Lyapunov-Razumikhin techniques for state-dependent DDEs

Tony Humphries*[1] and Felicia Magpantay[2,3]

[1]McGill University – Montréal QC, Canada
[2]University of Michigan – United States
[3]University of Manitoba – Canada

Abstract

We present theorems for Lyapunov and asymptotic stability of steady states for general state-dependent delay differential equations (DDEs) using Lyapunov-Razumikhin methods. The Lyapunov stability result applies to nonautonomous DDEs with multiple discrete state-dependent delays of the form

$$\begin{aligned} \dot{u}(t) &= f\big(t, u(t), u(t-\tau_1(t,u(t))), \ldots, u(t-\tau_N(t,u(t)))\big), \quad & t \geqslant t_0, \\ u(t) &= \varphi(t), & t \leqslant t_0, \end{aligned}$$

and is proved by a contradiction argument adapted from a previous result of Barnea for retarded functional differential equations (RFDEs). Our asymptotic stability result applies to autonomous DDEs with multiple state-dependent discrete delays. Its proof is entirely new, and is based on a contradiction argument together with the Arzelà-Ascoli theorem. This alleviates the need for an auxiliary function to ensure the asymptotic contraction. We apply our results to the state-dependent model equation

$$\begin{aligned} \dot{u}(t) &= \mu u(t) + \sigma u(t-a-cu(t)), \quad & t \geq 0, \\ u(t) &= \varphi(t), & t \leqslant 0, \end{aligned}$$

to directly establish asymptotic stability in parts of the stability domain along with lower bounds for the basin of attraction. We also generalise our techniques to derive a condition for global asymptotic stability of the zero solution to the model problem, and also to find bounds on the periodic solutions when the steady-state solution is unstable.

*Speaker