Modelling fungal hyphae growth: searching for travelling waves in an extension of the thin viscous sheet equations.

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Abstract

The cell growth of fungal hyphae typically proceeds as an elongating expansion in a single direction. It grows at a constant speed preserving the overall tube-like form. Hence, if we assume that it never stops growing then mathematically the growth can be described by a travelling wave. The existence of travelling wave solutions for certain hyphae growth models has already been proven. However, these models do not take into account the dynamics in the cell wall and consequently have to prescribe explicitly how the wall grows as new material arrives. Our new model overcomes this by modelling the cell wall as a thin viscous sheet where ageing of the cell wall results in hardening of the wall by making the wall more viscous over time. Existence of the desired travelling wave corresponds to finding an increasing concave bounded solution in a highly non-linear 5-dimensional first order ODE which behaves singular close to the boundaries of the desired solution. At the boundaries our local existence and uniqueness proofs yield solutions with the desired boundary behaviour. Furthermore, our numerical work suggests that globally the desired solutions exist.