c - nc - u \cdot \nabla c, \\ u_t = \Delta u - (u \cdot \nabla) u - \nabla P + n \nabla \phi, \\ \nabla \cdot u = 0, \end{cases} \tag{KSNS}$$

under the no-flux boundary conditions for n, c and the Dirichlet boundary conditions for u in a bounded domain $\Omega \subset \mathbb{R}^2$. $\phi \in W^{1,\infty}(\Omega)$ is a known function. We assume that the matrix valued position sensitivity $S(n,c,x) = (s_{ij}(n,c,x))_{i,j\in\{1,2\}}$ satisfies $|S(n,c,x)| \le \tilde{S}(c)$, where $\tilde{S}$ is a non-decreasing function on $[0,\infty)$.

When $S = I$ (identity matrix), Winkler (2012, 2014) proved global existence and stabilization in a semilinear system, moreover, Francesco-Lorz-Markowich (2010) obtained that degenerate systems (namely, Δn is replaced with Δn^m $(m > 1)$) have global solutions when $m \in (\frac{3}{2}, 2]$. In contrast, when $S \neq I$ Ishida (2015) found global bounded solutions of degenerate systems with $m \in (1,\infty)$. However, she used the condition $m > 1$, so global solvability of the semilinear system with $S \neq I$ is open. We are going to give an answer to it.

[a] Speaker

Multi-scale Dynamics in Microstructures

Annalisa Iuorio [a] [1], Christian Kühn [1], Peter Szmolyan [1]

[1] Institute for Analysis and Scientific Computing, Vienna University of Technology – Austria

We consider a singularly perturbed, non-convex variational problem describing microstructures in one space dimension (in particular, simple laminates). The minimizers of the functional exhibit a complicated multi-scale behavior. We propose a dynamical systems approach to analyse the critical points of the functional by applying methods from geometric singular perturbation theory to the corresponding Euler-Lagrange equation. Some microstructures can be interpreted as periodic solutions of the system obtained by writing the Euler-Lagrange equation as a 4-dimensional singularly perturbed Hamiltonian system of ODEs depending on a small parameter $0 < \varepsilon \ll 1$. Starting from a singular orbit ($\varepsilon = 0$), we prove for $\varepsilon \neq 0$ the existence of a family of periodic orbits parametrized by the value μ of the Hamiltonian for ε small. The periodicity of the solutions explains the observed multi-scale structures. Numerical computations based on AUTO are performed in order to study quantitative properties and bifurcations of solutions as key parameters ε and μ vary.

[a] Speaker

Strongly nonlinear internal wave model in a three-layer fluid system

Tae-Chang Jo [a] [1]

[1] Mathematics, Inha University – 100 Inha-ro Nam-Gu, Incheon 402-751, South Korea

Many observations of large amplitude internal waves in the ocean are recently reported due to advanced science and technology. Large amplitude waves are common phenomena for internal waves, but early theoretical works are based on KdV-type equation from weakly nonlinear assumption, which is, in general, not valid for describing large amplitude waves. A strongly nonlinear model in a two-layer fluid system, called MCC model, is suggested for studying large amplitude internal waves. It turns out that the model has excellent agreements with Euler equations and laboratory experiments. For real application of the model, a continuous density profile needs to convert to two constant densities. Even with proper parameters, the model set-up cannot explain recent observation of the second mode baroclinic internal waves. Thus, we investigate three-layer MCC model, which is the simplest set-up for understanding second baroclinic internal modes. We describe the formulation of three-layer model and identify mode-1 and mode-2 baroclinic internal waves from the model PDE. We also discuss some numerical result and its stability issues which come from the governing equations, coupled Euler equations.

[a] Speaker

Spectral stability of internal solitary waves in continuously stratified fluids

Andreas Klaiber [a] [1]

[1] Zentrum Mathematik [Muenchen] (TUM) – Boltzmannstrasse 3 D-85747 Garching, Germany

Internal solitary waves (ISWs) are ecologically important since they are involved in mixing and energy transport in lakes and oceans. From the mathematical point of view, ISWs are exact solutions of the 2D Euler equations for incompressible, inviscid fluids with non-constant density. This talk is concerned with the study of their spectral stability, which has not yet received attention at a rigorous mathematical level.

In the first part of the talk, after introducing the physical model, I will present an Evans-function approach to spectral stability of ISWs. Starting from the full Euler eigenvalue problem associated with an ISW, this approach is based on a formal spatial-dynamics formulation and a formal Galerkin procedure to obtain finite-dimensional truncations for which we can rigorously define Evans functions on the closed right complex half-plane. For small-amplitude ISWs, the Evans functions have no zeros in a neighbourhood of the origin apart from the origin itself.

In the second part of the talk, I will present recent investigations that allow to define a "moment of instability" for ISWs (of arbitrary amplitude), a classical tool in the stability theory of solitary waves, and directions of research.

[a]Speaker

Symmetric mountain pass lemma and sublinear elliptic equations

Ryuji Kajikiya [a] [1]

[1] Saga University – Japan

We study the p-Laplace elliptic equation

$$-\Delta_p u = f(x,u) \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega, \tag{8}$$

where $\Delta_p u := \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian and Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary. We assume that $f(x,u)$ is a continuous function which is odd with respect to u, i.e., $f(x,-u) = -f(x,u)$. Then equation (1) satisfies either (A) or (I) below:

(A) the zero solution is an accumulation point of the set of all solutions,

(I) the zero solution is an isolated point of the set of all solutions.

Applying the symmetric mountain pass lemma in our paper [Kajikiya, J. Funct. Anal., **225**, (2005) 352–370], we give a weak sufficient condition for (A). Moreover, we decide which type of (A) or (I) holds for some elliptic equations.

[a]Speaker

Criteria on contraction of admissible discontinuities for the systems of conservation laws

Moon-Jin Kang [a] [1]

[1] Laboratoire Jacques-Louis Lions (LJLL) – Université Pierre et Marie Curie [UPMC] - Paris VI – 4 place Jussieu 75005 Paris, France, France

In this talk, we present criteria on contraction of admissible discontinuities for the systems of conservation laws in the spatially inhomogeneous pseudo-distance. We first generalize the criterion developed by Serre and Vasseur, which can be applied to the contraction for the extremal shocks of the physical systems including the Euler systems. Moreover, we present a necessary condition for the contraction of admissible discontinuities. Considering the negation of the necessary condition, we construct two kinds of criteria to identify the invalidity of contraction of intermediate entropic shocks and contact discontinuities. This is joint work with Alexis Vasseur.

[a] Speaker

Partial Differential Equations in Banach Lattices Related to Size-Structured Population Models

Nobuyuki Kato [a] [1]

[1] Faculty of Electrical and Computer Engineering, Kanazawa University – Japan

We are concerned with size-structured population models with spatial diffusion expressed by partial differential equations of the population density $p(s, t, x)$ of size s, time t and position x with nonlocal boundary condition on $s = 0$, which describes the reproduction process and the Neumann boundary condition on the boundary of habitat, which means the individuals do not go outside through the boundary. Transforming the models into some function space with variable x, they become partial differential equations of $p(s, t)$ with values in the function space. In view of this, we will consider abstract partial differential equations in Banach lattices. We first show the existence of positive solutions of the abstract problems for positive initial data. Also, a comparison result and the boundedness properties of solutions are obtained in the Banach lattice setting.

Next, we consider the dual problems and show the existence of a bounded solutions to the dual problems. Through the dual problems, we introduce weak solutions and establish the uniqueness of weak solutions.

[a] Speaker

Abstract existence result for parabolic quasi-variational inequalities

Nobuyuki Kenmochi [a] [1]

[1] Chiba University – Japan

In this talk we discuss an abstract existence result for a parabolic quasi-variational inclusion in a Hilbert space H, which is governed by a functional F(u,v)on HxH, coupled with a sort of feedback control system. Our functional F(u,v) is proper, l.s.c. and convex in u, and v is a non-local parameter. Moreover, our set-up requires some relationship between two variables u and v, say v=Au; in most cases A is a nonlinear integral or integro-differential operator. Roughly speaking, our abstract problem is a parabolic inclusion generated by the subdifferential of F(u,v) with respect to variable u, depending the unknown parameter v=Au. This formulation is motivated by many real world problems, for instance, superconductivity phenomemon with gradient constraint, bacteria's activity in fluids and economic growth with technological innovation, etc..

[a]Speaker

Localized States in Periodically Forced Systems

Edgar Knobloch [a] [1]

[1] University of California at Berkeley – United States

The theory of stationary spatially localized patterns in dissipative systems driven by time-independent forcing is well developed. With time-periodic forcing, related but time-dependent structures may result. These may consist of breathing localized patterns, or states that grow for part of the cycle via nucleation of new wavelengths of the pattern followed by wavelength annihilation during the remainder of the cycle. These two competing processes lead to a complex phase diagram whose structure is a consequence of a series of resonances between the nucleation time and the forcing period [1]. The resulting diagram is computed for the periodically forced quadratic-cubic Swift–Hohenberg equation, and its details are interpreted in terms of the properties of the depinning transition for the fronts bounding the localized state on either side. Both sinusoidal and nonsinusoidal forcing is considered. The results are expected to shed light on localized states in a large variety of periodically driven systems.

[1] P. Gandhi, C. Beaume and E. Knobloch. Phys. Rev. Lett. 114, 034102 (2015) and SIADS (submitted)

[a]Speaker

Dynamics of bipolar disorders model: periodic and switching phenomenon

Ilona Kosiuk [a] [1], Peter Szmolyan [2], Ekaterina Kutafina [3]

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[2] The Vienna University of Technology – Austria
[3] AGH University of Science and Technology – Poland

Slow-fast dynamical systems are a prevalent class of singularly perturbed differential equations that appear in the modelling of many complex biological processes. In this talk an ODE system modeling oscillatory patterns of mood alternations in manic-depression, also known as bipolar disorder, is analyzed. The model is four-dimensional, contains many parameters of different orders of magnitude, and non-polynomial nonlinearities. Hence, the equations do not allow for a global split into slow and fast variables and a complete treatment of the model by means of singular perturbation arguments is not at all straightforward. In particular, identification of slow manifolds, which organize phase space becomes challenging. It turns out that to understand the global dynamics of this multi-parameter singular perturbation problem one needs to identify and use hierarchies of local approximations based on various — hidden -– forms of time scale separation.

I will highlight the main concepts from geometric singular perturbation theory and geometric desingularization based on the blow-up method in combination with standard techniques from dynamical systems theory which I use to understand this intriguing switching phenomenon.

[a] Speaker

Planar linkages, geodesic flows, billiards and chaos

Mickaël Kourganoff [a] [1]

[1] Unité de mathématiques pures et appliquées (UMPA) – École Normale Supérieure - Lyon – 46 allée d'Italie, 69364 Lyon Cedex 07, France

A mechanical linkage is a mechanism made of rigid rods linked together by flexible joints, in which some vertices are pinned down to the plane while others may move. If the system is given an initial speed, and if no external force applies, then its physical behavior corresponds to the geodesic flow on the configuration space endowed with some Riemannian metric. In 2003, Hunt and MacKay showed that the behavior of Thurston's "triple linkage" is an Anosov flow: it is expected that many linkages have this property, but no other example was known until now.
I will introduce a new example of a 5-rod linkage with Anosov behavior. It is obtained by a new method using the well-known chaoticity of billiards with dispersing walls (Sinai's billiards), and the approximation of the billiard flow by the geodesic flows of "flattened" surfaces. This is the first example of an Anosov linkage in which the lengths of the rods are given explicitly.

[a] Speaker

Nonlinear evolution equations arising from mathematical biology and medicine

Akisato Kubo [a] [1]

[1] School of Health Sciences, Fujita Health University (FHU) – 1-98 Dengakugakubo,Kutsukake-cho,Toyoake,Aichi.470-1192, Japan

In this talk we consider initial-Neumann boundary value problem of nonlinear evolution equations with strong dissipation and proliferation arising from mathematical biology and medicine formulated as

$$(NE)\begin{cases} u_{tt} = D\nabla^2 u_t + \nabla\cdot(\chi(u_t, e^{-u})e^{-u}\nabla u) + \mu_1 u_t(1-u_t) & in\ (x,t)\in\mathbf{\Omega}\times(0,\infty)\ (1.1) \\ \dfrac{\partial}{\partial\nu}u|_{\partial\mathbf{\Omega}} = 0 & on\ \partial\mathbf{\Omega}\times(0,\infty)(1.2) \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x) & in\ \mathbf{\Omega}(1.3) \end{cases}$$

where constants D, μ_1 are positive, $\mathbf{\Omega}$ is a bounded domain in R^n with a smooth boundary $\partial\mathbf{\Omega}$ and ν is the outer unit normal vector. Under some regularity and boundedness conditions of the coefficient $\chi(u_t, e^{-u})$ of (1.1), we derive the energy estimate of (NE), which enables us to show the global existence in time and asymptotic behavior of the solution. We deal with mathematical models of tumor migration as an extended case of (NE) and applying our result to them we discuss the behavior of cell migration.

[a]Speaker

Multiscale Oscillations in the Olsen Model

Christian Kuehn [a] [1], Peter Szmolyan [1]

[1] Vienna University of Technology – Austria

In this talk I am going to report on the geometric decomposition of nonlinear dynamics in the Olsen model. Although this model has been proposed by Olsen already in the late 1970s and has been investigated many times with different methods, a full understanding of the mechanisms that lead to oscillatory patterns was not available. Nonlinearity, several small parameters, higher-dimensionality and wide parameter ranges are the key difficulties in this context. However, using methods from the geometric theory of multiple time scale dynamical systems, it is possible to identify the main mechanisms. In particular, I am going to illustrate the main steps to prove the existence of non-classical relaxation oscillations and explain how one may deal with mixed-modes and chaotic solutions from the same viewpoint.

[a]Speaker

A mathematical model describing concrete carbonation process

Kota Kumazaki [a] [1]

[1] Department of Natural and Physical Sciences, Tomakomai National College of Technology – Japan

In this talk, we consider a mathematical model for concrete carbonation phenomenon. This model consists of moisture transport and carbon dioxide transport. Moisture transport is described as a quasilinear parabolic equation with a hysteresis effect between the relative humidity and the degree of saturation, and carbon dioxide transport is a parabolic equation considered the concentration of carbon dioxide in air and in water. In this talk, we discuss the existence and uniqueness of a solution for an initial boundary value problem of this model.

[a]Speaker

Equilibrating effect of Maxwell-type boundary condition in highly rarefied gas

Hung-Wen Kuo [a] [1]

[1] National Cheng Kung University – Taiwan

We study the equilibrating effects of the boundary and intermolecular collision in the kinetic theory for rarefied gases. We consider the Maxwell-type boundary condition with variable boundary temperature, which has weaker equilibrating effect than the commonly studied diffuse reflection boundary condition. The gas region is the spherical symmetric domain in $\mathbb{R}^d$, $d = 1, 2$. First, without the equilibrating effect of the collision, we obtain the algebraic convergence rates to the steady state of free molecular flow. The convergence behavior has intricate dependence on the accommodation coefficient of the Maxwell-type boundary condition. Then we couple the boundary effect with the intermolecular collision and study their interaction. We are able to construct the steady state solutions of the full Boltzmann equation for large Knudsen numbers and small boundary temperature variation. We also establish the nonlinear stability with exponential rate of the stationary Boltzmann equation. Our analysis is based on the explicit formulations of the boundary condition for symmetric domains.

[a]Speaker

The discrete Swift-Hohenberg equation

Rudy Kusdiantara [a] [1], Hadi Susanto[1]

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We investigate the discretization effect on the bifurcation behavior of the Swift-Hohenberg equation with cubic and quintic nonlinearity. The discrete Swift-Hohenberg equation is obtained by discretizing the spatial derivatives using a center finite difference. This introduces the discretization parameter h. There are regions of the parameter wherein the discrete Swift-Hohenberg equation behaves either similarly or differently from the continuous version. When $1 \leq h \leq 2$, multiple Maxwell points can occur for the periodic solutions and may cause irregular snaking and isolas. The offset stability of the uniform solution and the pinning region are shifting for large discretization $h > 2$. Numerical continuation is used to obtain and analyze localized and periodic solutions for each case.

