Sharp asymptotic profiles for singular solutions to an elliptic equation with a sign-changing nonlinearity

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Abstract

We consider a positive solution to a nonlinear elliptic equation on a punctured ball. The linear part is the classical Laplacian. When the nonlinear part is positive and critical, this is similar to the classical problem studied by Caffarelli-Gidas-Spruck. When the nonlinear part is negative and a pure power, the problem is associated to a natural convex functional and the singularities are completely understood. In the present work, we mix the two nonlinearities. We show the existence of several potential behaviors. Two of them are natural extensions of the case of constant-sign nonlinearity. Two other behaviors are arising from the interaction of the two nonlinearity. In this talk, I will describe all the possible behaviors and I will show how the methods of apriori analysis in nonlinear elliptic problems are helping understanding this problem.