On the appearance of gaps in the spectrum of a Dirichlet problem in a double-periodic perforated plane

Francesco Ferraresso*[1]

[1]Dipartimento di Matematica Pura e Applicata [Padova] – Via Trieste, 63 35121 Padova, Italy

Abstract

We consider a problem of surface wave propagation around a double-periodic grid of cylindrical obstacles whose section have the shape

$$\{|x_1|^\mu + |x_2|^\mu \leq r\},$$

where $\mu \in]1, \infty[$. Applying separation of variables and Floquet-Bloch theory leads us to consider an eigenvalue problem for the Laplace operator in a periodicity cell with Dirichlet boundary conditions on a large part of the boundary and some compatibility conditions on the edges of the cell.

Our main theorem states that for r sufficiently close to $1/2$ the whole of the spectrum of the operator associated with the problem is essential and assumes a "band-gap" structure. More precisely, each eigenvalue of the limit problem corresponds to a spectral interval (a band) in the perturbed problem, and these bands can be made disjoint for r close to $1/2$. As a byproduct, we prove that the eigenfunctions of the limit problem are smooth up to the boundary, with exponential decay at the cusp vertexes. Our results generalize those obtained in [1] for circular holes. Joint work with Jari Taskinen, see [2].
[1] S.Nazarov, K.Ruotsalainen, J.Taskinen : "Spectral gaps in the Dirichlet and Neumann problems on the plane perforated by a double-periodic family of circular holes", J. Math. Sci. (N. Y.), 181(2), 2012.

[2] F.Ferraresso, J.Taskinen : "Singular perturbation Dirichlet problem in a double-periodic perforated plane", to appear in Annali dell'università di Ferrara.

*Speaker