[a]Speaker

A few domain perturbation problems for the eigenvalues of the Reissner-Mindlin system

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[2] Dipartimento di Matematica, Universita' degli Studi di Padova – Via Trieste, 63 - 35121 Padova, Italy

Let Ω be a bounded open set in $\mathbb{R}^2$, $t > 0$. According to the Reissner-Mindlin model, the free vibration modes of a moderately thin clamped plate $\Omega \times (-t/2, t/2)$ with midplane Ω and thickness t, are given by the system

$$\begin{cases} -\frac{\mu}{12}\Delta\beta - \frac{\mu+\lambda}{12}\nabla\mathrm{div}\beta - \frac{\mu k}{t^2}(\nabla w - \beta) = \frac{\gamma t^2}{12}\beta, & \text{in } \Omega, \\ -\frac{\mu k}{t^2}(\Delta w - \mathrm{div}\beta) = \gamma w, & \text{in } \Omega, \\ \beta = 0, \quad w = 0, & \text{on } \partial\Omega, \end{cases}$$

in the unknowns $(\beta, w) = (\beta_1, \beta_2, w)$ (the eigenvector) and γ (the eigenvalue). Here λ and μ are the Lamé constants, $k > 0$ the correction factor, w represents the transverse displacement of the midplane, $\beta = (\beta_1, \beta_2)$ the fiber rotation and γt^2 the vibration frequency. The above problem has a divergent sequence of positive eigenvalues of finite multiplicity $\gamma_{n,t}[\Omega]$, $n \in \mathbb{N}$. In this talk we discuss some results obtained in [1] concerning the dependence of $\gamma_{n,t}[\Omega]$ on Ω. [1] D. Buoso and P.D. Lamberti, Shape sensitivity analysis of the eigenvalues of the Reissner-Mindlin system. SIAM J. Math. Anal. 47 (2015), no. 1, 407–426.

[a]Speaker

Recent resuts on the Kuramoto–Sakaguchi equation

Mikhail Lavrentiev [a] [1], Renato Spigler [2]

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The Kuramoto-Sakaguchi (or just Kuramoto) equation is a Fokker-Planck equation, where the space variable is an angle, and the drift is quadratically nonlinear through an integral which accounts for the action of the infinitely many oscillators on any given one of them. This PDE provides a model which describes a variety of phenomena, in particular self-synchronization of chemical and biological oscillations, hence in neurosciences, as well as in physical and social systems. An additional integration over all frequencies appears, which makes the model even more nonstandard from the mathematical viewpoint. It is the parabolic equation with non-standard non-linear integral term. Integration is calculated with respect to the problem coefficient over unbounded interval. The results available in the literature concerning parabolic equations, or even integroparabolic equations, cannot be applied to this case. The Kuramoto-Sakaguchi equation is here considered when the "frequency distribution", the frequency being an independent variable in the model equation, has an *unbounded* support. This case, interesting in view of applications, have not been considered earlier. Problem solvability is established.

[a] Speaker

Global properties of solutions to the Einstein-Boltzmann system with Bianchi I symmetry

Ho Lee [a] [1], Ernesto Nungesser [2]

[1] Kyung Hee University (KHU) – South Korea
[2] Trinity College Dublin [Dublin] –, College Green, Dublin 2, Ireland, Ireland

In this talk we consider the Einstein-Boltzmann system with Bianchi I symmetry. A suitable assumption will be made on the scattering kernel, and initial data will be assumed to be small, in the sense that the universe is almost isotropic and collisions between particles are very small. We obtain global-in-time existence and asymptotic behaviour of classical solutions, and will see that solutions behave as dust at late times, like the Einstein-Vlasov system. For the Einstein part, we use the bootstrap argument, which has recently been applied to the Einstein-Vlasov system with Bianchi I symmetry by Nungesser, and for the Boltzmann part, we use the decomposition argument, which has been developed by Guo and Strain for the relativistic Boltzmann equation. This is a jointwork with Ernesto Nungesser.

[a] Speaker

Bifurcation without parameters

Stefan Liebscher [a] [1]

[1] Free University Berlin – Institute of Mathematics, Arnimallee 3, 14195 Berlin, Germany

We study dynamical systems with manifolds of equilibria near points at which normal hyperbolicity of these manifolds is violated. Manifolds of equilibria appear frequently in classical bifurcation theory by continuation of a trivial equilibrium. Here, however, we are interested in manifolds of equilibria which are not caused by additional parameters. In fact we require the absence of any flow-invariant foliation transverse to the manifold of equilibria at the singularity. We therefore call the emerging theory bifurcation without parameters.
Albeit the apparent degeneracy of our setting (of infinite codimension in the space of all smooth vectorfields) there is a surprisingly rich and diverse set of applications ranging from networks of coupled oscillators, viscous and inviscid profiles of stiff hyperbolic balance laws, standing waves in fluids, binary oscillations in numerical discretizations, population dynamics, cosmological models, and many more.

[a]Speaker

Uniqueness and stability results on steady water waves with vorticity

Evgeniy Lokharu [a] [1], Vladimir Kozlov[1], Nikolay Kuznetsov[2]

[1] Linköping University - LIU (SWEDEN) – Sweden
[2] Institute for Problems in Mechanical Engineering, Russian Academy of Sciences – Russia

We consider the two-dimensional nonlinear problem describing steady gravity water waves with vorticity in a channel of a finite depth. The water motion is assumed to be unidirectional and the surface tension is neglected. It is well known that among small-amplitude waves only Stokes and Solitary waves exist provided the Bernoulli's constant is close to its critical value. We complete this result by proving that all near-critical waves are necessary small. Furthermore, we prove a stability estimate that imply uniqueness for small-amplitude waves with a prescribed Cauchy data of the profile at some point and provide a parametrization by the amplitude for the family of waves with near-critical values of the Bernoulli's constant. Using similar methods, we study solitary type waves for arbitrary Bernoulli's constants and prove that they are necessary supported by sub-critical shear flows. In particularly, this means that there are no waves that oscillate and decay at the same time.

[a]Speaker

Long time behavior of the quadratic Klein-Gordon equation in the nonrelativistic limit regime

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We consider the following Cauchy problem of the Klein-Gordon equation

$$\begin{cases} \varepsilon^2 \partial_{tt} u - \Delta u + \frac{1}{\varepsilon^2} u + f(u) = 0, \quad t \geq 0, \quad x \in \mathbb{R}^d, \\ u(0) = u_{0,\varepsilon}, \quad (\partial_t u)(0) = \frac{1}{\varepsilon^2} u_{1,\varepsilon}. \end{cases} \tag{9}$$

Here $u = u(t,x)$ is a real-valued (or complex-valued) field, and $f(u)$ is a real-valued function (or $f(u) = g(|u|^2)u$ if u is complex-valued). The non-dimensional parameter ε is proportional to the inverse of the speed of light.
We study the asymptotic behavior of the Klein-Gordon equation in the nonrelativistic limit regime $\varepsilon \to 0$. By employing the techniques in geometric optics, we show that the solution of the quadratic Klein-Gordon equation can be approximately described by a linear Schr'odinger equation with an error of order $O(\varepsilon)$ over a long time interval of order $O(\varepsilon^{-1})$. With general nonlinearities, we show that the Klein-Gordon equation can be approximated by nonlinear Schrödinger equations over time of order $O(1)$.

[a] Speaker

Positive solutions for a system of fractional differential equations with coupled integral boundary conditions

Rodica Luca Tudorache [a] [1], Johnny Henderson [2]

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We investigate the existence and nonexistence of positive solutions for a system of nonlinear Riemann-Liouville fractional differential equations with parameters, subject to coupled Riemann-Stieltjes integral boundary conditions. In the proof of our existence results, we use the Guo-Krasnosel'skii fixed point theorem.

[a] Speaker

Periodic solutions for parabolic equations on R^N

Aleksander Cwiszewski [1], Renata Lukasiak [a] [1]

[1] Nicolaus Copernicus University – Poland

We consider a nonautonomous parabolic equation

$$u_t(x,t) = \Delta u(x,t) + f(t,x,u(x,t)), \qquad x \in R^N, \ t > 0, \tag{10}$$

with continuous and T-periodic in time nonlinearity $f : [0,+\infty) \times R^N \times R \to R$. By using a translation along trajectories approach we derive criteria for the existence of T-periodic solutions of (10). Here, the translation operator is not compact on bounded sets. Therefore we apply tail estimates to show that it is ultimately compact. This will allow us to compute topological indices of the translation operator and use continuation together with averaging techniques to show that branches of periodic solutions emanate from nontrivial solutions of

$$-\Delta u = \widehat{f}(x,u),$$

where $\widehat{f} : R^N \times R \to R$ is the time-average function of f.

[a]Speaker

Spherically symmetric solutions to the Euler-Poisson equations and the Einstein-Euler equations

Tetu Makino [a] [1]

[1] Yamaguchi University – Japan

We can construct spherically symmetric solutions to the evolution equations of gaseous stars in the non-relativistic and relativistic theory. The solutions are close to an equilibrium, governed by the Lane-Emden or the Tolman-Oppenheimer-Volkoff equation, plus a small time periodic solution to the linearized equations at the equilibrium, and touch the vacuum with the so called physical vacuum boundary. Detailed discussions can be found at arXiv:1210.5769 and arXiv:1410.1234. But the study already done assumes that $\gamma/(\gamma-1)$ is an integer, where γ is the adiabatic exponent of the gas near the vacuum. To extend the result at least to the case when $\gamma = 5/3$ and so on is an important open problem.

[a]Speaker

Stability of front solutions in a model for a surfactant driven flow on an inclined plane

Vahagn Manukian [a] [1], Anna Ghazaryan [1], Stephane Lafortune [2]

[1] Department of Mathematics, Miami University – United States
[2] Department of Mathematics, College of Charleston – United States

We consider a model for the flow of a thin liquid film down an inclined plane in the presence of a surfactant. The model is known to possess various families of traveling wave solutions. We use a combination of analytical and numerical methods to study the stability of the traveling waves. We show that for at least some of these waves the spectra of the linearization of the system about them are within the closed left-half complex plane.

[a] Speaker

On the Asymptotic Properties of Piecewise Contracting Maps

Arnaud Meyroneinc [a] [1], Eleonora Catsigeras [2], Pierre Guiraud [3], Edgardo Ugalde [4]

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We are interested in the phenomenology of the asymptotic dynamics of piecewise contracting maps. They appear for instance as Poincaré maps in the study of bifurcations of stable heteroclinic cycles of some C^1 (non-equivariant) vector fields on $\mathbb{R}^n$, in the characterization of Cherry flows on compact two-manifolds, or in the characterization of the asymptotic stability of some piecewise continuous vector fields. They can also be found as discrete time models of regulatory networks with thresholds. In either way, they naturally appear in the modelling of some biological systems (neural, genetic and ecological) and in engineering (electro-mechanical and switched arrival-server systems), where the number of variables can be significantly high.
We consider here a wide class of such maps, i.e. Lipschitz contracting when restricted to any piece of a finite and dense union of disjoint open pieces in a compact metric space X. We give sufficient conditions to ensure some general basic properties, such as the periodicity, the total disconnectedness or the zero Lebesgue measure (when $X \subset \mathbb{R}^n$, $n \geq 1$) of the attractor. These conditions show in particular that a non-periodic attractor necessarily contains discontinuities of the map. Under this hypothesis, we obtain numerous examples of attractors, ranging from finite to connected and chaotic, contrasting with the (quasi-)periodic asymptotic behaviours observed so far.

[a] Speaker

Everywhere discontinuous anisotropy of thin periodic composite plates

Cherdantsev Mikhail [a] [1], Cherednichenko Kirill [2]

[1] Cardiff University – United Kingdom
[2] University of Bath – United Kingdom

We consider an elastic periodic composite plate in the full bending regime, i.e. when the displacement of the plate is of finite order. Both the thickness of the plate h and the period of the composite structure ε are small parameters. We start from the non-linear elasticity setting. Passing to the limit as $h, \varepsilon \to 0$ we carry out simultaneous dimension reduction and homogenisation to obtain an effective limit elastic functional which describes the asymptotic properties of the composite plate. We show, in particular, that in the regime $h << \varepsilon^2$ the limit elastic functional is discontinuously anisotropic in every direction of bending. This remarkable property (suggesting that the corresponding composite plate can be referred to as metamaterial) is due to the in-limit linearisation of the bending deformations and the multi scale interaction.

[a]Speaker

Mathematical modelling of cell adhesion forces : from delay to friction, an instantaneous limit.

Vuk Milisic [a] [1]

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This is a joint work with Dietmar OELZ from the Courant Institute, NYU. In this talk we present the starting mechanical model of the lamellipodial actin-cytoskeleton meshwork. The model is derived starting from the microscopic description of mechanical properties of laments and cross-links and also of the life-cycle of cross-linker molecules [2]. We introduce a simplified system of equations that accounts for adhesions created by a single point on which we apply a force. We present the adimensionalisation that led to a singular limit that motivated our mathematical study. Then we explain the mathematical setting and results already published [2, 3]. In the last part we present the latestdevelopments : we give results for the fully coupled system with unbounded non-linear off-rates [1]. This leads to two possible regimes : under certain hypotheses on the data there is global existence, out of this range we are able to prove blow-up in nite time.

References :

1. V. Milisic and D. Oelz. Unbounded o-rates in the structured model for friction mediated by transient elastic linkages. in preparation.
2. V. Milisic and D. Oelz. On the asymptotic regime of a model for friction mediated by transient elastic linkages. J. Math. Pures Appl. (9), 96(5):484-501, 2011.
3. V. Milisic and D. Oelz. On a structured model for the load dependent reaction kinetics of transient elastic linkages mediating nonlinear friction. 2015. Accepted for publication in SIAM SIMA.

[a]Speaker

Existence of a weak solution to a fluid-elastic structure interaction problem with the slip boundary condition

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We study a nonlinear, moving boundary fluid-structure interaction problem between the $2D$ Newtonian fluid and $1D$ elastic structure of a shell/plate type. The structure and the fluid velocities are coupled via the Navier's slip boundary condition. The slip boundary condition is more realistic then the classical no-slip boundary condition is some realistic situations, for example in describing the dynamics near a contact, or describing the multi-layered blood vessel. We prove existence of a weak solution in a quite general setting. The proof is constructive and we propose the corresponding convergent numerical scheme for solving the problem.

[a]Speaker

Symmetry results for a family of Caffarelli-Kohn-Nirenberg inequalities

Matteo Muratori [a] [1]

[1] Università degli Studi di Milano – Italy

We investigate optimal functions for the following inequalities:

$$\left(\int_{\mathbb{R}^d} |w(x)|^{2p}\, |x|^{-\gamma} \mathrm{d}x\right)^{\frac{1}{2p}} \leq C_\gamma \left(\int_{\mathbb{R}^d} |\nabla w(x)|^2\, \mathrm{d}x\right)^{\frac{\vartheta}{2}} \left(\int_{\mathbb{R}^d} |w(x)|^{p+1}\, |x|^{-\gamma} \mathrm{d}x\right)^{\frac{1-\vartheta}{p+1}} ,$$

where $d \geq 3$, $\gamma \in (0,2)$, $p \in (1,(d-\gamma)/(d-2))$, $\vartheta = \vartheta(d,\gamma,p) \in (0,1)$ and $C_\gamma > 0$. For $\gamma = 0$ they coincide with $b(x) := (1+|x|^2)^{-\frac{1}{p-1}}$ [Del Pino, Dolbeault, JMPA (2002)]. If $\gamma \in (0,2)$ they continue to exist, and our main concern is their radial symmetry. Indeed, as soon as optimal functions are radial, they coincide with $b_\gamma(x) := (1+|x|^{2-\gamma})^{-\frac{1}{p-1}}$. However, Schwarz symmetrization techniques fail. We then let $\gamma \to 0$ and exploit known results at $\gamma = 0$. By means of a concentration-compactness analysis we prove that optimal functions converge to b. Afterwards we use a perturbation argument by contradiction, which involves angular derivatives of possibly nonradial optimal functions. Such an argument allows us to prove that optimal functions do coincide with b_γ for γ small. The talk is based on [Dolbeault, M., Nazaret, Symmetry in weighted interpolation inequalities, preprint].

[a]Speaker

Singular perturbation problems in perforated domains

Paolo Musolino [a] [1]

[1] Dipartimento di Matematica, Università degli Studi di Padova – Italy

The asymptotic behaviour of the solutions of boundary value problems in domains with small holes has been investigated by many authors with different approaches. In this talk, we consider a Dirichlet problem for the Laplace operator in a bounded domain Ω^o of $\mathbb{R}^n$ containing the origin, where we remove a small set whose size is determined by a parameter ϵ and which collapses to 0 for $\epsilon = 0$. Then for $\epsilon \neq 0$ we denote the solution to such a problem by u_ϵ. If $p \in \Omega^o$ and $p \neq 0$, then it makes sense to consider for $\epsilon \neq 0$ and 'small' the value of the solution u_ϵ at the point p. It is natural to ask what can be said on the map which takes ϵ small and positive to $u_\epsilon(p)$ around the degenerate value $\epsilon = 0$. One can try to answer to this question in several ways. By the approach proposed by Lanza de Cristoforis, one can show that, if $n \geq 3$, then there exist $\epsilon_p > 0$ and a real analytic function U_p from $]-\epsilon_p, \epsilon_p[$ to $\mathbb{R}$ such that $u_\epsilon(p) = U_p[\epsilon]$ for all $\epsilon \in]0, \epsilon_p[$, and one can then investigate the validity of such an equality for ϵ negative. After an introductory part on the case of dimension $n \geq 3$, we will turn to consider the two-dimensional case.
Based on joint works with M. Dalla Riva (CIDMA, Universidade de Aveiro) and S.V. Rogosin (Belarusian State University).

