
The Clausius-Mossotti formulas and beyond

Mitia Duerinckx^{*1}

¹Universite Libre de Bruxelles – Belgium

Abstract

We study the behavior of the homogenized coefficients associated with some random stationary ergodic medium under a Bernoulli perturbation. More precisely, a stationary family of possible inclusions is considered, and each of them is chosen independently according to a Bernoulli process, thus yielding perturbed inclusions in a reference stationary medium. Introducing a notion of derivative with respect to the perturbation of the medium at each possible inclusion – very reminiscent of the vertical derivatives used in stochastic homogenization, or of the randomized derivatives used e.g. in the context of Stein’s method –, we prove for the perturbed homogenized coefficients a proxy for a Taylor expansion in terms of these derivatives. This justifies the so-called cluster expansions formally used by physicists, and proves the analyticity of the homogenized coefficients with respect to the Bernoulli parameter. Our approach holds under the minimal assumptions of stationarity and ergodicity, both in the scalar and vector cases. In particular, the first-order term yields the celebrated (electric and elastic) Clausius-Mossotti formulas for isotropic spherical random inclusions in an isotropic reference medium. This work constitutes the first rigorous proof of these formulas in the case of random inclusions.

^{*}Speaker