A selection approach in stochastic homogenization: Special Quasirandom Structures

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Abstract

In this work, we introduce a selection approach for the homogenization of a random, linear elliptic second order partial differential equation set on a bounded domain in $\mathbb{R}^d$. The random diffusion coefficient matrix field is assumed to be uniformly elliptic, bounded and stationary ("periodic in law"). In the limit when $\varepsilon \to 0$, the solution of the equation converges to that of a homogenized problem of the same form, the coefficient field of which is a deterministic and constant matrix $A^\star$ given by an average involving the so-called corrector function that solves a random auxiliary problem set on the entire space.

In practice, the corrector problem is approximated on a bounded domain Q_N as large as possible. A by-product of this truncation procedure is that the deterministic matrix $A^\star$ is approximated by a random, apparent homogenized matrix $A^\star_N(\omega)$. We select only random realizations that satisfy special conditions (e.g. in a bi-composite material with equal probability for each phase, we enforce that each phase is present with equal volume in the finite supercell).

We prove, under mild hypothesis (symmetry of A), that our approach is convergent. We demonstrate in special cases that it is efficient. A significant part is devoted to the introduction of the conditioning we used, since the efficiency is related to the conditioning. The method is illustrated with numerical results.

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