[a] Speaker

Existence and blowing up character of solutions to Volterra type integral equations related to ordinary differential equations with monotonic nonlinearities

Wojciech Mydlarczyk [a] [1]

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We are going to discuss a nonlinear Abel type integral equation of the

$$u(t) = \int_{-\infty}^{t} (t-s)^{\alpha-1} r(s) g(u(s)) ds \quad (-\infty < t, \quad 0 < \alpha),$$

where $r(s)$ and $g(u)$ (with $g(0) = 0$) are nonnegative nondecreasing functions. For an integer α, it is an integral formulation of the initial value problem for the ordinary differential equation

$$u^{(n)}(t) = r(t)g(u(t)), \;\; u(-\infty) = u'(-\infty) = \cdots = u^{(n-1)}(-\infty) = 0.$$

Such equations arise in the investigation of one-dimensional models of a diffusive medium which can experience explosive behavior. It is easily observed that these equations have trivial solution $u(t) \equiv 0$. However, from physical point of view only positive solutions are interesting. We analyze nonnegative nondecreasing solutions $u(t)$, $t > -\infty$ and show when they starts with some $t_0 > -\infty$, i.e. $u(t) = 0$ for $t \leq t_0$ and $u(t) > 0$ for $t > t_0$. We also provide conditions under which the solution $u(t)$ is blowing up.

[a] Speaker

Separation structure of positive radial solutions for semilinear elliptic equations

Yuki Naito [a 1]

[1] Yuki Naito – Department of Mathematics, Ehime University, Matsuyama 790-8577, Japan

We consider positive radial solutions of the semilinear elliptic equation

$$-\Delta u = f(u) \quad \text{in } \mathbf{R}^N,$$

where $N \geq 3$, $f \in C^1$, and $f(u)$ is non-negative, non-decreasing, and convex for $u \geq 0$. We are interested in separation phenomena of positive radial solutions. In a typical case $f(u) = u^p$, it is well known that, if $N \geq 11$ and

$$p \geq \frac{(N-2)^2 - 4N + 8\sqrt{N-1}}{(N-2)(N-10)},$$

then any two radial solutions do not intersect each other. In this talk, for general nonlinearity, we give a classification of the solution structures, and then we show separation and intersection properties of solutions. In particular, we find that the equation changes its nature drastically according to the behavior and some other properties of $f(u)$.

[a]Speaker

Inviscid incompressible limits on expanding domains

Sarka Necasova [a 1], Eduard Feireisl, Yongzhong Sun

[1] Institute of Mathematics, Academy of Sciences (IM ASCR) – Zitna 25, 11567 Praha 1, Czech Republic

We consider the inviscid incompressible limit of the compressible Navier-Stokes system on large domain, the radius of which becomes infinite in the asymptotic limit. We show that the limit solutions satisfy the incompressible Euler system on the whole physical space R^3 as long as the radius of the domain is larger than the speed of acoustic waves inversely proportional to the Mach number. It is a joint work with E. Feireisl and Y. Sun.

[a]Speaker

L^∞-stability of traveling waves to a hyperbolic-elliptic coupled system

Masashi Ohnawa [a] [1]

[1] Waseda University – Japan

In this talk, we discuss L^∞-stability of traveling waves or shock waves to a hyperbolic-elliptic coupled system of $u_t + uu_x + q_x = 0,\ -q_{xx} + q + u_x = 0$ over $x \in \mathbb{R}$, where u and q are scalar functions. This is called the Hamer model, derived by simplifying a system of a gas dynamics with radiation. S. Kawashima and S. Nishibata(1998) clarified that the system admits traveling waves uniquely up to shifts if $\delta := u_- - u_+$ is positive, and remarkably, the profile is discontinuous if and only if $\delta > \sqrt{2}$ where $u_\pm := \lim_{x\to\pm\infty} u(t,x)$.
We first show that subcritical shock waves i.e. the case with $\delta < \sqrt{2}$ are stabilized by radiation, while an arbitrary small perturbation could give rise to a blow up of u_x in a finite time for a critical shock wave ($\delta = \sqrt{2}$). Then we show supercritical shock waves ($\delta > \sqrt{2}$) regain stability thanks to the presence of discontinuity and the contribution of convection. These results could be understood as a rationalization of the difference in continuity of traveling waves from the view point of L^∞-stability, making a stark contrast to the powerful results by D. Serre(2003): all traveling waves are L^1-stable to arbitrary large perturbations.

[a]Speaker

Analytic solutions of a class of nonlinear partial differential equations

Eugenia Petropoulou [a] [1]

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A general class of nonlinear partial differential equations, which can be connected with wave-type equations and Laplace-type equations, is studied by use of a functional-analytic technique. The results establish primarily the existence and uniqueness of bounded solutions in the two-dimensional Hardy–Lebesque space of analytic functions with independent variables lying in the open unit disc, although they can be simply modified in order to increase the definition domain of the established solution. The proofs have a constructive character enabling the determination of concrete and easily verifiable conditions, as well as the determination of the coefficients appearing in the power series solution. Illustrative examples are given concerning the sine–Gordon equation, the Klein–Gordon equation, as well as equations with nonlinear terms of algebraic, exponential and logistic type.

[a]Speaker

On the steady compressible Navier–Stokes–Fourier system with temperature dependent viscosities

Milan Pokorny [a] [1]

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We consider in $\Omega \subset R^3$

$$\mathrm{div}\,(\varrho\mathbf{u}) = 0, \tag{11}$$

$$\mathrm{div}(\varrho\mathbf{u}\otimes\mathbf{u}) - \mathrm{div}\mathbf{S} + \nabla p = \varrho\mathbf{f}, \tag{12}$$

$$\mathrm{div}(\varrho E\mathbf{u}) = \varrho\mathbf{f}\cdot\mathbf{u} - \mathrm{div}(p\mathbf{u}) + \mathrm{div}(\mathbf{S}\mathbf{u}) - \mathrm{div}\mathbf{q} \tag{13}$$

which models steady flow of a heat conducting compressible fluid. We consider (11)–(13) together with the boundary conditions at $\partial\Omega$

$$\mathbf{u} = \mathbf{0}, \tag{14}$$

$$-\mathbf{q}\cdot\mathbf{n} + L(\vartheta - \Theta_0) = 0. \tag{15}$$

We consider Newtonian fluids, i.e. $\mathbf{S} = \mu(\vartheta)(\nabla\mathbf{u} + \nabla\mathbf{u}^T - \frac{2}{3}\,\mathrm{div}\;\mathbf{u}\mathbf{I}) + \xi(\vartheta)\mathrm{div}\,\mathbf{u}\mathbf{I}$, with the pressure $p \sim \varrho\vartheta + \varrho^\gamma$ and the heat flux $\mathbf{q} = -\kappa(\vartheta)\nabla\vartheta$. We study existence of a solution to our problem (11)–(15) in dependence on γ, α and m, where $\mu(\vartheta)$, $\xi(\vartheta) \sim (1+\vartheta)^\alpha$, $\kappa(\vartheta) \sim (1+\vartheta)^m$.

[a] Speaker

Existence results to the nonlinear peridynamic model in nonlocal elastodynamics

Dimitri Puhst [a] [1]

[1] Technische Universität Berlin – Germany

Peridynamics is a nonlocal elasticity theory based on differences in the deformation instead of the deformation gradient. It is therefore suitable to describe long range forces as well as material failure.

In this talk, we will give an introduction into the theory of peridynamics and consider its equation of motion as a nonlinear second order evolution equation. We present results on existence of weak and measure-valued solutions in the absence of any monotonicity assumption on the peridynamic operator. The method of proof also applies to other nonlocal partial differential equations such as evolution equations involving the fractional p-Laplacian.

This is joint work with Etienne Emmrich.

[a] Speaker

Stability and bifurcation investigation of discrete-time nonlinear systems by realization theory methods

Volker Reitmann [a] [1], Yulia Abdalova

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Realization theory for linear input-output operators and frequency-domain methods for the solvability of Riccati operator equation are used for the stability and bifurcation investigation of nonlinear discrete-time non-autonomous difference equations. The key idea is ([3]) to consider a time-invariant control system (discrete-time Boltzmann transport equation) generated by the shift operator and some control operator in a weighted Sobolev space which has the same stability properties as the given difference equation.
As an example we investigate a discrete-time difference equation of the cardiac conduction problem ([1,2]).

References [1] Y. Abdalova: Local bifurcation for discrete-time non-autonomous systems connected with a heart model. Proc. of the International Student Conference "Science and Progress", St. Petersburg, 2014.
[2] A. Maltseva and V. Reitmann: Global stability and bifurcation of invariant measures for the discrete cocycles of the cardiac conduction system's equations. Differential equations, vol. 50/13, pp. 1718 – 1732, 2014.
[3] V. Reitmann: Realization theory methods for the stability investigation of non-linear infinite-dimensional input-output systems: MATHEM-BOHEMICA, vol. 136/2, pp. 185-194, 2011.

[a]Speaker

Sturm global attractors as regular CW-complexes

Carlos Rocha [a] [1], Bernold Fiedler [2]

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We consider global attractors for dissipative semiflows generated by scalar reaction-diffusion equations defined on an interval with Neumann boundary conditions. Assuming hyperbolicity of all equilibrium solutions, the global attractor possesses a dynamical decomposition as a finite disjoint union of unstable manifolds of the equilibria. We use this decomposition to describe the global attractor as a finite regular CW-complex. This is based on a joint work with B. Fiedler.

[a]Speaker

Sonic lines arising from 2-D Riemann problems of the compressible Euler system

Kyungwoo Song [a] [1]

[1] Department of Mathematics, Kyung Hee University – 26 Kyungheedae-ro, Dongdaemun-gu, Seoul 130-701, South Korea

We study the sonic lines to a Riemann problem for the two-dimensional Euler system in the self-similar plane. In the plane, the governing equation becomes quasilinear and changes its type. The type of the flow in the far-field is hyperbolic and the type of the flow near the origin is mixed. The semi-hyperbolic patches and sonic lines are located inside the mixed area. We are specially interested in the property of the sonic line where the degeneracy of hyperbolicity occurs. This type of solution patch and the sonic line appear in the transonic flow over an airfoil and the Guderley reflection, and is common in the numerical configurations of Riemann problems. The exact behavior of solutions in the semi-hyperbolic patches near the sonic lines are studied and the C1 regularity of the sonic line is obtained.

[a]Speaker

Quasilinearization and resonant boundary value problems

Felix Sadyrbaev [a] [1], Nadezhda Sveikate [2]

[1] Institute of Mathematics and Computer Science, University of Latvia – Latvia
[2] Daugavpils University – Latvia

We consider boundary value problems in the form

$$(l_2x)(t) = \varphi(t, x, x'), \quad x(a) = A,\ x(b) = B, \tag{16}$$

where $(l_2x)(t) := x'' + p(t)x' + q(t)x$ is a resonant linear differential expression (resonant means that the homogeneous problem $(l_2x)(t) = 0$, $x(a) = 0$, $x(b) = 0$ has a nontrivial solution). Our approach is based on quasilinearization idea (M. Dobkevich, F. Sadyrbaev, N. Sveikate, I. Yermachenko. On Types of Solutions of the Second Order Nonlinear Boundary Value Problems. Abstract and Applied Analysis, Volume 2014 (2014), Article ID 594931) which is employed to convert the problem into that with a nonresonant linear part.

[a]Speaker

A critical problem for fractional Laplacians in contractible domains

Naoki Shioji [a] [1]

[1] Yokohama National University – Japan

We study the problem

$$\begin{cases} (-\Delta)^s u = u^{\frac{N+2s}{N-2s}} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega \end{cases}$$

involving the fractional Laplacian $(-\Delta)^s$. Here, $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ is a bounded domain with smooth boundary,

$$(-\Delta)^s u(x) = C(N,s) \lim_{\varepsilon \to 0} \int_{\complement B_\varepsilon(x)} \frac{u(x) - u(y)}{|x-y|^{N+2s}}\, dy \quad \text{for } u \in C^\infty(\mathbb{R}^N) \cap \dot{H}^s(\mathbb{R}^N),$$

where $s \in (0,1)$ and $C(N,s)$ is a positive constant. Recently, such kind of problems are studied by many researchers. Ros-Oton and Serra [ARMA 213 (2014)] showed if Ω is star-shaped, then the problem does not admit solutions. Secchi, Squassina and the speaker [DIE 28 (2015)] showed that if Ω has a small hole then the problem has a solution. Servadei and Valdinoci [TAMS 365 (2015)] studied the Brezis-Nirenberg problem, which is closely related to the problem. In this talk, we will show that if $N \geq 3$ and $s \in (0,1)$, or $N = 2$ and $s \in (0, 1/2]$, then there is a contractible domain Ω such that the problem has a solution. We will give such a domain. This result is based on a joint work with Mosconi and Squassina.

[a] Speaker

Energy dissipations for non-isothermal models of grain boundary motions

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[2] Salesian Polytechnic – 4-6-8 Oyamagaoka, Machida, Tokyo, 194-0215, Japan

This study is based on recent jointworks with Prof. Moll, J. S., University of Valencia, Spain. In this talk, systems of parabolic type PDEs are considered. These are based on phase field models of grain boundary motions, proposed in [Warren, J. A., Kobayashi, R., Lobkovsky, A. E. and Carter, W. C.: Acta Materialia, 51 (2003), 6035–6058]. Each of our systems consists of:
(1st.eq.) heat equation,
(2nd.eq.) nonstandard parabolic equation for phase transition,
(3rd.eq.) singular diffusion equation for crystalline orientation,
and in particular, the latter two equations (2nd.eq.)-(3rd.eq.) are derived as a gradient system of a governing energy, called free-energy. Hence, in the light of modelling methods, we can expect that our systems should be to reproduce some energy-dissipations, and the energy-dissipations should bring in some stability for the corresponding dynamical systems.
The aim in this talk is to have certain positive answers for the expectation. To this end, we focus on a special kind of solution, named "energy-dissipative solution", which somehow achieves the energy-dissipation together with the compatibilities with (1st.eq.)-(3rd.eq.). Under suitable assumptions, several theorems concerned with:
(A) the existence of energy-dissipative solutions,
(B) the large-time behavior for energy-dissipative solutions,
will be demonstrated as the main results of this talk.

[a] Speaker

Attractors for Damped Semilinear Wave Equations with a Robin–Acoustic Boundary Perturbation

Sergio Frigeri [1], Joseph Shomberg [a] [2]

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Under consideration is the damped semilinear wave equation

$$u_{tt} + u_t - \Delta u + u + f(u) = 0$$

on a bounded domain Ω in $\mathbb{R}^3$ with a perturbation parameter $\varepsilon > 0$ occurring in an acoustic boundary condition, limiting ($\varepsilon = 0$) to a Robin boundary condition. With minimal assumptions on the nonlinear term f, the existence and uniqueness of global weak solutions is shown for each $\varepsilon \in [0, 1]$. Also, the existence of a family of global attractors is shown to exist (re: J. Ball's generalized semiflows). After proving a general result concerning the upper-semicontinuity of a one-parameter family of sets, the result is applied to the family of global attractors. No further regularity from the global attractors is needed in order to obtain this upper-semicontinuity result.
With more relaxed assumptions on the nonlinear term f, we are able to show the global attractors possess optimal regularity and prove the existence of an exponential attractor, for each $\varepsilon \in [0, 1]$. This result insures that the corresponding global attractor inherits finite (fractal) dimension; however, the dimension is *not* necessarily uniform in ε.

[a] Speaker

Weakly nonlinear waves in non-ideal fluids

Triveni Shukla [a] [1], V. Sharma

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We study the propagation of weakly nonlinear waves in non-ideal fluids in which the local value of the fundamental derivative changes sign. Perturbation expansions with multiple scales are used to study the behavior of the flow governed by the usual Navier-Stokes equations with viscosity and thermal conduction, supplemented by the equation of state for a van der Waals fluid; the transport equation is a parabolic evolution equation that includes both quadratic and cubic nonlinearities inherent in the governing hyperbolic system. Riemann's solutions of the associated kinematic equation are presented with a rectangular pulse initial distribution. The interaction time of shocks and wave-fans, speeds and strengths of shocks, widths of wave-fans, and the eventual decay rate of merged shocks, influenced by the van der Waals parameters, are investigated. The parabolic evolution equation, in the hyperbolic limit, is solved numerically using the fifth order WENO scheme and the results are compared with the exact analytical solutions of the associated kinematic equation.

[a] Speaker

Standing and Traveling Waves in Two-Layer Systems with Heat Release/Consumption at the Interface

Ilya Simanovskii [a] [1]

[1] Technion – Israel

Convective phenomena in systems with interfaces has been a subject of an extensive investigation in the past few decades (for a review, see I. Simanovskii, A. Nepomnyashchy, "Convective Instabilities in Systems with Interface", (Gordon and Breach, London, 1993); A. Nepomnyashchy, I. Simanovskii, J. C. Legros,"Interfacial Convection in Multilayer Systems", Second Edition, (Springer, New York, 2012).
We consider a system of two horizontal layers of immiscible viscous fluids with different physical properties. A constant heat release is set on the interface. Nonlinear oscillatory convective regimes, developed under the joint action of buoyant and thermocapillary effects in a two-layer system with periodic boundary conditions on the lateral walls, have been investigated. The computational regions with different lengths, have been considered. It is shown that in the case of periodic boundary conditions we get completely different nonlinear regimes than those, which have been obtained in the case of closed cavities by I. Simanovskii, Phys. Fluids, 25, 072106 (2013). Specifically, regimes of traveling waves and modulated traveling waves, have been found. It is shown that for sufficiently small values of the modified Grashof number, corresponding to the case of heat sinks, the phase velocity of the traveling wave changes in a non-monotonic way. Nonlinear oscillatory flows exist in a finite interval of the Grashof number values, bounded from below and from above.

