A family of fourth-order porous medium equations

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Abstract

We analyze a family of nonlinear fourth-order diffusion equations, which includes the well known thin-film and the Derrida-Lebowitz-Speer-Spohn (DLSS) equation on the one-dimensional torus, i.e. subject to periodic boundary conditions, and prove the existence of global nonnegative weak solutions to these equations. Contrary to the gradient flow approach proposed by Matthes, McCann and Savaré (2009), our method relies on dissipation property of the corresponding entropy functionals called Tsallis entropies, which provide required a priori estimates. Moreover, our method extends the existence result from Matthes, McCann and Savaré (2009) to a wider range of the family members. Applying generalized Beckner-type functional inequalities yield an exponential decay rate of (relative) Tsallis entropies, which in further, using Csiszár-Kullback-Pinsker inequalities implies the exponential stability in the $L^1$-norm of the constant steady state.

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