A geometric approach for sharp Local well-posedness of quasilinear wave equations

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Abstract

I will present my work on the problem of optimal well-posedness for general quasi-linear wave equations in $\mathbb{R}^{1+3}$. In general, equations of this type are ill-posed with H^s data for $s \leq 2$. The optimal result of the well-posedness with data in $H^s, s > 2$ was proved by Smith-Tataru by constructing parametrices using wave packets. I will present the proof by the vectorfield approach. This approach is initiated by Klainerman, developed by him and Rodnianski to achieve the result of $s > 2 + \frac{2-\sqrt{3}}{2}$ and then applied to the Einstein vacuum equations to achieve the result of $s > 2$. To achieve the optimal result for the general quasi-linear wave equations, one has to face the major hurdle caused by the Ricci tensor of the metric. This posed a question that if the geometric approach can provide the sharp result for the non-geometric equations. The optimal result is achieved based on geometric normalization and new observations on the regularity properties of the eikonal equation associated to the quasi-linear wave equations.

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