Existence and stability of a solution with a new prescribed behavior for a heat equation with a critical nonlinear gradient term

Hatem Zaag*[1]

[1]Laboratoire Analyse, Géométrie et Applications (LAGA) – Université Sorbonne Paris Cité (USPC), Institut Galilée, université Paris 13, CNRS : UMR7539 – Institut Galilée, Université Paris 13, 99 avenue Jean-Baptiste Clément, F-93430, Villetaneuse, France, France

Abstract

We consider the nonlinear heat equation with a nonlinear gradient term: $\partial_t u = \Delta u + \mu|\nabla u|^q + |u|^{p-1}u,\ \mu > 0,\ q = 2p/(p+1),\ p > 3,\ t \in (0,T),\ x \in \mathbb{R}^N$. We construct a solution which blows up in finite time $T > 0$. We also give a sharp description of its blow-up profile and show that it is stable with respect to perturbations in initial data.

The construction relies on the reduction of the problem to a finite dimensional one, and uses the index theory to conclude. The stability is a by-product of the existence proof, thanks to the interpretation of the finite dimensional problem in terms of the blow-up time and point.

The blow-up profile does not scale as $(T-t)^{1/2}|\log(T-t)|^{1/2}$, like in the standard nonlinear heat equation, i.e. $\mu = 0$, but as $(T-t)^{1/2}|\log(T-t)|^{\beta}$ with $\beta > 1/2$. We also show that u and ∇u blow up simultaneously and at a single point, and give the final profile. In particular, the final profile is more singular than the case of the standard nonlinear heat equation. This is a joint work with Slim Tayachi from the University of Tunis El Manar.

*Speaker