Existence and symmetry of least energy nodal solutions for Hamiltonian elliptic systems

Hugo Tavares*[1], Miguel Ramos[2], Ederson Dos Santos[3], and Denis Bonheure[4]

[1]Instituto Superior Tecnico [Lisboa] (IST) – Av. Rovisco Pais, 1 1049-001 Lisboa, Portugal
[2]Faculdade de Ciências [Lisboa] – Universidade de Lisboa, Campo Grande, Edificio CS, 7749-076 Lisboa, Portugal
[3]Universidade de São Paulo - São Carlos (USP) – Universidade de São Paulo - Campus de São Carlos Av. Trabalhador São-carlense, 400, Arnold Schimidt São Carlos - São Paulo CEP 13566-590, Brazil
[4]Département de Mathématique [Bruxelles] (ULB) – Avenue Franklin Roosevelt 50 - 1050 Bruxelles, Belgium

Abstract

In this talk we will discuss recent existence results of least energy nodal solutions for the Hamiltonian elliptic system with Hénon–type weights

$$-\Delta u = |x|^{\beta}|v|^{q-1}v, \quad -\Delta v = |x|^{\alpha}|u|^{p-1}u \quad \text{in } \Omega, \qquad u = v = 0 \text{ on } \partial\Omega,$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$, $N \geq 1$, $\alpha, \beta \geq 0$ and the nonlinearities are superlinear and subcritical, namely

$$1 > \frac{1}{p+1} + \frac{1}{q+1} > \frac{N-2}{N}.$$

For the proof, we use a dual method applied to a perturbed system. Moreover, when Ω is either a ball or an annulus centred at the origin and $N \geq 2$, we show that these solutions display foliated Schwarz symmetry. It is natural to conjecture that these solutions are not radially symmetric. We provide such a symmetry breaking in a range of parameters where the solutions of the system behave like the solutions of a single equation.

*Speaker