Dirichlet problem in billiard spaces

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Abstract

An elementary observation is that the dynamical system of a billiard type with a uniform motion can be modeled by the simple impulsive second-order system

$$\begin{cases} \ddot{x}(t) = 0, & \text{for } t \geq 0, \\ \dot{x}(s+) = \dot{x}(s) + I(x(s), \dot{x}(s)), & \text{if } x(s) \in \partial K, \end{cases} \tag{1}$$

where $K = \overline{intK} \subset \mathbb{R}^n$ is a compact subset, and I is an impulse function describing the impact law. If we have a crocked table the acceleration becomes nontrivial and the problem becomes nonlinear and complicated.

We shall discuss the constrained Dirichlet boundary value problem

$$\begin{cases} \ddot{x}(t) = f(t, x(t)), & \text{for } t \in [0, T], x(t) \in int\, K, \\ \dot{x}(s+) = \dot{x}(s) + I(x(s), \dot{x}(s)), & \text{if } x(s) \in \partial K, \\ x(0) = x(T) = 0, \end{cases} \tag{2}$$

where I describes the impact law of the equality of the angle of incidence and angle of reflection and the equality of a length of the velocity vector before and after the impact. The results presented in the talk develop the research on impulsive Dirichlet problems with state-dependent impulses. The main part will be devoted to both the existence and multiplicity results in one dimensional billiards ($K = [-r, r]$). Several observations concerning the multidimensional case and important open problems will also be given.

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