The Ginzburg-Landau Justification in the Quasilinear Case

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Abstract

We are interested in the behaviour of pattern forming systems close to the first instability. By multiscale analysis we find that in lowest order the slow time and space modulations of propagating patterns have to satisfy a Ginzburg-Landau equation.
This derivation of the Ginzburg-Landau equation is only formal in the sense that it is unclear that the so constructed approximate solutions make correct predictions about the full problem.
To make these calculation rigorous, we have to show that the error, i.e., the difference between the formal approximation and a true solution stays small over the relevant time scale.
In the semilinear case, the long-time estimate for the error is done with the help of the variation of constants formula and a simple application of Gronwall's inequality.
In the quasilinear case, however, this method fails, since a direct estimate is not possible due to an appearing non-integrable singularity.
We demonstrate how to overcome this difficulty by combining ideas from maximal regularity with the classical approach.

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