Regularization by noise for the stochastic transport equation

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Abstract

We discuss several aspects of weak (L^∞-) solutions to the stochastic transport equation

$$du = b \cdot Du\, dt + \sigma Du \circ dW_t$$

with Stratonovich multiplicative noise. Here, b is a time dependent vector field (the drift), u is the unknown, σ a real number, and W_t a Brownian motion.

For the deterministic equation ($\sigma = 0$) it is well-known that multiple solutions may exist and that solutions may blow up from smooth initial data in finite time provided that the drift is not regular enough (basically less than Lipschitz in space). For the stochastic equation ($\sigma \neq 0$) instead, a suitable integrability condition on the drift is sufficient to prevent the formation of non-uniqueness and of singularities.

In my talk I will explain some ideas of a joint project with F. Flandoli, M. Gubinelli and M. Maurelli, which concern this phenomenon of regularization by noise. While a similar result was achieved via stochastic characteristics, we have obtained the conservation of Sobolev regularity of the initial data relying on PDE techniques.

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