On attractors of a differential system arising in the network control theory

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Abstract

We study the system

$$x_1' = \frac{1}{1 + e^{-\mu(x_2 - \Theta)}} - x_1, \quad x_2' = \frac{1}{1 + e^{-\mu(x_1 - \Theta)}} - x_2 \tag{1}$$

which appears in description of a gene regulatory network (Y. Koizumi et al. Application of attractor selection to adaptive virtual network topology control, in Proc. of BIONETICS, pp. 1 – 8, Nov. 2008) . We provide a number of results concerning the structure of attracting sets for a system (1). A sample of the results follows. **Theorem.** The system has exactly one attracting critical point (stable node) for $\mu \in (0, 4)$, $\Theta \in R^+ := (0, +\infty)$. For $\mu = 4$ and for all positive Θ, except the special value $\Theta = 0.5$, there is exactly one critical point of the same type. For $\mu > 4$ there are two values of Θ, $\Theta_1 < \Theta_2$, with the property: if $\Theta \in exterior[\Theta_1, \Theta_2] \bigcap R^+$, then there is one attracting critical point (stable node); if $\Theta \in (\Theta_1, \Theta_2)$, then there are two attracting critical points (stable nodes) and a saddle point between them; if $\Theta = \Theta_1$ or $\Theta = \Theta_2$ then there are two critical points, namely, a stable node and an attracting degenerate critical point (the matrix of coefficients of the linearized system is degenerate).

Acknowledgement. The support of the ESF project 2013/0024/1DP/1.1.1.2.0/13/APIA/VIAA/045 is kindly acknowledged.

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