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# On Whitham's conjecture of a highest cusped wave for a nonlocal shallow water wave equation

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## Abstract

We consider the Whitham equation  $u_t + 2uu_x + Lu_x = 0$ , where  $L$  is the non-local (Fourier multiplier) operator given by the symbol  $m(\xi) = (\frac{\tanh(\xi)}{\xi})^{1/2}$ . G. B. Whitham conjectured that for this equation there would be a highest, cusped, travelling-wave solution. We find this wave as a limiting case at the end of the main bifurcation curve of  $P$ -periodic solutions, and prove that it belongs to the Hölder space  $C^\alpha$  for all  $\alpha < 1/2$ , but to no Hölder space  $C^\alpha$  with  $\alpha > 1/2$ . Further properties of the wave, and of traveling-wave solutions of the Whitham equation in general, are given. An essential part of the proof consists in a precise analysis of the integral kernel corresponding to the symbol  $m$ .

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