[a]Speaker

Arnold's diffusion and formally gradient dynamics of the action functional

Sinisa Slijepcevic [a] [1]

[1] University of Zagreb – Croatia

We prove existence of Arnold's diffusion in the well-known Arnold's example of a two and a half degrees of freedom Hamiltonian, for all values of the parameters; i.e. in the entire range from arbitrarily close to far from integrable. We introduce a new technique of considering formally gradient dynamics of the action functional. The approach relies on the emerging theory of extended gradient systems, including their asymptotics and stability, combined with a version of the Morse-Sard theorem, and geometric and variational information of the original equation. We compare the approach with some recent results on Arnold's diffusion with either geometric or variational approach.

[a]Speaker

Holomorphic normal form of nonlinear perturbations of nilpotent vector fields

Laurent Stolovitch [a] [1]

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We consider the problem of transformation to a normal form of germ of holomorphic nonlinear perturbations of a nilpotent linear vector field in $(\mathbb{C}^n, 0)$, $n \geq 2$. In dimension $n = 2$, every such vector field is analytically conjugated to its Takens normal form (Stróżyna-Żoładek theorem) and such a statement is known to be false in dimension $n > 2$. We give a sufficient condition that ensures that there exists a germ of holomorphic transformation to a normal form of a germ of holomorphic nonlinear perturbation of a nilpotent linear vector field in $(\mathbb{C}^n, 0)$, $n > 2$. Our proof is based on $\mathfrak{sl}_2(\mathbb{C})$-representations and a general notion of normal form.

[a] Speaker

Glimpses on Lipschitz truncations and regularity

Bianca Stroffolini [a] [1]

[1] Dipartimento di Matematica e Applicazioni "Renato Caccioppoli" – Via Cintia, Monte S. Angelo I-80126 Napoli, Italy, Italy

It is well known that in the vectorial case solutions of a nonlinear elliptic system are of class $C^{1,\alpha}(\Omega \setminus \Sigma, \mathbb{R}^N)$ where Σ has n-dimensional Lebesgue measure zero. A recent approach for proving partial regularity is based on the so called $\mathcal{A}$-harmonic approximation method, that is, obtaining a good approximation of functions$u \in W^{1,2}(B, \mathbb{R}^N)$ with the solution h of a suitable linearized system. The solution u is shown to be *almost $\mathcal{A}$-harmonic*:

$$|\frac{1}{|B|}\int_B \mathcal{A}\nabla u \cdot \nabla \xi \, dx| \leq \delta\Big(\frac{1}{|\widetilde{B}|}\int_{\widetilde{B}} |\nabla u| \, dx\Big)\|\nabla \xi\|_{L^\infty} \tag{17}$$

for all $\xi \in C_0^\infty(B, \mathbb{R}^N)$. The idea now is that the good regularity estimates available for h are inherited by u. Originally, the closeness of the function to its $\mathcal{A}$-harmonic approximation was stated in terms of the L^2-distance and for nonlinear problem, in terms of the L^p-distance. Based on a refinement of the Lipschitz truncation technique, we have shown that also the distance in terms of the gradients is small. I will present a suitable version of the $\mathcal{A}$-harmonic approximation lemma in the Orlicz setting. Next, for parabolic systems, I will state a p-caloric approximation Lemma and related partial regularity results.

[a] Speaker

White noise perturbation of the autoresonance model

Oskar Sultanov [a] [1]

[1] Institute of Mathematics with Computer Center of the Ufa Science Center of the Russian Academy of Sciences (IMCC USC RAS) – Russia

A system of two first-order ordinary differential equations describing the initial stage of a capture into a resonance in forced nonlinear oscillatory systems is considered. Unboundedly growing solutions in time at infinity correspond to autoresonance phenomenon. By the reason of the nonlinearity of the considered equations the explicit formulas for the solutions cannot be obtained. However, it is possible to construct an asymptotic expansion for some particular solutions in the form of power series. We study the stability of such solutions under perturbations of white noise type. Analysis of stability is based on the construction an appropriate Lyapunov function. We propose a method for constructing of such functions that can be used for more general nonlinear systems.

[a] Speaker

Existence and asymptotic behavior of strongly monotone solutions of nonlinear differential equations

Tomoyuki Tanigawa [a] [1]

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Two types of nonlinear differential systems

$$x' + p(t)y^{\alpha} = 0, \qquad y' + q(t)x^{\beta} = 0; \tag{A}$$

$$x' - p(t)y^{\alpha} = 0, \qquad y' - q(t)x^{\beta} = 0 \tag{B}$$

are considered under the assumption that α and β are positive constants such that $\alpha\beta < 1$ and $p(t)$ and $q(t)$ are continuous regularly varying functions on a neighborhood of infinity. An attempt is made to obtain precise information on the existence and asymptotic behavior of strongly monotone regularly varying solutions $(x(t), y(t))$ of (A) and (B) whose x-components or y-components are slowly varying. It is shown that the results thus obtained are applied to the generalized Thomas-Fermi equations of the form $(p(t)|x'|^{\alpha-1}x')' = q(t)|x|^{\beta-1}x$ to provide new useful knowledge of their strongly monotone solutions. The present paper is designed to supplement the pioneering results on the asymptotic analysis of (A) and (B) by means of regular variation developed in the paper [1].

References

[1] J. Jaroš and T. Kusano, Existence and precise asymptotic behavior of strongly monotone solutions of systems of nonlinear differential equations, *Differ. Equ. Appl.* 5 (2013), 185 – 204.

[a] Speaker

Lifespan of solutions to nonlinear Cauchy problems with small analytic data

Hidetoshi Tahara [a] [1]

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The effect of small initial data on the solutions of Cauchy problems has been extensively studied by many authors. Most of the research focus on the interrelated notions of lifespan, finite-time blowup, and global solvability.
In this talk, I will treat this problem from the standpoint of Cauchy-Kowalevsky theorem. Namely, I will consider the lifespan of solutions to the Cauchy problem for general analytic nonlinear partial differential equations with small analytic data. The main result is as follws: the lifespan of the solution becomes longer as the initial data become smaller, and the dependence of the lifespan on the smallness of the data is sharply described by the property of the equation. This gives a generalization of results in Gourdin-Mechab (Temps de vie des solutions d'un probleme de Cauchy non lineaire, C. R. Acad. Sci. Paris, t. 328, Serie I (1999)), and Yamane (Nonlinear Cauchy problems with small analytic data and the lifespan of their solutions, Banach Center Publ., 97 (2012)).

[a]Speaker

Uniqueness of sign-changing radial solutions for scalar field equations in some ball and annulus

Satoshi Tanaka [a] [1]

[1] Okayama University of Science – Japan

The following Dirichlet problem is considered: $\Delta u - u + |u|^{p-1}u = 0$ in Ω; $u = 0$ on $\partial\Omega$. Here, Ω is either an annulus or a ball in $\mathbf{R}^N$ and $p > 1$. The uniqueness of radial solutions having exactly $k - 1$ nodes is shown in some cases.

[a]Speaker

Determination of one unknown thermal coefficient through the one-phase fractional Stefan problem

Domingo Tarzia [a] [1]

[1] Universidad Austral – Paraguay 1950, S2000FZF Rosario, Argentina

We obtain explicit expressions for one unknown thermal coefficient (among the conductivity, mass density, specific heat and latent heat of fusion) of a semi-infinite material through the one-phase fractional Stefan problem with an overspecified boundary condition on the fixed face $x = 0$. The partial differential equation and a condition on the free boundary include a time Caputo's fractional derivative of order less than one. Moreover, we get the necessary and sufficient conditions on data in order to have a unique solution by using recent results obtained for the fractional diffusion equation given in: Roscani - Santillan Marcus, Fract. Calc. Appl. Anal., 16 (2013), 802-815; Roscani-Tarzia, Adv. Math. Sci. Appl., 24 (2014), 237-249 and Voller, Int. J. Heat Mass Transfer, 74 (2014), 269-277 by using the Wright and Mainardi functions. This work generalizes the method developed for the determination of unknown thermal coefficients given for the classical Stefan problem in Tarzia, Adv. Appl. Math., 3 (1982), pp. 74-82 which are recovered by taking the limit when the order goes to one.

[a] Speaker

Continuous dependence of solutions to the free boundary problem describing adsorption phenomena on boundary data

Aiki Toyohiko [a] [1], Murase Yusuke [2]

[1] Japan Women's University – Japan
[2] Meijo university – Japan

Recently, we have proposed a one-dimensional free boundary problem as a mathematical model for adsorption phenomena appearing in porous media. More precisely, the relationship between the relative humidity and the saturation of degree is represented by the relationship between the boundary data and the position of the free boundary. By numerical simulations we can say that our free boundary problem is one of good mathematical description for the adsorption phenomena. On this free boundary problem we already established the existence and the uniqueness of a solution. Also, under some suitable conditions we obtained the large time behavior result of the free boundary. As a next step we consider a concrete carbonation problem in a three-dimensional domain with adsorption phenomena. In order to apply our free boundary problem to the concrete carbonation problem it is necessary to investigate continuous dependence of the solution on boundary data, because the concrete carbonation problem consists of some partial nonlinear differential equations with coefficients given by a gradient of saturation. In this talk we show modeling, mathematical results and graphs from numerical simulations on adsorption phenomena.

[a] Speaker

Approximate current-vortex sheets near the onset of instability

Paola Trebeschi [a] [1], Alessandro Morando [1], Paolo Secchi [1]

[1] DICATAM - University of Brescia – via Branze, 43 - 25123 Brescia, Italy

In this talk I present a recent result about the free boundary problem for 2D current-vortex sheets in ideal incompressible magneto-hydrodynamics near the transition point between the linearized stability and instability. In order to study the dynamics of
the discontinuity near the onset of the instability, Hunter and Thoo have introduced an asymptotic quadratically nonlinear integro-differential equation for the amplitude of small perturbations of the planar discontinuity. We study such amplitude equation
and prove its nonlinear well-posedness under a stability condition given in terms of a longitudinal strain of the fluid along the discontinuity. This is a joint work with A.Morando and P. Secchi.

[a] Speaker

A kinetic equation for modelling irrationality and herding effects

Lara Trussardi [a] [1], Ansgar Jüngel [1], Bertram Düring [2]

[1] Technical University of Vienna (TU WIEN) – Austria
[2] Department of Mathematics [Sussex] – University of Sussex Sussex House Brighton BN1 9RH United Kingdom, United Kingdom

In financial crises, often herding behaviour, which is characterized by prize bubbles and crashes, can be observed. This behaviour may be caused by an irrational behaviour of the market participants, e.g. driven by emotion. We suggest a simple kinetic equation based on binary interactions of the market agents.
The goal is to describe the evolution of the distribution function of the value of a given product in a large market taking into account the effect of a herding behaviour (which is not possible with classical linear financial models) and the rationality of the individuals. We derive the corresponding nonlocal Fokker-Planck equation in the crazing collisions limit and we prove the existence of weak solutions. Finally, we present some numerical simulations.

[a] Speaker

Existence and uniqueness for heat equations with hysteresis coupled with Navier-Stokes equations in 2D and 3D domains

Yutaka Tsuzuki [a] [1]

[1] Tokyo University of Science (TUS) – 1-3 Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan

There was no result on existence and uniqueness for a system of heat equations with hysteresis and Navier-Stokes equations, which describes phenomenon by thermostat devices with incompressible fluid. Recently, an existence result for the problem in a 2D domain was obtained in [1], provided that the Navier-Stokes equation was dealt with in a weak sense. However this result does not assert uniqueness for the problem nor the 3D case. In this talk, by turning our eyes to a new inequality, we establish not only existence but also uniqueness for the problem in both the 2D and 3D cases. Here we deal with the Navier-Stokes equation in a stronger sense by introducing the fractional power of the Stokes operator A^{α}. In particular, we give a sufficient condition for α to solve the problem.

[1] Y. Tsuzuki, *Existence of solutions to heat equations with hysteresis coupled with Navier-Stokes equations in 2D domains*, J. Math. Anal. Appl., **423** (2015), 877–897.

[a] Speaker

A Mathematical Justification of the Thin Film Approximation for the Flow down an Inclined Plane

Hiroki Ueno [a] [1], Tatsuo Iguchi [1]

[1] Department of Mathematics, Faculty of Science and Technology, Keio University – 3-14-1 Hiyoshi Kohoku Yokohama 223-8522, Japan

We consider a two-dimensional motion of a thin film flowing down an inclined plane under the influence of the gravity and the surface tension. The motion is mathematically formulated as a free boundary problem for the incompressible Navier–Stokes equations. It is hard to analyze the Navier–Stokes equations directly in order to investigate the stability and the instability of the surface waves, so that we often use a thin film approximation. It is an approximation obtained by the perturbation expansion with respect to the aspect ratio δ of the film under the thin film regime $\delta \ll 1$. The famous examples of the approximate equations are the Burgers equation, the Kuramoto–Sivashinsky equation, the KdV–Burgers equation, the KdV–Kuramoto–Sivashinsky equation, the Benney equation, and so on. We give a mathematically rigorous justification of a thin film approximation by establishing an error estimate between the solution of the Navier–Stokes equations and those of approximate equations under the assumptions that the Reynolds number, the angle of inclination, and initial data are sufficiently small.

[a] Speaker

Otto calculus for measures of variable mass and a model from spatial ecology

Dmitry Vorotnikov [a] [1]

[1] Universidade de Coimbra (PORTUGAL) – Portugal

We introduce a new metric on the set of non-negative finite measures which is related to a sort of optimal transport with the mass changing according to certain intrinsic rules. The underlying structure turns out to be much richer than a metric one, and one may, e.g., develop a variable mass version of the celebrated Otto calculus for probability measures [1]. We then consider a degenerate reaction-diffusion model from spatial ecology which is a gradient flow in our calculus, and prove exponential convergence of solutions to the steady state. This is a joint work with S. Kondratyev and L. Monsaingeon.

[1] C. Villani. Optimal transport: old and new. Springer, 2008.

[a] Speaker

Solvability of some degenerate parabolic equations with convective terms

Hiroshi Watanabe [a] [1]

[1] Salesian Polytechnic – 4-6-8 oyamagaoka, machida, Tokyo zip 194-0215, Japan

In this talk, we consider some strong degenerate parabolic equations and systems. The equations which are called strongly degenerate parabolic equations are regarded as a linear combination of the conservation laws and the porous medium type equations. Thus, these equations have both properties of hyperbolic equations and those of parabolic equation. Moreover, they describe various nonlinear convective diffusion phenomena such as filtration problems, Stefan problems and so on. In this talk, we consider strongly degenerate parabolic equations and systems. Our purpose is to prove the existence and uniqueness of entropy solutions to the problem.

[a] Speaker

A nonlinear age-structured model of semelparous species

Radosław Wieczorek [a] [1]

[1] University of Silesia – Katowice, Poland

A species is called semelparous if its specimen reproduces only once in the lifetime, and usually dies afterwards. We consider a population such that individuals may give birth only at a given age. Discrete-time models of semelparous population have been intensively studied recently, because they have unexpected asymptotic properties, such as the extinction of all but one year classes. It is an interesting question if a continuous time age-structured model may exhibit similar properties.
We present a non-linear McKendrick-type age-structured model given by a linear partial differential equation with a nonlinear boundary condition of the form
Properties of measure-valued periodic solutions of this system are investigated. We observe that there exists a unique nonnegative stationary distribution which is often unstable. We show that in some cases the age profile of the population tends to a Dirac measure, which means that the population asymptotically consists of individuals at the same age. This phenomenon is observed in nature in some insects populations.

[a]Speaker

Wall law for a non-Newtonian incompressible fluid flow over an oscillating surface

Aneta Wroblewska-Kaminska [a] [1]

[1] Institute of Mathematics, Polish Academy of Sciences (IMPAN) – ul. Śniadeckich 8, 00-656 Warszawa, Poland

We analyse how small irregularities of the solid surface effects the steady flow of a general viscous fluid at larger scales. In particular we consider generalised Stokes system for incompressible non-Newtonian fluids of power-law type with zero Dirichlet boundary conditions when the surface of boundary is rough. Namely, it contains microscopic surface irregularities and an amplitude, and a wavelength of oscillations is described by a small parameter which converges to zero. Our aim is to derive effective boundary conditions - a wall law - on a smoothed boundary which gives a small approximation error. To this end we study corresponding boundary layer problem and work in frame of weak solutions. This is a result of a joint research with David Gérad-Varet.

