
On the continuation of orbits “after infinity” in Newtonian dynamics

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Abstract

Consider a hyperbolic motion of the n -body problem. The asymptotic expansion of each cartesian coordinate of each body begins with a linear term in t . The next term is in $\log t$. If we eliminate the time t and express the coordinates of each body as a function of one of them, there are infinitely many terms with a log, except in a remarkable case. If the limiting configuration is a central configuration, there are no term in log. Chazy discovered this property and related it with the possibility of an analytic continuation of the orbit after infinity, which he claimed to be possible only when the limiting configuration is central. In the example $n = 2$, the orbit is a hyperbola. The continuation after infinity makes it a closed orbit, an ellipse in the projective space. This continuation is clear if we know a remark due to Appell: The Kepler problem in the plane corresponds by central projection with the Kepler problem on the sphere. In the second problem, “after infinity” means below the equator. The hyperbolas project on closed spherical ellipses. We will state precisely Chazy’s remarks and prove them by using Appell’s central projection, i.e., by the principles of projective dynamics.

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