A selection approach in stochastic homogenization: Special Quasirandom Structures

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Abstract

In this work, we introduce a selection approach for the homogenization of a random, linear elliptic second order partial differential equation set on a bounded domain in $\mathbb{R}^d$. The random diffusion coefficient matrix field is assumed to be uniformly elliptic, bounded and stationary (“periodic in law”). In the limit when $\varepsilon \to 0$, the solution of the equation converges to that of a homogenized problem of the same form, the coefficient field of which is a deterministic and constant matrix $A^*$ given by an average involving the so-called corrector function that solves a random auxiliary problem set on the entire space. In practice, the corrector problem is approximated on a bounded domain $Q_N$ as large as possible. A by-product of this truncation procedure is that the deterministic matrix $A^*$ is approximated by a random, apparent homogenized matrix $A_N^*(\omega)$. We select only random realizations that satisfy special conditions (e.g. in a bi-composite material with equal probability for each phase, we enforce that each phase is present with equal volume in the finite supercell).

We prove, under mild hypothesis (symmetry of $A$), that our approach is convergent. We demonstrate in special cases that it is efficient. A significant part is devoted to the introduction of the conditioning we used, since the efficiency is related to the conditioning. The method is illustrated with numerical results.

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