Mean-field limit and fluctuations for interacting oscillators with singular spatial interaction

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Abstract

There has been recently a growing interest in generalizations of mean-field models (e.g. Kuramoto or FitzHugh-Nagumo oscillators) to the case where interactions between particles are no longer uniform or do not follow the complete graph. We analyze here the large population behavior of mean-field interacting diffusions $(\theta_i(t))_{1\leq i\leq N}$ with spatial geometry: each oscillator θ_i is at a fixed position x_i on a regular lattice and the interaction between particles i and j depends on a spatial kernel $\Psi(x_i, x_j)$. Interesting examples include P-nearest-neighbor interactions or interactions with polynomial decay.

We first address the convergence as $N \to \infty$ of the empirical measure of the particles to a deterministic process of McKean-Vlasov type. We then discuss the finite-size fluctuations of the system around its limit, with a special consideration given to the influence of the geometry on the fluctuations. In the case of polynomial decay, the system exhibits a phase transition: for a strong interaction, the fluctuations are Gaussian, solutions to a linear SPDE as $N \to \infty$ whereas for weak interaction the fluctuations are deterministic. The main difficulty of the analysis lies in the singularity of the kernel Ψ and requires the introduction of an auxiliary process capturing the dependence of the system in the space variable.
This is joint work with W. Stannat (TU Berlin).

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