[a]Speaker

Existence of Chapman-Jouguet detonation and deflagration waves

Johannes Wächtler [a] [1], Peter Szmolyan [2], Ingenuin Gasser [1]

[1] FB Mathematik, Universität Hamburg – Germany
[2] Institute for Analysis and Scientific Computing, Vienna University of Technology – Austria

We study the existence of profiles for Chapman-Jouguet detonation and deflagration waves in the Navier-Stokes equations for a reacting gas. In the limit of small viscosity, heat conductivity, and diffusion, the profiles correspond to heteroclinic orbits of a system of singularly perturbed ordinary differential equations. The burned end state of the waves, however, is a nonhyperbolic equilibrium of the associated, purely gas dynamic layer problem, and standard methods from geometric singular perturbation theory hence fail. We show how to resolve this degeneracy by combining a center manifold reduction with the blow-up method. The main result is the existence of viscous profiles for various types of Chapman-Jouguet processes. In addition, we obtain results on the spatial decay rates of these waves which are expected to be relevant for the stability analysis of the waves.

[a]Speaker

Singular limit of Allen–Cahn equation with constraints and dynamic boundary condition and its Lagrange multiplier

Noriaki Yamazaki [a] [1], Mohammad Hassan Farshbaf-Shaker [2], Fukao Takeshi [3]

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[2] Weierstrass Institute for Applied Analysis and Stochastics – Mohrenstrasse 39, 10117 Berlin, Germany
[3] Department of Mathematics, Kyoto University of Education – 1 Fujinomori, Fukakusa, Fushimi-ku, Kyoto, 612-8522, Japan

We consider an Allen–Cahn equation with constraints and dynamic boundary condition. Our constraint is a subdifferential of an indicator function on the closed interval [-1,1], which is the multivalued function. In this talk we call the pair of elements of constraint a Lagrange multiplier to our problem. Then, we give the characterization of the Lagrange multiplier to our problem. Moreover, we consider the singular limit of our system as the parameter goes to 0. Then, we clarify the limit of the solution and the Lagrange multiplier to our problem as the parameter goes to 0.

[a]Speaker

Global existence and boundedness in a quasilinear degenerate Keller-Segel system

Tomomi Yokota [a] [1]

[1] Tokyo University of Science – 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan

We study global existence and boundedness of weak solutions to the quasilinear degenerate Keller–Segel system

$$\begin{cases} u_t = \nabla \cdot (\nabla u^m - u^{q-1}\nabla v) & \text{in } \mathbb{R}^N \times (0,\infty), \\ v_t = \Delta v - v + u & \text{in } \mathbb{R}^N \times (0,\infty), \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x), & x \in \mathbb{R}^N, \end{cases} \tag{KS}$$

where $N \in \mathbb{N}$, $m \geq 1$, $q \geq 2$ and the initial data (u_0, v_0) is assumed to be a pair of nonnegative functions. Global existence of weak solutions to (KS) was first established under the condition $q \leq m$ by Sugiyama and Kunii [2]. The condition for q was relaxed into the condition $q < m+2/N$ by a joint work [1] with Sachiko Ishida. Both results gave only global existence and it is still open whether (KS) admits a weak solution that is uniformly-in-time bounded. We would like to give an answer to this open question.

[1] S. Ishida, T. Yokota, *Global existence of weak solutions to quasilinear degenerate Keller-Segel systems of parabolic-parabolic type*, J. Differential Equations **252** (2012), 1421–1440.

[2] Y. Sugiyama, H. Kunii, *Global existence and decay properties for a degenerate Keller-Segel model with a power factor in drift term*, J. Differential Equations **227** (2006), 333–364.

[a] Speaker

Multi-solitons interaction with higher nonlinearity effects in an extended Schrödinger equation

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[2] California State University at Northridge (CSUN) – 18111 Nordoff St. Northridge, CA, United States

We considered an extended nonlinear Schrödinger with the third and fourth order space-derivatives that is known to have one-soliton and rogue solutions. We have transformed the model to a homogeneous model that is utilized to show the existence of interacting multi-soliton solutions. We have analyzed, and gave specific forms of these new class of N-solitons in a new simple form and discussed the interactions of these multi-solitons. We have shown the existence of three types of head-on collisions, head-on collisions with or without over-taking or splitting into two solitons.

[a] Speaker

Existence of weak solutions to a parabolic-elliptic Keller-Segel system with nonlinear degenerate diffusion

Noriaki Yoshino [a] [1]

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We consider the parabolic-elliptic chemotaxis system

$$\begin{cases} \dfrac{\partial b}{\partial t} - \Delta D(b) + \nabla \cdot (K(b,c)b\nabla c) = 0 \quad \text{in } (0,\infty)\times\Omega, \\ -\Delta c + c = b \quad \text{in } (0,\infty)\times\Omega. \end{cases}$$

Marinoschi [1] established an abstract approach to give existence of local solutions to this system with sufficiently small initial data in the case $0 < D_0 \leq D'(r) \leq D_\infty < \infty$ and $(r_1, r_2) \mapsto K(r_1, r_2)r_1$ is Lipschitz continuous on $\mathbb{R}^2$. The smallness assumption on the initial date was recently removed by the authors [2]. However the case of non-Lipschitz and degenerate diffusion, such as $D(r) = r^m$, is left incomplete. This talk presents the local solvability with non-Lipschitz and degenerate diffusion by applying the previous work [2] to an approximate system.

[1] G. Marinoschi, *Well-posedness for chemotaxis dynamics with nonlinear cell diffusion*, J. Math. Anal. Appl. **402** (2013), 415–439.

[2] . Yokota, N. Yoshino, *Existence of solutions to chemotaxis dynamics with Lipschitz diffusion and superlinear growth*, J. Math. Anal. Appl. **419** (2014), 756–774.

[a] Speaker

A positive decomposition of the entropy production for the ellipsoidal BGK model

Seok-Bae Yun [a] [1]

[1] Sungkyunkwan University [Suwon] (SKKU) – Suwon Gyeonggi-Do 440-746, South Korea

The BGK model is widely used in place of the Boltzmann equation since it reproduces various qualitative properties of the Boltzmann equation at much lower computational cost. But it's well known that the BGK approximation of the Navier Stokes equation does not give the correct Prandtl number, which is defined as the ratio between the viscosity and the thermal conductivity. In an effort to cook up a variant of the BGK model that gives the correct Prandtl number, Holway introduced the ellipsoidal BGK model (ES-BGK model) where the collision operator of the Boltzmann equation is replaced by a relaxation operator involving a non-isotrophic Gaussian designed to match the Prandle number of the Navier-Stokes equation [1]. In this talk, we consider the entropy dissipation structure of the ES-BGK model. More precisely, we show that the entropy production functional of ES-BGK model is decomposed into two sub-production terms, one of which shares similar structure with the original BGK entropy production functional, whereas the other involves a novel positivity structure of its own interest. As an application of this newly revealed structure, we prove the existence of weak solutions for the ES-BGK model with fixed collision frequency. [1] Holway, L.H.: Kinetic theory of schock structure using and ellipsoidal distribution function. Rarefied Gas Dynamics, Vol. I

[a]Speaker

Solvability of mathematical model for brewing process of Japanese Sake and its numerical simulations

Murase Yusuke [a] [1]

[1] Meijo university – Japan

The main purpose of our research is analysing a mathematical model for brewing process of Japanese Sake, and disclosing the brewing process. Japanese sake is brewed by using a complicated brewing technique so called "Multiple parallel fermentation" with 5 brewing steps. We configured a mathematical model to represent the brewing phenomena. Our model is formulated with 14 reaction-diffusion equations with homogeneous Neumann boundary condition, a heat equation with Robin boundary condition, and a constraint condition. In first step of fermenting, the mathematical model can be simplified 5 reaction-diffusion equations, a heat equation, and constraint condition.

The constraint in the model is determined by fixing unknown functions. It's one of the difficulties to analyse this model. From variational inequality point of view, we can say that constraint set depends upon unknown functions self. We call this type variational inequality "Quasi-variational inequality".

In this talk, We discuss solvability of approximated model and original model in first fermenting step, and some numerical results, especially.

[a]Speaker

Almost Periodicity of State-dependent Impulsive Neural Networks

Enes Yılmaz [a] [1]

[1] Gazi University – Polatlı Faculty of Science and Arts, Department of Mathematics, 06900 Ankara, Turkey

In this talk, we address a new model of neural networks related to the discontinuity phenomena which is called impulsive recurrent neural networks with variable moments of time. Sufficient conditions for existence and uniqueness of exponentially stable almost periodic solution are investigated. An example is given to illustrate our theoretical results. The main novelty of the talk is to investigate sufficient conditions ensuring the existence and uniqueness of almost periodic solution. To solve the problem, we should develop the technique of the reduction of the considered system to system with fixed moments of impulses. That is, B-equivalence method, which was studied for bounded domain in the phase space, is utilized. Equations with non fixed moments of discontinuity create a great number of opportunities for theoretical inquiry as well as theoretical challenges. The proposed new involvements have an important role for the real world problems. Exceptional practical interest is connected with discontinuities, which appear at non-prescribed moments of time.

[a]Speaker

On local and nonlocal variational constants of motion

Gaetano Zampieri [a] [1]

[1] Department of Computer Science / Dipartimento di Informatica [Verona] – University of Verona - Strada le Grazie 15 I-37134 Verona Italy, Italy

Let $q(t)$ be a solution to Euler-Lagrange equation for a smooth Lagrangian $L(t, q, \dot{q})$, with q in an open set of R^n, and let $q_\lambda(t)$, $\lambda \in R$, be a smooth family of perturbed motions, such that $q_0(t) \equiv q(t)$. Then the following function is constant:

$$t \mapsto \partial_{\dot{q}} L\big(t, q(t), \dot{q}(t)\big) \cdot \partial_\lambda q_\lambda(t)\big|_{\lambda=0} - \int_{t_0}^{t} \frac{\partial}{\partial \lambda} L\big(s, q_\lambda(s), \dot{q}_\lambda(s)\big)\Big|_{\lambda=0} ds$$

($\partial_{\dot{q}}$ gradient with respect to the vector $\dot{q}$ and $\cdot$ scalar product in R^n). This constant of motion is generally *nonlocal* and trivial.
We can get genuine first integrals as for $L = \frac{1}{2}\|\dot{q}\|^2 - U(q)$ with U homogeneous of degree -2, in particular Calogero's potential, and $q_\lambda(t) = e^\lambda\, q\big(e^{-2\lambda} t\big)$. This example is taken from:
G. Gorni, G. Zampieri. Revisiting Noether's theorem on constants of motion. Journal of Nonlinear Mathematical Physics 21 (2014), 43-73.
We also find nonlocal constants of motion which give global existence and asymptotic estimates for the solutions of $\ddot{q} = -k\dot{q} - \partial_q U(q)$, when $k > 0$ and $U : R^n \to R$ is bounded from below.

Finally, we show a nonlocal constant of motion for the Maxwell-Bloch system in Caşu's Lagrangian formulation which leads to separation of one of the variables.

[a]Speaker

Two-velocity formulation of the degenerate Navier-Stokes equations

Ewelina Zatorska [a] [1]

[1] Institute of Mathematics, Polish Academy of Sciences (IMPAN) – ul. Śniadeckich 8, 00-656 Warszawa, Poland

This talk will be devoted to existence result for the low Mach number limit system obtained from the full compressible Navier-Stokes model with density-dependent viscosities. We will first present derivation of the incompressible system called Kazhikhov-Smagulov model. Under special compatibility condition between the viscous tensor and the diffusive term we will prove the existence of global in time weak solutions [1]. The proof relies on use of relative velocity, which is divergence free and which allows to reduce the coupling between particular subsystems. We will also mention possible generalizations of constraints appearing in the system and present an application to the full compressible system [2], which together with [3] yields the existence of solutions. References:

1. D. Bresch, V. Giovangigli, E. Zatorska. Two-velocity hydrodynamics in fluid mechanics: Part I Well posedness for zero Mach number systems, to appear in JMPA, (2015).
2. D. Bresch, B. Desjardins, E. Zatorska.Two-velocity hydrodynamics in fluid mechanics: Part II Existence of global κ–entropy solutions to compressible Navier-Stokes systems with degenerate viscosities, to appear in JMPA, (2015).

3. A. Vasseur, C. Yu. Existence of Global Weak Solutions for 3D Degenerate Compressible Navier-Stokes Equations, arXiv:1501.06803, (2015).

[a] Speaker

Linear inviscid damping for monotone shear flows

Christian Zillinger [a] [1]

[1] Mathematisches Institut der Universität Bonn – Endenicher Allee 60 53115 Bonn, Germany

Recently there has been much interest in damping phenomena for kinetic equations following the seminal works of Mouhot-Villani on Landau damping and of Bedrossian-Masmoudi on inviscid damping around Couette flow.
In this talk I present a proof of linear inviscid damping for the 2D Euler equations around general monotone shear flows in the framework of Sobolev regularity. Here I consider both the settings of an infinite periodic and a finite periodic channel with impermeable walls.
In the latter case I explain the non-negligible effect of boundary conditions on the attainable regularity and stability results.

[a] Speaker

Convergence properties of numerical solutions for nonlinear integro-differential equations.

Barbara Zubik-Kowal [a] [1]

[1] Boise State University (BSU) – 1910 University Drive Boise, ID 83725, United States

We are concerned with iterative schemes for the numerical integration of nonlinear Volterra integro-differential equations applied in neuroscience. We introduce numerical algorithms which can be easily implemented in parallel computing environments and efficiently applied to conduct fast numerical simulations. We analyze the errors of the approximate solutions and derive error bounds that show their convergence to the exact solutions of general dynamical systems. The theoretical results validate that the numerical simulations produce reliable solutions.

[a]Speaker

Characteristic box dimension of Poincaré map of nilpotent focus

Vesna Županović [a] [1], Lana Horvat Dmitrović [2]

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[2] University of Zagreb Faculty of Electrical Engineering and Computing (UNIZG FER) – Croatia

We show how box dimension of an orbit generated by the Poincaré map or by the unit-time map, can be used in studying the cyclicity problem of nilpotent singularities of planar vector fields. On the characteristic curve of nilpotent singularity we define characteristic map and characteristic box dimension in order to obtain multiplicity of the singularity. In the case of Hopf-Takens bifurcation or saddle-node bifurcation, there is already known connection between the multiplicity of singularity and the box dimension of the Poincaré map or unit-time map. We generalize that results for nilpotent singularities.

The box dimension of the Poincaré map near the nilpotent focus on the characteristic curve reveals the upper bound for cyclicity.

[a]Speaker

Posters

On attractors of a differential system arising in the network control theory

Svetlana Atslega [a] [1]

[1] Institute of Mathematics and Computer Science, University of Latvia – Latvia

We study the system

$$x_1' = \frac{1}{1+e^{-\mu(x_2-\Theta)}} - x_1, \quad x_2' = \frac{1}{1+e^{-\mu(x_1-\Theta)}} - x_2 \tag{18}$$

which appears in description of a gene regulatory network (Y. Koizumi et al. Application of attractor selection to adaptive virtual network topology control, in Proc. of BIONETICS, pp. 1 – 8, Nov. 2008) . We provide a number of results concerning the structure of attracting sets for a system (18). A sample of the results follows. **Theorem.** The system has exactly one attracting critical point (stable node) for $\mu \in (0, 4)$, $\Theta \in R^+ := (0, +\infty)$. For $\mu = 4$ and for all positive Θ, except the special value $\Theta = 0.5$, there is exactly one critical point of the same type. For $\mu > 4$ there are two values of Θ, $\Theta_1 < \Theta_2$, with the property: if $\Theta \in exterior[\Theta_1, \Theta_2] \bigcap R^+$, then there is one attracting critical point (stable node); if $\Theta \in (\Theta_1, \Theta_2)$, then there are two attracting critical points (stable nodes) and a saddle point between them; if $\Theta = \Theta_1$ or $\Theta = \Theta_2$ then there are two critical points, namely, a stable node and an attracting degenerate critical point (the matrix of coefficients of the linearized system is degenerate).
Acknowledgement. The support of the ESF project 2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045 is kindly acknowledged.

[a]Speaker

Resolution in Little Hölder Space of a Mixed Boundary Value Problem Set on Cusp Domain.

Chaouchi Belgacem [a] [1]

[1] khemis miliana university – Algeria

This work is devoted to the study of Laplace equation set on planar cusp domains. A mixed boundary conditions of Dirichlet-Neumann type are imposed . The study is performed in the little Hölder spaces framework.

[a]Speaker

Behavior of strong solutions to the oblique derivative problem for elliptic linear and quasilinear equations in a domain with boundary conical point

Mariusz Bodzioch [a] [1]

[1] Faculty of Mathematics and Computer Science, University of Warmia and Mazury in Olsztyn – Poland

We study the behaviour of strong solutions to the oblique derivative problem for linear and quasilinear second-order elliptic equations in a neighborhood of a conical boundary point of an n-dimensional bounded domain. We establish an exponent of the solution's decreasing rate near the conical boundary point, i.e. we show that $|u(x)| = O(|x|^{\lambda})$ with an exact exponent λ.

