Oscillatory orbits in the restricted elliptic planar three body problem

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Abstract

The restricted planar elliptic three body problem models the motion of a massless body under the Newtonian gravitational force of two bodies evolving in Keplerian ellipses.The possible motions the massless body can perform were already known by Chazy (1922), who gave a complete classification of all possible states that the body $q(t)$ can approach as time tends to infinity. The possible final states are reduced to four:

- Hyperbolic: $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to c > 0$ as $t \to \pm\infty$.
- Parabolic: $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to 0$ as $t \to \pm\infty$.
- Bounded: $\limsup_{t\to\pm\infty} \|q\| < +\infty$.
- Oscillatory: $\limsup_{t\to\pm\infty} \|q\| = +\infty$ and $\liminf_{t\to\pm\infty} \|q\| < +\infty$

Examples of all these types of motion, except the oscillatory ones, were already known by Chazy. In this talk, we prove the existence of oscillatory motions for any value of the masses of the primaries, assuming they move in ellipses whose exectricity is small enough. The key idea is to look at the restricted elliptic three body problem as a small perturbation of the circular case. Using a previous result about the transversal intersection of the stable and unstable manifolds of infinity in the circular case, the elliptic one can be seen as “a priory chaotic”. Studing the corresponding scattering map with classical perturbative arguments, one can show the existence of transition chains of periodic orbits with bounded angular momentum, and a suitable shadowing lemma provides the existence of oscillatory motions.

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