Continuation and codimension-two bifurcations for a delay-differential equation with two state-dependent delays

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Abstract

We consider the model equation

$$\varepsilon\dot{u}(t) = -\gamma u(t) - \kappa_1 u(t - a_1 - cu(t)) - \kappa_2 u(t - a_2 - cu(t)),$$

with two delays which has no nonlinearity except for state-dependency of the delays, and for which the delays are merely linearly state-dependent. We investigate the bifurcations that arise in both the singularly perturbed $\varepsilon \to 0$ and regular ($\varepsilon = 1$) cases. In the singular limit an algebraic construction allows us to construct candidate large amplitude solutions and find fold and cusp-like bifurcations. Numerically computed bifurcation diagrams using DDE-Biftool confirm the location of these bifurcations and that the bifurcation structures found also persist for $\varepsilon > 0$, as well as allowing us to locate other bifurcations including period-doubling and Hopf bifurcations.

Double-Hopf bifurcations give rise to invariant tori, including at least one stable torus, when $\varepsilon = 1$, that we investigate in some detail. We propose a method for computing the normal form of the double Hopf bifurcation that gives rise to the torus, and show that the resulting normal form corresponds to the bifurcation curves computed numerically. We believe this to be the first normal form computation for a double-Hopf bifurcation in a DDE with state-dependency in the nonlinearity. We also consider the eventual break-up of the torus.

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