
The singular Nirenberg problem

Francesca De Marchis*¹ and Rafael Lopez Soriano²

¹Università degli Studi di Roma Sapienza – Italy

²Universidad de Granada – Spain

Abstract

I will consider the problem of prescribing the Gaussian curvature (under pointwise conformal change of the metric) on surfaces with conical singularities. This question has been first raised by Troyanov and it is a generalization of the Kazdan-Warner problem for regular surfaces, known as the Nirenberg problem on the sphere.

Answer this question amounts to solve the following differential problem on a surface Σ

$$-\Delta_g u + 2K_g = 2Ke^u - 4\pi \sum_{j=1}^m \alpha_j \delta_{p_j}, \quad (1)$$

where K_g is the Gaussian curvature of the background metric g , K is the curvature we want to prescribe, p_j are the singular points and α_j the orders of the singularities of the new metric we are looking for: $\tilde{g} = e^u g$, verifying $K_{\tilde{g}} = K$.

This equation has been studied first in the case $K > 0$.

I will present some new results (obtained in collaboration with R. López-Soriano) in the case K sign-changing.

When $\Sigma = S^2$, under some mild conditions on the nodal set of K we derived some sufficient conditions on K and on the singularities for the existence of solutions of (1). Even if we do not expect these conditions to be necessary, I will explain why they are somehow sharp.

*Speaker