Anisotropic interactions in a first-order aggregation model

Razvan Fetecau*[1], Joep Evers[2], and Lenya Ryzhik[3]

[1]Simon Fraser University – Canada
[2]Eindhoven University of Technology – Netherlands
[3]Stanford University – United States

Abstract

We consider the following anisotropic extension of a well-studied first-order ODE model for collective behaviour:

$$\frac{dx_i}{dt} = v_i,$$

$$v_i = -\frac{1}{N}\sum_{j\neq i} \nabla_{x_i} K(|x_i - x_j|) w_{ij},$$

where x_i $(i = 1, \ldots, N)$ denote the positions of N particles (individuals) in $\mathbb{R}^d$, K is an aggregation potential which incorporates inter-individual social interactions, and w_{ij} represent weights that model limitations in sensorial perception (e.g., limited field of vision). We assume that the weights w_{ij} depend on the direction of motion of individuals (i.e., on $v_i/|v_i|$) and consequently, the equations that determine the velocities v_i become *implicit*. This fact brings major new analytical issues, such as non-uniqueness and jump discontinuities in velocities. We study the well-posedness of the anisotropic model and discuss its modes of breakdown. To extend solutions beyond breakdown we propose a relaxation system containing a small parameter ε, which can be interpreted as a small amount of inertia or response time. We show that the limit $\varepsilon \to 0$ can be used as a jump criterion to select the physically correct velocities. We illustrate the results with numerical simulations in two dimensions.

*Speaker