Asymptotic behavior of positive solutions of a kind of Lanchester-type system

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Abstract

Let us consider the system

$$\begin{cases} x' = -a(t)xy, \\ y' = -b(t)xy, \end{cases} \tag{1}$$

where $a, b \in C[0, \infty)$ are positive functions satisfying

$$0 < \inf_{t \geq 0} a(t) \leq \sup_{t \geq 0} a(t) < \infty, \quad 0 < \inf_{t \geq 0} b(t) \leq \sup_{t \geq 0} b(t) < \infty. \tag{2}$$

If the initial values $x(0), y(0)$ are positive, then the solution $(x(t), y(t))$ of (1) exists on $[0, \infty)$, and remains positive there. So $\lim_{t\to\infty} x(t)$ and $\lim_{t\to\infty} y(t)$ exist as nonnegative numbers.

Denote the solution of (1) with the initial value $(x(0),\ y(0)) = (\alpha,\ \beta)$, $\alpha, \beta > 0$, by $(x(t; \alpha, \beta),\ y(t; \alpha, \beta))$. When we fix α and move $\beta > 0$, we want to examine how $\lim_{t\to\infty}(x(t; \alpha, \beta),\ y(t; \alpha, \beta))$ varies according to β.

Theorem 1. *There are constants* $\beta_1 = \beta_1(\alpha)$ *and* $\beta_2 = \beta_2(\alpha)$ $(0 < \beta_1 \leq \beta_2)$ *such that:*

(i) *if* $\beta < \beta_1$, *then* $\lim_{t\to\infty} x(t; \alpha, \beta) > 0$, $\lim_{t\to\infty} y(t; \alpha, \beta) = 0$;

(ii) *if* $\beta_1 \leq \beta \leq \beta_2$, *then* $\lim_{t\to\infty} x(t; \alpha, \beta) = \lim_{t\to\infty} y(t; \alpha, \beta) = 0$;
(iii) *if* $\beta > \beta_2$, *then* $\lim_{t\to\infty} x(t; \alpha, \beta) = 0$, $\lim_{t\to\infty} y(t; \alpha, \beta) > 0$.

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