[a] Speaker

A family of fourth-order porous medium equations

Mario Bukal [a] [1]

[1] University of Zagreb Faculty of Electrical Engineering and Computing (UniZg-FER) – Unska 3, 10000 Zagreb, Croatia

We analyze a family of nonlinear fourth-order diffusion equations, which includes the well known thin-film and the Derrida-Lebowitz-Speer-Spohn (DLSS) equation on the one-dimensional torus, i.e. subject to periodic boundary conditions, and prove the existence of global nonnegative weak solutions to these equations. Contrary to the gradient flow approach proposed by Matthes, McCann and Savaré (2009), our method relies on dissipation property of the corresponding entropy functionals called Tsallis entropies, which provide required a priori estimates. Moreover, our method extends the existence result from Matthes, McCann and Savaré (2009) to a wider range of the family members. Applying generalized Beckner-type functional inequalities yield an exponential decay rate of (relative) Tsallis entropies, which in further, using Csiszár-Kullback-Pinsker inequalities implies the exponential stability in the L^1-norm of the constant steady state.

[a] Speaker

Shape differentiability of the eigenvalues of the biharmonic operator

Davide Buoso [a] [1], Pier Domenico Lamberti [2], Luigi Provenzano [2]

[1] Dipartimento di Scienze Matematiche, Politecnico di Torino – Corso Duca degli Abruzzi, 24 - 10129 Torino, Italy

[2] Dipartimento di Matematica, Universita' degli Studi di Padova – Via Trieste, 63 - 35121 Padova, Italy

In this poster we will present some old and new results concerning the shape differentiability of the eigenvalues of the biharmonic operator subject to different homogeneous boundary conditions. After having recalled the problems under consideration (Dirichlet, Neumann, intermediate and Steklov boundary value problems), we will show that all the elementary symmetric functions of the eigenvalues are real analytic. Then we provide Hadamard-type formulas for all the above mentioned problems. Finally, after having recalled the known results in eigenvalue shape optimization, we will show how to use the Hadamard-type formulas to prove that the ball is a critical domain under volume contstraint for any elementary symmetric function of the eigenvalues, and for all the problems considered. Based on joint works with P.D. Lamberti and L. Provenzano.

[a]Speaker

Traveling wave fronts in a coupled delayed reaction-diffusion and difference system

Mostafa Adimy [1], Abdennasser Chekroun [a] [1], Bogdan Kazmierczak [2]

[1] DRACULA (INRIA Grenoble Rhône-Alpes / Institut Camille Jordan) – INRIA, Institut Camille Jordan – Institut Camille Jordan Université Claude Bernard Lyon 1 43 boulevard du 11 novembre 1918 69622 Villeurbanne cedex France, France

[2] Institute of Fundamental Technological Research Polish Academy of Sciences, Warsaw (Poland) – Poland

The formation and development of blood cells (red blood cells, white cells and platelets) is a very complex process, called hematopoiesis. This process involves a small population of cells called hematopoietic stem cells (HSCs). We propose a mathematical model describing the dynamics of HSC population, taking into account their spatial distribution and diffusion. The resulting model is an age-structured reaction-diffusion system. The method of characteristics can be used to reduce this model to an unstructured time-delayed reaction-diffusion equation coupled with a difference equation. We investigated mathematical studies of the model and showed the existence of travelling wave front solutions connecting the zero equilibrium with the positive uniform steady state. We used the classical monotone iteration technique coupled with the sub- and super-solutions method. A numerical simulations carried out to show the propagation of the solution in a travelling wave front.

[a]Speaker

Viscosity-stratified flow in a Hele-Shaw cell

Alexander Chesnokov [a] [1,2], Valery Liapidevskii [1,2]

[1] Novosibirsk State University (NSU) – Pirogova str. 2, 630090 Novosibirsk, Russia
[2] Lavrentyev Institute of Hydrodynamics (LIH) – 630090, Novosibirsk, Russia pr. ac. Lavrentyeva, 15 LIH SB RAS Phone: +7(383-2) 33-16-12 Fax: +7(383-2) 33-16-12 E-mail: igil@hydro.nsc.ru, Russia

A hierarchy of mathematical models describing viscosity-stratified flow in a Hele-Shaw cell is constructed. Numerical modelling of jet flow and development of viscous fingers with the influence of inertia and friction is carried out. One-dimensional multi-layer flows are studied. In the framework of three-layer flow scheme the interpretation of the Saffman–Taylor instability is given. Two kinematic-wave models of viscous fingering are proposed. The first one includes friction between the fluid layers. The second model takes into account the formation of the intermediate mixing layer. Comparison with calculations on the basis of two-dimensional equations shows that these models allow one to determine the velocity of propagation and the thickness of the viscous fingers. Some details can be found here: arxiv.org/abs/1501.00366.

[a]Speaker

Droplets spreading under contact line friction

Maria Chiricotto [a] [1], Lorenzo Giacomelli [2]

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[2] Università degli Studi di Roma "La Sapienza" [Rome] – Piazzale Aldo Moro 5, 00185 Roma, Italy

The lubrication approximation for a liquid droplet spreading on a plane solid surface leads to a class of free boundary problems for fourth order degenerate parabolic equations. The focus here is on effective conditions which relate the speed of the contact line (where liquid, solid and vapor meet) to the microscopic contact angle. One such condition has been recently proposed by Weiqing Ren and Weinan E (Physics of Fluids 19 (2007), 022101): it includes into the model the effect of frictional forces which arise at the contact line from unbalanced components of the Young's stress, leading to an additional dissipation term in the energy balance. We are interested in the well-posedness of weak solutions as well as for a class of traveling-wave solutions. For speed-dependent contact angle conditions of rather general form, a matched asymptotic study is worked out, relating the macroscopic contact angle to the speed of the contact line. This is joint work with Lorenzo Giacomelli (La Sapienza - University of Rome).

[a]Speaker

A construction of two different solutions to an elliptic system

Jacek Cyranka [a] [1]

[1] Institut of Applied Mathematics - University of Warsaw – Poland

The paper aims at constructing two different solutions to an elliptic system

$$u \cdot \nabla u + (-\Delta)^m u = \lambda F$$

defined on the two dimensional torus. Here $u = (u^1, u^2)$ is sought as a vector function. The operator $(-\Delta)^m$ is elliptic homogenous of order $2m$. It can be viewed as an elliptic regularization of the stationary Burgers 2D system. A motivation to consider the above system comes from an examination of unusual propetries of the linear operator

$$\lambda \sin y d_x w + (-\Delta)^m w.$$

Roughly speaking the term with λ effects in a special stabilization of the norms of the operator. We shall underline that the special features of this operator were found firstly via numerical analysis. Our proof is valid for a particular force F and for $\lambda > \lambda_0$, $m > m_0$ sufficiently large. The main steps of the proof concern finite dimension approximation of the system and concentrate on analysis of features of large matrices, which resembles standard numerical analysis. Our analytical results are illustrated by numerical simulations. Experiments are agreed with the conjecture : for small m, in particular for $m = 1$ – for the classical Burgers equation with diffusion, the system does not admit solutions for large λ.

[1] Jacek Cyranka, Piotr Boguslaw Mucha, *A construction of two different solutions to an elliptic system*, arXiv:1502.03363.

[a] Speaker

Modelling fungal hyphae growth: searching for travelling waves in an extension of the thin viscous sheet equations.

Thomas De Jong [a] [1,2], Georg Prokert [1], Joost Hulshof [2]

[1] Centre for Analysis, Scientific computing and Applications, Technische Universiteit Eindhoven (CASA) – Netherlands
[2] Department of Mathematics, VU Amsterdam – Netherlands

The cell growth of fungal hyphae typically proceeds as an elongating expansion in a single direction. It grows at a constant speed preserving the overall tube- like form. Hence, if we assume that it never stops growing then mathematically the growth can be described by a travelling wave. The existence of travelling wave solutions for certain hyphae growth models has already been proven. However, these models do not take into account the dynamics in the cell wall and consequently have to prescribe explicitly how the wall grows as new material arrives. Our new model overcomes this by modelling the cell wall as a thin viscous sheet where ageing of the cell wall results in hardening of the wall by making the wall more viscous over time. Existence of the desired travelling wave corresponds to finding an increasing concave bounded solution in a highly non-linear 5-dimensional first order ODE which behaves singular close to the boundaries of the desired solution. At the boundaries our local existence and uniqueness proofs yield solutions with the desired boundary behaviour. Furthermore, our numerical work suggests that globally the desired solutions exist.

[a]Speaker

Unsteady non-isothermal and non-Newtonian flow problem with mixed boundary conditions

Hanene Debbiche [a] [1], Mahdi Boukrouche [1], Laetitia Paoli [1]

[1] Institut Camille Jordan, UMR CNRS 5208, Université de Saint Etienne (ICJ) – ICJ, Université de Saint Etienne – Faculté des Sciences, 23 rue Michelon, 42023 Saint Etienne Cedex 2, France

We consider the non-stationary flow of an incompressible, non-Newtonian and non-isothermal fluid in a bounded domain $\Omega \subset \mathbb{R}^3$. We assume non-standard mixed boundary conditions with a given time dependent velocity on a part of the boundary and Tresca's friction law on the other part.From the latter condition, we obtain that the variational formulation of the problem is given by a parabolic variational inequality.
The originality in this work is that the fluid viscosity depends on temperature and also on the modulus of strain rate tensor and the velocity of the fluid.
We prove the existence of a solution by using Schauder fixed point theorem, the notion of semi-group and monotony methods. Then, we conclude by applying De Rham theorem to construct the pressure term.

[a]Speaker

To study fractional impulsive integro-differential equation with an integral boundary conditions

Jaydev Dabas [a] [1]

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In this work, we have established the existence and uniqueness solution for a class of impulsive fractional integro-differential equations with integral boundary conditions. The existence and uniqueness results are obtained by applying the classical fixed point theorems. An application is given to verify our results.

[a] Speaker

3-D flow of a compressible viscous micropolar fluid with cylindrical symmetry: a local existence theorem

Ivan Drazic [a] [1], Nermina Mujakovic [2]

[1] Faculty of Engineering, University of Rijeka – Vukovarska 58, 51000 Rijeka, Croatia
[2] Department of Mathematics, University of Rijeka – Radmile Matejčić 2, 51000 Rijeka, Croatia

In this work we consider the nonstationary 3D flow of a compressible viscous and heat-conducting micropolar fluid, which is in the thermodynamical sense perfect and polytropic. The fluid domain is the subset of $\mathbf{R}^3$ bounded with two coaxial cylinders that present solid thermoinsulated walls. We assume that the initial mass density, temperature, as well as the velocity and microrotation vectors are radially dependent only and analyze the corresponding initial-boundary value problem. With the additional assumption that the initial mass density and temperature are strictly positive we prove that for smooth enough initial data there exists a cylindrically symmetric generalized solution locally in time. The proof is based on the Faedo-Galerkin method.

[a] Speaker

On the appearance of gaps in the spectrum of a Dirichlet problem in a double-periodic perforated plane

Francesco Ferraresso [a] [1]

[1] Dipartimento di Matematica Pura e Applicata [Padova] – Via Trieste, 63 35121 Padova, Italy

We consider a problem of surface wave propagation around a double-periodic grid of cylindrical obstacles whose section have the shape

$$\{|x_1|^\mu + |x_2|^\mu \leq r\},$$

where $\mu \in]1, \infty[$. Applying separation of variables and Floquet-Bloch theory leads us to consider an eigenvalue problem for the Laplace operator in a periodicity cell with Dirichlet boundary conditions on a large part of the boundary and some compatibility conditions on the edges of the cell.

Our main theorem states that for r sufficiently close to $1/2$ the whole of the spectrum of the operator associated with the problem is essential and assumes a "band-gap" structure. More precisely, each eigenvalue of the limit problem corresponds to a spectral interval (a band) in the perturbed problem, and these bands can be made disjoint for r close to $1/2$. As a byproduct, we prove that the eigenfunctions of the limit problem are smooth up to the boundary, with exponential decay at the cusp vertexes. Our results generalize those obtained in [1] for circular holes. Joint work with Jari Taskinen, see [2].

[1] S.Nazarov, K.Ruotsalainen, J.Taskinen : "Spectral gaps in the Dirichlet and Neumann problems on the plane perforated by a double-periodic family of circular holes", J. Math. Sci. (N. Y.), 181(2), 2012.

[2] F.Ferraresso, J.Taskinen : "Singular perturbation Dirichlet problem in a double-periodic perforated plane", to appear in Annali dell'università di Ferrara.

[a] Speaker

Bifurcations and applications of low-dimensional polynomial dynamical systems

Valery Gaiko [a] [1]

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We carry out the global qualitative analysis of low-dimensional polynomial dynamical systems. Using new bifurcational and topological methods, we solve first Hilbert's Sixteenth Problem on the maximum number and distribution of limit cycles for the general two-dimensional Liénard polynomial system with an arbitrary number of singular points. Then, applying a similar approach, we study three-dimensional polynomial systems and complete the strange attractor bifurcation scenario for the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles. We discuss also how to apply our approach for studying global limit cycle bifurcations of discrete polynomial (and rational) dynamical systems which model the population dynamics in biomedical and ecological systems. This work was partially supported by the Simons Foundation of the International Mathematical Union and the Department of Mathematics and Statistics of the Missouri University of Science and Technology.

[a] Speaker

Well-posedness and regularity for a class of thin-film free boundary problems

Manuel Gnann [a] [1]

[1] University of Michigan [Ann Arbor] – 500 Church Street Ann Arbor, MI 48109-1090, United States

We investigate a free boundary problem for a thin-film equation with quadratic mobility and a zero contact angle condition at the triple point where air, liquid, and solid meet. This problem can be derived by a lubrication approximation from the Navier-Stokes system with a Navier-slip condition at the substrate. By treating the model problem of source-type solutions, we motivate why general solutions to this problem are generically singular. The method for proving well-posedness therefore requires to suitably subtract the leading-order singular expansion at the free boundary in the maximal regularity estimates for the linearized evolution. We also discuss the regularizing effect of the degenerate-parabolic operator to arbitrary orders of the singular expansion. Many of the presented results are joint with Lorenzo Giacomelli, Hans Knüpfer, and Felix Otto.

[a] Speaker

Solution dynamics in a class of differential delay equations

Anatoli Ivanov [a] [1]

[1] Pennsylvania State University (PSU) – P.O. Box PSU, Lehman, PA 18627, United States

We consider a class of neutral differential delay equations of the form

$$\varepsilon \left[x'(t) + ax'(t-1)\right] + x(t) = f(x(t-1)),\ t \geq 0, \tag{19}$$

where $f : \mathbb{R} \mapsto \mathbb{R}$ is a continuous function, $a \in \mathbb{R}$ is a constant, and $\varepsilon > 0$ is a small parameter. Such equations find various applications in modelling diverse real life phenomena. The neutral type ($a \neq 0$) equation (19) appears as an exact reduction of certain boundary value problems for hyperbolic PDEs. The limiting case $\varepsilon = 0$ in equation (19) results in a continuous type difference equation of the form

$$x(t) = f(x(t-1)),\ t \geq 0, \tag{20}$$

which dynamics are largely determined by relevant properties of the map f as one-dimensional dynamical system. We establish continuous dependence results between solutions of equations (19) and (20) as $\varepsilon \to 0^+$. We address the problem of existence of periodic solutions in equation (19) for several classes of the nonlinearity f, such as the Farey-type nonlinearity and a nonlinearity with the negative feedback. Some of the results of this work, as well as a number of related open problems and conjectures, are supported by numerical simulations.

[a]Speaker

The existence of solutions for the perfect plasticity model with time dependent constraints

Risei Kano [a] [1]

[1] Risei Kano – Japan

In this talk, We discuss problems with the plastic deformation of perfect materials. This problem has been discussed by many scholars. In particular, G.Duvaut and J.L.Lions showed the solutions of the evolution problem that has the constraints on threshold of stress, in 1976. We think about the solvability of the extended problem having the function f(t,x) of the threshold. Here follows the way of Duvaut-Lions, we talk about the existence of solutions of the parabolic approximation problem.

