Mean-field limit and fluctuations for interacting oscillators with singular spatial interaction

Eric Lucon∗

Mathématiques appliquées Paris 5 (MAP5) – CNRS : UMR8145, Université Paris V - Paris Descartes – UFR de Maths et informatique 45 rue des Saints Pères 75270 PARIS CEDEX 06, France

Abstract

There has been recently a growing interest in generalizations of mean-field models (e.g. Kuramoto or FitzHugh-Nagumo oscillators) to the case where interactions between particles are no longer uniform or do not follow the complete graph. We analyze here the large population behavior of mean-field interacting diffusions \((\theta_i(t))_{1\leq i \leq N}\) with spatial geometry: each oscillator \(\theta_i\) is at a fixed position \(x_i\) on a regular lattice and the interaction between particles \(i\) and \(j\) depends on a spatial kernel \(\Psi(x_i, x_j)\). Interesting examples include \(P\)-nearest-neighbor interactions or interactions with polynomial decay.

We first address the convergence as \(N \to \infty\) of the empirical measure of the particles to a deterministic process of McKean-Vlasov type. We then discuss the finite-size fluctuations of the system around its limit, with a special consideration given to the influence of the geometry on the fluctuations. In the case of polynomial decay, the system exhibits a phase transition: for a strong interaction, the fluctuations are Gaussian, solutions to a linear SPDE as \(N \to \infty\) whereas for weak interaction the fluctuations are deterministic. The main difficulty of the analysis lies in the singularity of the kernel \(\Psi\) and requires the introduction of an auxiliary process capturing the dependence of the system in the space variable.

This is joint work with W. Stannat (TU Berlin).

∗Speaker