Asymptotic stability of constant solutions in delay differential equations with a constant delay and transcendental equations with complex coefficients

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Abstract

In this talk, we investigate the asymptotic stability of a constant solution for a delay differential equation (DDE) $x'(t) = f(x(t), x(t-\tau))$, where $f\colon \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a smooth function, and τ is a positive constant. This is an equation with a single constant delay.
In general, the asymptotic stability of a constant solution of DDEs is determined by the location of the roots of the characteristic equation of the linearized equation. For the above DDE, the characteristic equation becomes $\det(\lambda I + A - \mathrm{e}^{-\lambda\tau} B) = 0$, where A and B are $n \times n$ real matrices. If A and B are simultaneously triangularizable, then this equation is reduced to a transcendental equation $z + a - b\mathrm{e}^{-z} = 0$, where parameters a and b are generally complex.
We see that by using the "graph-like" expression of the Lambert W function in some coordinate system of the complex plane $\mathbb{C}$, a necessary and sufficient condition on a and b for which all the roots of the above transcendental equation have negative real parts can be obtained. Here the Lambert W function is the multi-valued inverse of a complex function $z \mapsto z\mathrm{e}^z$, and the set of the roots is equal to $W(b\mathrm{e}^a) - a$. We also give an application of this result to the stabilization problem of unstable constant solutions by the delayed feedback control proposed by Pyragas.

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