
Cahn-Hilliard approach to Stefan problem with dynamic boundary condition and related topics

Takeshi Fukao*¹

¹Kyoto Univeristy of Education (KUE) – 1 Fujinomori-cho, Fukakusa, Fushimi-ku, Kyoto 612-8522, Japan

Abstract

In this presentation, the well-posedness of the Stefan problem with dynamic boundary condition is studied. The dynamic boundary condition is a sort of differential equation which represents the dynamics on the boundary. As the condition for solving the partial differential equation, the time derivative is included and it is well treated like, Dirichlet, Neumann and Robin type conditions with various problems. About the dynamic boundary condition, the existence and uniqueness of the Stefan problem was studied by Aiki from the subdifferential approach. More general setting, we can find the results in Andreu, Mazón, Toledo and Igbida. Arranging the condition from the previous two results, the well-posedness is studied. In order to show the existence of the weak solution, the original problem is approximated by a limit of some equation and dynamic boundary condition of the Cahn-Hilliard type, it was introduced by Goldstein-Miranville-Schimperna in 2011 and studied by many from the various view point. By this Cahn-Hilliard approach, it becomes clear that the state of the mushy region of the Stefan problem can be characterized mathematically by an asymptotic limit of the fourth order system which has a double-well structure. This fact also gives the possibility that we can apply the result for the various model of degenerate parabolic equations with dynamic boundary condition, porous medium equations, Hele-Shaw cell etc, the Stefan problem is one of them.

*Speaker