Existence and stability of a solution with a new prescribed behavior for a heat equation with a critical nonlinear gradient term

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Abstract

We consider the nonlinear heat equation with a nonlinear gradient term:
\[ \partial_t u = \Delta u + \mu |\nabla u|^q + |u|^{p-1}u, \quad \mu > 0, \quad q = \frac{2p}{p+1}, \quad p > 3, \quad t \in (0, T), \quad x \in \mathbb{R}^N. \]

We construct a solution which blows up in finite time \( T > 0 \). We also give a sharp description of its blow-up profile and show that it is stable with respect to perturbations in initial data.

The construction relies on the reduction of the problem to a finite dimensional one, and uses the index theory to conclude. The stability is a by-product of the existence proof, thanks to the interpretation of the finite dimensional problem in terms of the blow-up time and point.

The blow-up profile does not scale as \( (T-t)^{1/2} |\log(T-t)|^{1/2} \), like in the standard nonlinear heat equation, i.e. \( \mu = 0 \), but as \( (T-t)^{1/2} |\log(T-t)|^{\beta} \) with \( \beta > 1/2 \). We also show that \( u \) and \( \nabla u \) blow up simultaneously and at a single point, and give the final profile. In particular, the final profile is more singular than the case of the standard nonlinear heat equation. This is a joint work with Slim Tayachi from the University of Tunis El Manar.

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