Oscillatory Integrals and Fractal Dimension

Domagoj Vlah*†[1], Jean-Philippe Rolin[2], and Vesna županović[1]

[1]University of Zagreb, Faculty of Electrical Engineering and Computing, Department of Applied Mathematics – Croatia

[2]Université de Bourgogne, Faculté des Sciences Mirande – Laboratoire I.M.B., Laboratoire IMB – France

Abstract

Theory of singularities has been closely related with the study of oscillatory integrals. More precisely, the study of critical points is closely related to the study of asymptotic of oscillatory integrals.

In our work we investigate the fractal properties of a geometrical representation of oscillatory integrals. We are motivated by a geometrical representation of Fresnel integrals by a spiral called the clothoid, and the idea to produce a classification of singularities using fractal dimension. Fresnel integrals are a well known class of oscillatory integrals. We consider oscillatory integral

$$I(\tau) = \int_{\mathbb{R}^n} e^{i\tau f(x)}\phi(x)dx,$$

for large values of real parameter τ, where f is the analytic phase and ϕ is the smooth amplitude with a compact support. We measure the oscillatority by Minkowski dimension of the planar curve parameterized by functions X and Y that are the real and imaginary parts of the integral I, respectively. Also, the oscillatory dimension is defined as Minkowski dimension of the graph of function $x(t) = X(1/t)$, near $t = 0$, and analogously for Y. We provide explicit formulas connecting these Minkowski dimensions and associated Minkowski contents with asymptotics of the integral I and the type of the critical point of the phase f. Techniques used include Newton diagrams and the resolution of singularities.

*Speaker

†Corresponding author: domagoj.vlah@fer.hr