Krasnosel'skii Formula for constrained semilinear differential inclusion

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Abstract

The well-known Krasnosel'skii formula concerns an ODE $\dot{x} = f(t, x)$, $x \in \mathbb{R}^N$, $t \in [0, 1]$. Roughly speaking, it asserts that the Brouwer degrees of $-f(0, \cdot)$ and $\mathrm{Id} - P_t$ are equal, where P_t is the associated Poincaré t-operator. We consider a constrained semilinear evolution inclusions of parabolic type

$$\begin{cases} \dot{u}(t) \in Au(t) + F(t, u(t)), & t \in [0, 1], u \in \mathcal{K}, \\ u(0) = x \in \mathcal{K}, \end{cases} \tag{1}$$

in the infinite dimension Banach/Hilbert space and topological properties of the solution map. The set of constraints $\mathcal{K}$ is assumed to be closed and convex. A counterpart of Krasnosel'skii formula concerning (1), namely a relation between the constrained fixed point index of the Krasnosel'skii–Poincaré operator of translation along trajectories associated with (1) and the constrained topological degree of the right-hand side $A + F(0, \cdot)$ will be presented. The connection joining the problem (1) and partial differential inclusion on the open bounded subset of $\mathbb{R}^N$ will be discussed.

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