On Whitham’s conjecture of a highest cusped wave for a nonlocal shallow water wave equation

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Abstract

We consider the Whitham equation $u_t + 2uu_x + Lu_x = 0$, where $L$ is the non-local (Fourier multiplier) operator given by the symbol $m(\xi) = (\tanh(\xi)/\xi)^{1/2}$. G. B. Whitham conjectured that for this equation there would be a highest, cusped, travelling-wave solution. We find this wave as a limiting case at the end of the main bifurcation curve of $\mathcal{P}$-periodic solutions, and prove that it belongs to the Hölder space $C^\alpha$ for all $\alpha < 1/2$, but to no Hölder space $C^\alpha$ with $\alpha > 1/2$. Further properties of the wave, and of traveling-wave solutions of the Whitham equation in general, are given. An essential part of the proof consists in a precise analysis of the integral kernel corresponding to the symbol $m$. 

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