The Nonlinear Schrödinger equation on a tadpole graph

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Abstract

A tadpole graph is the structure given by a ring with a halfline attached at a point (vertex). A Nonlinear Schrödinger dynamics (NLS) is given on this carrier structure by imposing a boundary condition at the vertex which makes the dynamics hamiltonian. Here it is considered the so called Kirchhoff boundary condition, the most common in applications. The linear analogue is the simplest topologically nontrivial quantum graph and the nonlinear model is an example of interaction between a confined and an unbounded NLS dynamics. The NLS on a tadpole graph has a surprisingly rich family of standing waves, and comprises a) branches of standing waves bifurcating from embedded linear eigenvalues; b) edge solitons, i.e. branches of standing waves bifurcating from the threshold of the essential spectrum c) family of standing waves without linear analogue undergoing saddle-node bifurcation. In this talk the above classification is described and the linearized and orbital stability of standing waves bifurcating from the essential spectrum is studied.