On the eigenvalues of Steklov-type problems

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Abstract

We consider the Steklov eigenvalue problem for the Laplace operator and its natural fourth order generalization on bounded domains Ω of class C^1 in $\mathbb{R}^N$ which reads

$$\begin{cases} \Delta^2 u - \tau \Delta u = 0, & \text{in } \Omega, \\ \frac{\partial^2 u}{\partial \nu^2} = 0, & \text{on } \partial\Omega, \\ \tau \frac{\partial u}{\partial \nu} - \mathrm{div}_{\partial\Omega}\big(D^2 u.\nu\big) - \frac{\partial(\Delta u)}{\partial \nu} = \lambda u, & \text{on } \partial\Omega. \end{cases}$$

The Steklov problem for the Laplacian is classical (Steklov, 1902). This problem arises in the study of a free vibrating membrane with mass concentrated at the boundary. As for the biharmonic operator, we show that the boundary conditions that we introduce are naturally obtained through a physical model involving the vibration of a free plate with mass concentrating at the boundary. We do the same analysis also for the second order problem. We study the asymptotic behavior of the eigenvalues of the two problems in this mass concentration phenomenon.
Finally, we prove that an analogue of the Brock-Weinstock inequality for the Steklov Laplacian holds for our biharmonic Steklov problem, i.e., the ball maximizes the first non-trivial eigenvalue among all domains of class C^1 in $\mathbb{R}^N$ of given measure.

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