Dirichlet problem in billiard spaces

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Abstract

An elementary observation is that the dynamical system of a billiard type with a uniform motion can be modeled by the simple impulsive second-order system

\[
\begin{align*}
\ddot{x}(t) &= 0, & \text{for } t \geq 0, \\
\dot{x}(s+) &= \dot{x}(s) + I(x(s), \dot{x}(s)), & \text{if } x(s) \in \partial K,
\end{align*}
\]

where \( K = \overline{\text{int}K} \subset \mathbb{R}^n \) is a compact subset, and \( I \) is an impulse function describing the impact law. If we have a crooked table the acceleration becomes nontrivial and the problem becomes nonlinear and complicated.

We shall discuss the constrained Dirichlet boundary value problem

\[
\begin{align*}
\ddot{x}(t) &= f(t, x(t)), & \text{for } t \in [0, T], x(t) \in \text{int} K, \\
\dot{x}(s+) &= \dot{x}(s) + I(x(s), \dot{x}(s)), & \text{if } x(s) \in \partial K, \\
x(0) &= x(T) = 0,
\end{align*}
\]

where \( I \) describes the impact law of the equality of the angle of incidence and angle of reflection and the equality of a length of the velocity vector before and after the impact. The results presented in the talk develop the research on impulsive Dirichlet problems with state-dependent impulses. The main part will be devoted to both the existence and multiplicity results in one dimensional billiards \((K = [-r, r])\). Several observations concerning the multidimensional case and important open problems will also be given.