About attractors for 3D Bingham model

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Abstract

The motion of an incompressible fluid of a constant density is described by the Cauchy momentum equation

\[
\frac{\partial \mathbf{v}}{\partial t} + \sum_{i=1}^{n} \mathbf{v}_i \frac{\partial \mathbf{v}}{\partial x_i} + \nabla p = \text{Div} \sigma + f, \quad \text{Div} \mathbf{v} = 0.
\] (1)

The system of equations describing the motion of the Bingham fluid is obtained by adding to the general equation (1) the rheological relation between the deviator of the stress tensor \(\sigma\) and the strain rate tensor \(\varepsilon(\mathbf{v})\) = \(\frac{1}{2}(\nabla \mathbf{v} + \nabla^T \mathbf{v})\)

\[
\sigma = 2\mu \varepsilon(\mathbf{v}) + \tau^* \frac{\varepsilon(\mathbf{v})}{|\varepsilon(\mathbf{v})|} \text{ for } |\varepsilon(\mathbf{v})| \neq 0 \text{ and } |\sigma| \leq \tau^* \text{ for } |\varepsilon(\mathbf{v})| = 0,
\] (2)

where \(\mu, \tau^* = \text{const} > 0\).

The existence of solutions of this model has been studied in details. However, the behavior of solutions for this model at infinity (attractors) have been studied only in two dimensions. In the report we present the theorem of an attractor existence for the Bingham model in three dimension. We note that in its proof the theory of trajectory attractors was used, since it is impossible to use the theory of dynamical systems due to the fact that the uniqueness of weak solution in the three-dimensional case for this system is unknown.