Solution dynamics in a class of differential delay equations

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Abstract

We consider a class of neutral differential delay equations of the form

$$\varepsilon\,[x'(t) + ax'(t-1)] + x(t) = f(x(t-1)),\ t \geq 0, \tag{1}$$

where $f : \mathbb{R} \mapsto \mathbb{R}$ is a continuous function, $a \in \mathbb{R}$ is a constant, and $\varepsilon > 0$ is a small parameter. Such equations find various applications in modelling diverse real life phenomena. The neutral type ($a \neq 0$) equation (1) appears as an exact reduction of certain boundary value problems for hyperbolic PDEs. The limiting case $\varepsilon = 0$ in equation (1) results in a continuous type difference equation of the form

$$x(t) = f(x(t-1)),\ t \geq 0, \tag{2}$$

which dynamics are largely determined by relevant properties of the map f as one-dimensional dynamical system. We establish continuous dependence results between solutions of equations (1) and (2) as $\varepsilon \to 0^+$. We address the problem of existence of periodic solutions in equation (1) for several classes of the nonlinearity f, such as the Farey-type nonlinearity and a nonlinearity with the negative feedback. Some of the results of this work, as well as a number of related open problems and conjectures, are supported by numerical simulations.

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