Solutions for nonlinear systems on unbounded domains with p-Laplacian-like operators

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Abstract

We study the existence of at least one solution to the following systems of resonant boundary value problems

\[(\varphi(x'))' = f(t, x, x'), \quad x'(0) = 0, \quad x'(\infty) = 0,\]

where \(f : \mathbb{R}_+ \times \mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}^k\) is continuous,

\[
\varphi(s) = (\varphi_1(s_1), \ldots, \varphi_k(s_k)),
\]

\(s \in \mathbb{R}^k\), and \(\varphi_i : \mathbb{R} \to \mathbb{R}\) is an increasing homeomorphism such that \(\varphi_i(0) = 0, i = 1, \ldots, n\).

We give conditions for the existence of a solution for this BVP using the generalization of the Miranda Theorem [1]:

Let \(M_i > 0, i = 1, \ldots, k\), and \(F\) be an admissible map from \(\prod_{i=1}^k [-M_i, M_i]\) to \(\mathbb{R}^k\), i.e. there exist a Banach space \(E\), \(\dim E \geq k\), a linear, bounded and surjective map \(\varphi : E \to \mathbb{R}^k\) and an \(R_\delta\)-map \(\Phi\) from \(\prod_{i=1}^k [-M_i, M_i]\) to \(E\) such that \(F = \varphi \circ \Phi\).

If for any \(i = 1, \ldots, k\) and every \(y \in F(x)\), where \(|x_i| = M_i\), we have

\[x_i \cdot y_i \geq 0,\]

then there exists \(x\) such that \(0 \in F(x)\).


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