Densities for the Navier-Stokes equations with noise

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Abstract

We present a proof of existence of the density with respect to the Lebesgue measure, as well as of regularity in Besov spaces, for the solution of stochastic differential equations with "non-smooth" data.

As an application we show existence of densities for the finite dimensional marginal distributions of the law of solutions of the 3D Navier-Stokes equations forced by Gaussian noise. Classical methods, such as the Malliavin calculus, do not work in this setting for reasons that are strongly related to the three dimensional case.

The same method provides also regularity in time of the densities, as well as absolute continuity of the laws of some quantities (energy and dissipation rate, for instance) that depend on a infinite number of components.

A stronger result, namely Hölder continuity of the densities, is available through a suitable conditioned Fokker-Planck equation.

When the random forcing has no full support in Fourier space the problem is open. In this case we can prove the mere existence of a density, without any regularity property, by using the backward local smoothness of trajectories, and weak-strong uniqueness.

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