[a]Speaker

Adaptive time-splitting methods for nonlinear Schrödinger equations in the semiclassical regime

Winfried Auzinger [1], Thomas Kassebacher [a 2], Othmar Koch [3], Mechthild Thalhammer [2]

[1] Institute for Analysis and Scientific Computing – Wiedner Hauptstr. 8-10, 1040 Wien, Austria
[2] Department of Mathematics, University of Innsbruck – Faculty of Mathematics, Computer Science and Physics Technikerstraße 13 A-6020 Innsbruck, Austria
[3] Faculty of Mathematics, University of Vienna – Oskar-Morgenstern-Platz 1, 1090 Wien, Austria

We consider nonlinear Schrödinger equations involving a small parameter $\epsilon > 0$, a quadratic potential U, and a cubic nonlinearity,

$$\begin{aligned} i\partial_t \psi(t) &= -\epsilon\,\Delta\psi(t) + \tfrac{1}{\epsilon}(U + |\psi(t)|^2)\psi(t), \\ \psi(0) &= \psi_0, \end{aligned}$$

and study the error behavior of time-splitting methods, extending the work from Descombes and Thalhammer (2012). By suitable integral representations for the local error $\mathcal{L}(t)$ of the Lie–Trotter and Strang splitting methods, we deduce estimates that reflect the dependence on the time step t and the parameter ϵ,

$$\|\mathcal{L}_{Lie}(t)\|_{L^2} \le C\big(t^2 + t^3(\tfrac{1}{\epsilon} + \epsilon) + t^4(\tfrac{1}{\epsilon^2} + 1)\big) + t^2\mathcal{O}\big((\tfrac{t}{\epsilon})^3\big)\big(1 + \mathcal{O}(\varepsilon^2)\big),$$
$$\|\mathcal{L}_{Strang}(t)\|_{L^2} \le C\big(t^3(\tfrac{1}{\epsilon} + \epsilon) + t^4(\tfrac{1}{\epsilon^2} + 1)\big) + t^2\mathcal{O}\big((\tfrac{t}{\epsilon})^3\big)\big(1 + \mathcal{O}(\varepsilon^2)\big).$$

Numerical examples for the regimes $t > \epsilon$ and $t < \epsilon$ confirm these bounds. We also introduce a posteriori local error estimators and illustrate their performance, in particular for adaptive choice of the time steps.

[a] Speaker

Emergence of bi-cluster flocking for the Cucker-Smale model

Junghee Cho [1], Seung-Yeal Ha [1], Feimin Huang [2], Chunyin Jin [2], Dongnam Ko [a] [1]

[1] Seoul National University (Seoul National University) – 1 Gwanak-ro, Gwanak-gu, Seoul 151-742, South Korea

[2] Academy of Mathematics and Systems Science (AMSS) – Zhongguancun East Road No.55, Beijing 100190, China, China

We present the asymptotic emergence of bi-cluster flocking configurations for the Cucker-Smale model with short-range communication weights for well-prepared initial configurations. We derive a system of differential inequalities for the functionals that measure the local spatial and velocity fluctuations and differences of local spatial and velocity averages. We then derive the upper bound of spatial fluctuations and the lower bound of the difference between local velocity averages. We explicitly present an admissible class of initial configurations leading to the asymptotic emergence of bi-flocking configurations. Unlike global flocking (a mono-flocking configuration in velocity), where the convergence rate is always exponential, the asymptotic convergence to bi-flocking configurations is affected by the far-field decay rate of communication weights. Over a short period of time, the local velocity fluctuations relax to local average velocities. Then, in the long-time scale, the local average velocities converge to their asymptotic value for the rate of the Cucker-Smale communication weight.

[a]Speaker

On the bulk velocity of Brownian ratchets

Stanislav Kondratyev [a] [1]

[1] Universidade de Coimbra (PORTUGAL) – Portugal

Brownian ratchet is a generic term for a few micro-level mechanisms in physics and biology that are capable of producing unidirectional transport of matter in systems without apparent bias to a particular direction. We study the transport phenomenon for rachets modeled by a Fokker–Planck-type equation on the real axis. We establish a relation between the bulk transport velocity and a bi-periodic solution of the equation. This relation allows to characterise the transport for a few specific models such as adiabatic and semiadiabatic limits for tilting ratchets, generic ratchets with small diffusion, and the multi-state chemical ratchets. This includes qualitative results concerning the direction of transport as well as explicit asymptotic formulas for the bulk velocity. This is a joint work with J.M. Urbano and D. Vorotnikov (University of Coimbra).

[a]Speaker

Regularization of piecewise smooth border-collision bifurcations

Kristian Uldall Kristiansen [a] [1], John Hogan [2]

[1] Technical University of Denmark [Lyngby] (DTU) – Anker Engelunds Vej 1 Building 101A 2800 Kgs. Lyngby, Denmark

[2] Department of Engineering Mathematics, University of Bristol ((EMUOB)) – Merchant Venturer's Building, Woodland Road, Bristol BS8 1UB - United Kingdom, United Kingdom

This poster will present results on the Sotomayor-Teixeira regularization of border collision bifurcations in planar piecewise smooth systems. We will describe the perturbation of associated limit cycles and pseudo-equilibria. In particular, we will show how the regularization can induce additional bifurcations not present in the piecewise smooth system.

[a]Speaker

Dynamic Isoperimetry on Weighted Manifolds

Eric Kwok [a] [1]

[1] School of Mathematics and Statistics (UNSW) – Australia

Transport and mixing in dynamical systems are important mechanisms for many physical processes. We consider the detection of transport barriers using a recently developed geometric technique [1]: the dynamic isoperimetric problem. Solutions to the dynamic isoperimetric problem are sets with persistently small boundary size relative to interior volume, as the sets are evolved by the dynamics. In the presence of small diffusion these sets have very low dispersion over finite-times because of their lasting small boundary size, and thus are natural candidates for coherent sets, bounded by transport barriers. We construct a weighted dynamic Laplacian operator, and show corresponding results for a dynamic Cheeger inequality and dynamic Federer-Fleming theorem. We can handle general nonlinear dynamics, and weighted versions of area and volume. Finally, we formulate the connection between the present geometrical approach to recent probabilistic approaches to determining coherent sets using transfer operators.

[1] . G. Froyland, Dynamic Isoperimetry and the geometry of Lagrangian coherent structures.

[a]Speaker

Dynamics of cooperative neuronal networks depending on their associated graphs.

Pilar Lorenzo [a] [1], Marcos BarriosFlorencia Cubría

[1] Instituto de Matemática y Estadística Rafael Laguardia, Universidad de la República, Uruguay (IMERL, UdelaR) – Av Julio Herrera y Reissig 565, CP 11300, Montevideo, Uruguay

We investigate the dynamics of abstract neuronal networks modelled as impulsive differencial equations on arbitrarily large dimensions, mutually coupled by impulses. In particular, we study the dynamics that is induced by the graph of impulsive interactions, for certain classes of graphs. The methodology of research is mathematical with rigorous proofs based on the mathematical theory of dynamical systems and the theory of graphs.
The main tool we use is the consideration of the pulses ocurring in the whole network during certain time interval. For strongly connected graphs we prove that every neuron fires infinitely many times.
For complete graphs we find an optimal lower bound on the network size that ensures the existence of the so called "grand coalition". Also, if the graph is not complete but each neuron has enough connections, the grand coalition is still present. Finally, we construct extensions from the strongly connected and star-shaped graphs that also imply the existence the grand coalition.

[a]Speaker

Control and observation of gene regulatory networks by minimal number of molecules

Kazuki Maeda [a] [1], Atsushi Mochizuki [2]

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In modern biology, many complex biological networks have been determined by experiments. These networks describe regulations between a large number of species of molecules. Our goal is to understand the dynamics of these networks and to clarify the origin of biological functions. However, there is an obstacle: it is very difficult by the current technology to observe many molecules simultaneously with sufficient time resolution. Therefore, we cannot obtain sufficient data to describe the system as a concrete ODE system.
The Linkage Logic proposed by Mochizuki et al. and Fiedler et al. may give a solution to this problem. This theory enables us to identify the long-term dynamics of the system by observing only a subset of molecules called feedback vertex set (FVS) which is determined from the regulatory linkage alone. We will show that, as an example, cell differentiation in the development of Ciona intestinalis is identified or controlled by the activity of only five genes. We will also show that there is a classification of genes in the FVS: the genes related to intracellular dynamics and intercellular dynamics. We consider that the latter genes have an important role to make difference in differentiation between each cells.

[a]Speaker

Spectral properties of magnetic graphs

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We analyze spectral properties of a quantum graph with a δ coupling in the vertices exposed to a homogeneous magnetic field perpendicular to the graph plane. We find the band-and-gap structure of the spectrum in the case when the chain exhibits a translational symmetry. Then we study the discrete spectrum in the gaps resulting from compactly supported coupling, magnetic or geometric perturbations. The method we use is based on translating the spectral problem for the differential equation in question into suitable difference equations. These results were obtained in collaboration with Pavel Exner.

[a] Speaker

Numerical investigations on the stability of periodic waves

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This work is concerned with the stability theory of periodic traveling waves in some one-dimensional dispersive Hamiltonian PDEs pertaining to a general framework that contains well known cases like the generalized Korteweg-de Vries equation (gKdV) or the Euler-Korteweg system (EK). We focus on co-periodic stability, that is, stability with respect to perturbations with the same period as the wave.

We have obtained stability criteria in terms of an abbreviated action integral, which is a function of the wave's parameters. For periodic waves, this action integral plays the role of the celebrated Boussinesq moment of instability for solitary waves. Our stability criteria depend on the negative signature of the Hessian of the action. By comparison, stability criteria for the solitary waves only involve the second derivative of the Boussinesq moment of instability with respect to the speed of the solitary wave.

We will show how to compute the required Hessian matrices, and present some numerical investigations of those stability criteria for several PDEs including (gKdV) cases, as well as the generalized Boussinesq equation. This is a joint work with S. Benzoni-Gavage et L.M. Rodrigues.

[a] Speaker

Existence result to the rate type fluid model arising form crystal plasticity

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Looking at severe plastic deformation experiments, it seems that crystalline materials at yield behave as a special kind of anisotropic, highly viscous fluids flowing through an adjustable crystal lattice space. The flow through the lattice space is restricted to preferred crystallographic planes and directions causing anisotropy. In the deformation process the lattice is strained and rotated.
The derivation of a model is based on the rate form of the decomposition rule: the velocity gradient consists of the lattice velocity gradient and the sum of the velocity gradients corresponding to the slip rates of individual slip systems. We employ the Gibbs potential to obtain rate-type stress-strain constitutive response.

We propose a new regularisation to the stress evolution equation. Using the energy estimates we prove global in time existence of a weak solution by the Galerkin method in two space dimension. We apply the tensorial version of the logarithmic Sobolev inequality.
The considered crystal plasticity model allowing for large deformations is treated as the flow-adjusted boundary value problem. As a test example we analyse a micropillar compression. We propose finite element scheme for a numerical solution in the Arbitrary Lagrangian Eulerian (ALE) configuration.

[a]Speaker

A potential estimates approach to a priori estimates for elliptic equations

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Some a priori estimates for the solutions of certain classes of Dirichlet problems associated to non divergence structure elliptic equations will be presented. The bounds are achieved by means of a potential estimate obtained for the solutions of the same kind of problems, but with more regular datum.

[a]Speaker

Asymptotic stability of constant solutions in delay differential equations with a constant delay and transcendental equations with complex coefficients

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In this talk, we investigate the asymptotic stability of a constant solution for a delay differential equation (DDE) $x'(t) = f(x(t), x(t-\tau))$, where $f\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a smooth function, and τ is a positive constant. This is an equation with a single constant delay.
In general, the asymptotic stability of a constant solution of DDEs is determined by the location of the roots of the characteristic equation of the linearized equation. For the above DDE, the characteristic equation becomes $\det(\lambda I + A - \mathrm{e}^{-\lambda\tau} B) = 0$, where A and B are $n \times n$ real matrices. If A and B are simultaneously triangularizable, then this equation is reduced to a transcendental equation $z + a - b\mathrm{e}^{-z} = 0$, where parameters a and b are generally complex.
We see that by using the "graph-like" expression of the Lambert W function in some coordinate system of the complex plane $\mathbb{C}$, a necessary and sufficient condition on a and b for which all the roots of the above transcendental equation have negative real parts can be obtained. Here the Lambert W function is the multi-valued inverse of a complex function $z \mapsto z\mathrm{e}^z$, and the set of the roots is equal to $W(b\mathrm{e}^a) - a$. We also give an application of this result to the stabilization problem of unstable constant solutions by the delayed feedback control proposed by Pyragas.

[a] Speaker

Application of Extended Prelle-Singer Method for the Second-Order Ordinary Differential Equations

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In this work, we investigate how to obtain the general solutions of a nonlinear differential equations by using Prelle-Singer method. Then we consider Mathews-Lakshmanan oscillator equation, which possesses exact periodic solution. It has interesting features, namely the nonlinearity of the potential and the presence of a term that can be interpreted as a position-dependent mass. We obtain the time-independent integral of this equation by using Prelle-Singer method, the equation is integrable by these first integrals and we can find solution by using these first integrals. Further, the knowledge of λ-symmetry and integrating factor can be obtained by using functions S and R which are related to λ-symmetry and integrating factor, respectively. And the Lagrangian, the Hamiltonian and conjugate momentum functions can be identified. Then we interpret these results by drawing the graphics via phase portrait methods.

[a] Speaker

Segregation phenomena for some population models with three species

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We are concerned with the following population model;

$$\begin{cases} u_t = d_1\Delta u + u(a - u - kv) & \text{in} \quad \Omega \times (0,\infty), \\ v_t = d_2\Delta v + v(b - lu - v + mv) & \text{in} \quad \Omega \times (0,\infty), \\ w_t = d_3\Delta u + dw(1 - nv - w) & \text{in} \quad \Omega \times (0,\infty), \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = \dfrac{\partial w}{\partial \nu} = 0 & \text{on} \quad \partial\Omega \times (0,\infty) \end{cases}$$

where $d_i(i = 1,2,3)$, a, b, k, l, m, n and d are positive number. In special, d is a positive parameter. $\Omega \subset \mathbf{R^N}$ is a bounded domain with smooth boundary with $N \geqq 1$. Now u, v and w denotes each population density in Ω. We also put $u(x,0) = u_0(x), v(x,0) = v_0(x), w(x,0) = w_0(x)$ as non-negative initial function in Ω.
Under some special conditions on the coefficient, we can obtain a positive constant equilibrium point $\mathbf{u_p} = (\mathbf{u_*}, \mathbf{v_*}, \mathbf{w_*})$. By using usual bifurcation argument, we can constract Turing and Hopf bifurcation from u_p for some $d > 0$. Moreover, we can show some segregation phenomena by using Mathematica.

[a]Speaker

Perturbation-response relation and network topology in biochemical reaction systems

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In living cells, biochemical reaction pathways form complex networks, such as metabolic network. To elucidate the dynamics of these biochemical systems, one experimental approach is examining their sensitivities to perturbations where the amounts of enzymes, which catalyze the reactions, are increased/decreased. On the other hand, to study the systems mathematically, there is a limitation that we do not know quantitative details, such as reaction rate, of the dynamics.
In this work, we established a theoretical framework that determines the response to perturbations qualitatively from the network structure alone. It turns out that under a given perturbation (corresponding to enzyme knockout), only particular molecules can change their concentrations. We also found that there exists a topological condition that governs to what extent the effect of each perturbation can spread in the network.

[a]Speaker

Emergence of phase-locked states for the Winfree model in a large coupling regime

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We study the large-time behavior of the globally coupled Winfree model in a large coupling regime. The Winfree model is the first mathematical model for the synchronization phenomenon in an ensemble of weakly coupled limit-cycle oscillators. For the dynamic formation of phase-locked states, we provide a sufficient framework in terms of geometric conditions on the coupling functions and coupling strength. We show that in the proposed framework, the emergent phase-locked state is the unique equilibrium state and it is asymptotically stable in an ℓ^1-norm; further, we investigate its configurational structure.

[a]Speaker

Quantitative isoperimetric inequalities for log-convex probability measures on the line.

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We analyze the isoperimetric inequality for symmetric log-convex probability measures on the line. Using geometric arguments we first re-prove that extremal sets in the isoperimetric inequality are intervals or complement of intervals (a result due to Bobkov and Houdré). Then we give a quantitative form of the isoperimetric inequality, leading to a somehow anomalous behavior. Indeed, it could be that a set is very close to be optimal, in the sense that the isoperimetric inequality is almost an equality, but at the same time is very far (in the sense of the symmetric difference between sets) to any extremal sets!

References

F. Feo, M. Posteraro, Roberto C. (2014) *Quantitative isoperimetric inequalities for log-convex probability measures on the line* , Journal Of Mathematical Analysis And Applications, **420**, n.2, 879-907.

[a]Speaker

On the eigenvalues of Steklov-type problems

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We consider the Steklov eigenvalue problem for the Laplace operator and its natural fourth order generalization on bounded domains Ω of class C^1 in $\mathbb{R}^N$ which reads

$$\begin{cases} \Delta^2 u - \tau\Delta u = 0, & \text{in } \Omega, \\ \frac{\partial^2 u}{\partial \nu^2} = 0, & \text{on } \partial\Omega, \\ \tau\frac{\partial u}{\partial \nu} - \mathrm{div}_{\partial\Omega}\big(D^2 u.\nu\big) - \frac{\partial(\Delta u)}{\partial \nu} = \lambda u, & \text{on } \partial\Omega. \end{cases}$$

The Steklov problem for the Laplacian is classical (Steklov, 1902). This problem arises in the study of a free vibrating membrane with mass concentrated at the boundary. As for the biharmonic operator, we show that the boundary conditions that we introduce are naturally obtained through a physical model involving the vibration of a free plate with mass concentrating at the boundary. We do the same analysis also for the second order problem. We study the asymptotic behavior of the eigenvalues of the two problems in this mass concentration phenomenon.
Finally, we prove that an analogue of the Brock-Weinstock inequality for the Steklov Laplacian holds for our biharmonic Steklov problem, i.e., the ball maximizes the first non-trivial eigenvalue among all domains of class C^1 in $\mathbb{R}^N$ of given measure. Based on joint works with Pier Domenico Lamberti and Davide Buoso.

[a]Speaker

The porous medium equation on manifolds with conical singularities

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We consider the porous medium equation on manifolds with conical singularities. We show existence, uniqueness and maximal L^p-regularity of a short time solution. In particular, we obtain information on the short time asymptotics of the solution near the conical point. Our method is based on bounded imaginary powers results for cone differential operators on Mellin-Sobolev spaces and R-sectoriality perturbation techniques.

[a]Speaker

On the existence of periodic solutions to a free boundary problem for adsorption phenomenon

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We study a one-dimensional free boundary problem, which is a mathematical model for an adsorption phenomenon appearing in concrete carbonation process. This model was proposed in line of previous studies of three dimensional concrete arbonation process. We talked a result about the existence and uniqueness of a time-local (strong) solution to this system at the last meeting of Equadiff. This result was obtained by means of the abstract theory of nonlinear evolution equations and Banach's fixed point theorem, and especially, the maximum principle applied to our problem played a very important role to obtain the uniform estimate to approximate solutions. Aiki and Murase proved the existence of a time-global (strong) solution to this system by contradiction in 2014. In this talk we intend to report some results of the existence of periodic solution of the problem.

[a]Speaker

Systems of two delayed nonlinear equations

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This paper deals with systems of functional differential equations
considered for oscillators near Hopf bifurcation with constantly as well as linearly delayed coupling.
There are given numerical examples with the approximated solutions in the form of series with demanded accuracy obtained by differential transform method.

[a]Speaker

Robustness analysis of discontinuous neural networks

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Neural networks have been studied widely issuing from the fact that, they have many important applications in pattern recognition, signal processing, associative memory, and optimization problems. All of these applications tediously depend on dynamical behaviors of the network. In the present talk, different from the most existing results, we introduce a more general class of neural networks related to the impulsive phenomena that happen at nonprescribed moments of time. The aim of defining this new class is that in the real world problems the impulses of many systems do not occur at fixed times but depends on the states of the systems, for example, some circuit control systems, saving rate control systems and population control systems and so on. These types of systems are called state-dependent impulsive differential systems or impulsive systems with variable-time impulses. In the current talk, we discuss robustness of the neural networks having impulse times at the hyper surfaces $\Gamma_k : t = \theta_k + \tau_k(x), k \in \mathbb{Z}$, not on the planes $t = \theta_k$. In order to analyze global robust asymptotic stability of such systems, first we reduce the system to a fix time impulsive system by means of $B-$equivalence method, then we used an appropriate Lyapunov function and linear matrix inequality (LMI). An illustrative example is given to show the effectiveness of the theoretical results.

[a]Speaker

Modelling of spin-polarized transport in semiconductors

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[3] Laboratoire Paul Painlevé – Université Lille I - Sciences et technologies – France

Spintronics is a young science trying to subjugate spin of electron and make it work for the civilisation as well (and even better) as electron's charge works now in microelectronic devices. Some scientists speculate that semiconductor spin-based devices (e.g. spin field-effect transistor, spin diodes) can be more efficient than their counterparts based only on charge current.
We consider the matrix drift-diffusion model which except for charge densities and charge current considers three spin components and their currents. The model is derived from matrix Boltzmann equation. It consists of four continuity equations for charge and spin densities and Poisson equation for electric potential. The system is fully coupled and nonlinear.
We present some analytical results for continuous system as well as for its finite volume discretization: we proved existence of a unique bounded weak solution to the system and of a bounded numerical solution. The proves are based on different reformulations of the model. Furthermore free energy functions for continuous and discrete cases are presented and some results of numerical experiments for finite volume scheme are shown.

[a]Speaker

Krasnosel'skii Formula for constrained semilinear differential inclusion

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The well-known Krasnosel'skii formula concerns an ODE $\dot{x} = f(t,x)$, $x \in \mathbb{R}^N$, $t \in [0,1]$. Roughly speaking, it asserts that the Brouwer degrees of $-f(0,\cdot)$ and $\mathrm{Id} - P_t$ are equal, where P_t is the associated Poincaré t-operator. We consider a constrained semilinear evolution inclusions of parabolic type

$$\begin{cases} \dot{u}(t) \in Au(t) + F(t,u(t)), & t \in [0,1], u \in \mathcal{K}, \\ u(0) = x \in \mathcal{K}, \end{cases} \tag{21}$$

in the infinite dimension Banach/Hilbert space and topological properties of the solution map. The set of constraints $\mathcal{K}$ is assumed to be closed and convex. A counterpart of Krasnosel'skii formula concerning (21), namely a relation between the constrained fixed point index of the Krasnosel'skii–Poincaré operator of translation along trajectories associated with (21) and the constrained topological degree of the right-hand side $A + F(0,\cdot)$ will be presented. The connection joining the problem (21) and partial differential inclusion on the open bounded subset of $\mathbb{R}^N$ will be discussed.

[a]Speaker

Global Invertibility and Implicit Function Theorems by Mountain Pass Theorem

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We formulate some global invertibility and implicit function theorems. We extend the result of Idczak, Skowron and Walczak on the invertibility of the operators to the case of the operators with critical points. The proof relies on the Mountain Pass Theorem combined with the Palais-Smale condition guaranteeing the claim by the invertibility of the first or the third derivative.

[a]Speaker

Degeneracy in finite time of 1D quasilinear wave equations

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We consider the large time behavior of solutions of the Cauchy problem of the quasilinear wave equation: $\partial_t^2 u = \partial_x((1+u)^{2a}\partial_x u)$, which has richly physical backgrounds. If $1 + u(0, x)$ is bounded away from a positive constant, we can construct a local solution for smooth initial data. When $1 + u(t, x)$ is going to 0 in finite time, the equation degenerates. We give a sufficient condition that the equation degenerates in finite time. A known result on global existence and our main theorem determine a threshold of $\int_{\mathbb{R}} u_1(x)dx$ separating the global existence of solutions and the occurrence of the degeneracy.

[a]Speaker

Solutions for nonlinear systems on unbounded domains with p-Laplacian-like operators

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We study the existence of at least one solution to the following systems of resonant boundary value problems

$$(\varphi(x'))' = f(t, x, x'), \quad x'(0) = 0, \quad x'(\infty) = 0,$$

where $f : \mathbb{R}_+ \times \mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}^k$ is continuous,

$$\varphi(s) = (\varphi_1(s_1), \dots, \varphi_k(s_k)),$$

$s \in \mathbb{R}^k$, and $\varphi_i : \mathbb{R} \to \mathbb{R}$ is an increasing homeomorphism such that $\varphi_i(0) = 0$, $i = 1, \dots, n$.
We give conditions for the existence of a solution for this BVP using the generalization of the Miranda Theorem [1]:
Let $M_i > 0$, $i = 1, \dots, k$, and F be an admissible map from $\prod_{i=1}^k [-M_i, M_i]$ to $\mathbb{R}^k$, i.e. there exist a Banach space E, dim $E \geq k$, a linear, bounded and surjective map $\varphi : E \to \mathbb{R}^k$ and an R_δ-map Φ from $\prod_{i=1}^k [-M_i, M_i]$ to E such that $F = \varphi \circ \Phi$.
If for any $i = 1, \dots, k$ and every $y \in F(x)$, where $|x_i| = M_i$, we have

$$x_i \cdot y_i \geq 0,$$

then there exists x such that $0 \in F(x)$.

[1] K. Szymańska-Dębowska, On a generalization of the Miranda Theorem and its application to boundary value problems, J. Differential Equations, 258 (2015), 2686-2700.

[a]Speaker

Asymptotic behavior of positive solutions of a kind of Lanchester-type system

Hiroyuki Usami [a] [1]

[1] Gifu University – Japan

Let us consider the system

$$\begin{cases} x' = -a(t)xy, \\ y' = -b(t)xy, \end{cases} \tag{1}$$

where $a, b \in C[0, \infty)$ are positive functions satisfying

$$0 < \inf_{t \geq 0} a(t) \leq \sup_{t \geq 0} a(t) < \infty, \quad 0 < \inf_{t \geq 0} b(t) \leq \sup_{t \geq 0} b(t) < \infty. \tag{2}$$

If the initial values $x(0), y(0)$ are positive, then the solution $(x(t), y(t))$ of (1) exists on $[0, \infty)$, and remains positive there. So $\lim_{t\to\infty} x(t)$ and $\lim_{t\to\infty} y(t)$ exist as nonnegative numbers. Denote the solution of (1) with the initial value $(x(0), y(0)) = (\alpha, \beta), \alpha, \beta > 0$, by $(x(t; \alpha, \beta), y(t; \alpha, \beta))$. When we fix α and move $\beta > 0$, we want to examine how $\lim_{t\to\infty}(x(t; \alpha, \beta), y(t; \alpha, \beta))$ varies according to β.

Theorem 1. *There are constants* $\beta_1 = \beta_1(\alpha)$ *and* $\beta_2 = \beta_2(\alpha)$ $(0 < \beta_1 \leq \beta_2)$ *such that:*
(i) *if* $\beta < \beta_1$, *then* $\lim_{t\to\infty} x(t; \alpha, \beta) > 0$, $\lim_{t\to\infty} y(t; \alpha, \beta) = 0$;
(ii) *if* $\beta_1 \leq \beta \leq \beta_2$, *then* $\lim_{t\to\infty} x(t; \alpha, \beta) = \lim_{t\to\infty} y(t; \alpha, \beta) = 0$;
(iii) *if* $\beta > \beta_2$, *then* $\lim_{t\to\infty} x(t; \alpha, \beta) = 0$, $\lim_{t\to\infty} y(t; \alpha, \beta) > 0$.

[a] Speaker

Monomolecular reaction networks: a new proof of flux transitivity

Nicola Vassena [a] [1]

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We study the network response to perturbations of a reaction rate j*. Specifically, we describe which other reaction rates j' respond by nonzero reaction flux, at steady state. Nonzero responses of j' to j* are called flux influence of j* on j'. Mochizuki and Fiedler established transitivity of flux influence, for monomolecular reaction networks. We give a new, independent, and conceptually simplified proof of that intriguing fact. Our proof uses standard connectivity concepts from graph theory, and Menger's Theorem. Based on the network structure, only, this also leads to a simplified characterization of all flux influence sets.

[a] Speaker

Non-almost periodic and non-asymptotically almost periodic solutions of limit periodic and almost periodic difference systems

Michal Vesely [a] [1]

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This is a joint work with Petr Hasil. We study limit periodic and almost periodic homogeneous linear difference systems whose coefficient matrices belong to a group. We find such groups that the systems, which do not have any non-zero asymptotically almost periodic solution, form a dense subset in the set of all considered systems. Then, we find a very general condition on the groups under which the systems, whose fundamental matrices are not almost periodic, form a dense subset as well. The treated problem is analysed for the elements of the coefficient matrices from an infinite field with an absolute value. Nevertheless, the presented results are new even for the field of complex numbers.

[a]Speaker

Oscillatory Integrals and Fractal Dimension

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[2] Université de Bourgogne, Faculté des Sciences Mirande – Laboratoire I.M.B., Laboratoire IMB – France

Theory of singularities has been closely related with the study of oscillatory integrals. More precisely, the study of critical points is closely related to the study of asymptotic of oscillatory integrals.

In our work we investigate the fractal properties of a geometrical representation of oscillatory integrals. We are motivated by a geometrical representation of Fresnel integrals by a spiral called the clothoid, and the idea to produce a classification of singularities using fractal dimension. Fresnel integrals are a well known class of oscillatory integrals. We consider oscillatory integral

$$I(\tau) = \int_{\mathbb{R}^n} e^{i\tau f(x)} \phi(x) dx,$$

for large values of real parameter τ, where f is the analytic phase and ϕ is the smooth amplitude with a compact support. We measure the oscillatority by Minkowski dimension of the planar curve parameterized by functions X and Y that are the real and imaginary parts of the integral I, respectively. Also, the oscillatory dimension is defined as Minkowski dimension of the graph of function $x(t) = X(1/t)$, near $t = 0$, and analogously for Y. We provide explicit formulas connecting these Minkowski dimensions and associated Minkowski contents with asymptotics of the integral I and the type of the critical point of the phase f. Techniques used include Newton diagrams and the resolution of singularities.

[a]Speaker

Homogenized model of immiscible incompressible two-phase flow in double porosity media

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We prove the homogenization result for immiscible incompressible two-phase flow in double porosity media. The initial microscopic model consists of the usual equations derived from the mass conservation law for each fluid and the standard Darcy-Muskat law relating the velocities to the pressure gradients and gravitational effects. Our less restrictive assumptions yield the more general model, with the discontinuous global pressure and saturation functions allowed. The problem is written in the phase formulation, i.e. where the phase pressures and the phase saturations are primary unknowns. The fractured porous medium consists of periodically repeating homogeneous ordinary porous medium blocks and highly conductive fractures, with the discontinuous absolute permeability of the medium. We prove the convergence of the solutions and derive three macroscopic models of the problem corresponding to various ranges of contrast using the two-scale convergence method combined with the dilatation technique.

[a]Speaker

Asymptotic behavior of solutions to the drift-diffusion equation with critical dissipation

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The initial value problem for the drift-diffusion equation arising from the model of semiconductor devices is studied. The dissipation on the equation is given by the half Laplacian. Well-posedness, global in time existence and decay of solutions are known. The purpose in this poster is to show the asymptotic profile of the solution as the time variable tends to infinity.

[a]Speaker

q-Analogue of summability of formal solutions of linear q-difference-differential equations

Hiroshi Yamazawa [a 1]

[1] Hiroshi Yamazawa – Japan

Let $m \geq 1$, $q > 1$ and $\delta > 0$ be a real number. In this talk, I will consider the following q-difference-differential equations

$$\sum_{j+\delta|\alpha| \leq m} a_{j,\alpha}(t,z)(tD_q)^j \partial_z^\alpha X = F(t,z), \tag{EQ}$$

with the unknown function $X = X(t,z)$, where $a_{j,\alpha}(t,z)$ $(j + \delta|\alpha| \leq m)$ and $F(t,z)$ are holomorphic functions in a neighborhood of $(0,0) \in \mathbb{C}_t \times \mathbb{C}_z^d$

where we define $q-$difference operator D_q by

$$(D_q f)(t,z) = \frac{f(qt,z) - f(t,z)}{qt - t}.$$

The equation (EQ) has a formal solution which is q-summable in a suitable direction.

[a]Speaker

The Ginzburg-Landau Justification in the Quasilinear Case

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We are interested in the behaviour of pattern forming systems close to the first instability. By multiscale analysis we find that in lowest order the slow time and space modulations of propagating patterns have to satisfy a Ginzburg-Landau equation. This derivation of the Ginzburg-Landau equation is only formal in the sense that it is unclear that the so constructed approximate solutions make correct predictions about the full problem. To make these calculation rigorous, we have to show that the error, i.e., the difference between the formal approximation and a true solution stays small over the relevant time scale. In the semilinear case, the long-time estimate for the error is done with the help of the variation of constants formula and a simple application of Gronwall's inequality. In the quasilinear case, however, this method fails, since a direct estimate is not possible due to an appearing non-integrable singularity. We demonstrate how to overcome this difficulty by combining ideas from maximal regularity with the classical approach.

[a]Speaker

About attractors for 3D Bingham model

Victor Zvyagin [a] [1]

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The motion of an incompressible fluid of a constant density is described by the Cauchy momentum equation

$$\frac{\partial \upsilon}{\partial t} + \sum_{i=1}^{n} \upsilon_i \frac{\partial \upsilon}{\partial x_i} + \text{grad } p = \text{Div } \sigma + f, \ \text{div}\, \upsilon = 0. \qquad (1)$$

The system of equations describing the motion of the Bingham fluid is obtained by adding to the general equation (1) the rheological relation between the deviator of the stress tensor σ and the strain rate tensor $\varepsilon(\upsilon) = \frac{1}{2}\left(\nabla\upsilon + \nabla^T\upsilon\right)$

$$\sigma = 2\mu\varepsilon(\upsilon) + \tau^* \frac{\varepsilon(\upsilon)}{|\varepsilon(\upsilon)|} \text{ for } |\varepsilon(\upsilon)| \neq 0 \text{ and } |\sigma| \leq \tau^* \text{ for } |\varepsilon(\upsilon)| = 0, \qquad (2)$$

where $\mu, \tau^* = \text{const} > 0$.

The existence of solutions of this model has been studied in details. However, the behavior of solutions for this model at infinity (attractors) have been studied only in two dimensions. In the report we present the theorem of an attractor existence for the Bingham model in three dimension. We note that in its proof the theory of trajectory attractors was used, since it is impossible to use the theory of dynamical systems due to the fact that the uniqueness of weak solution in the three-dimensional case for this system is unknown.

[a] Speaker

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