Solutions for nonlinear systems on unbounded domains with p-Laplacian-like operators

Katarzyna Szymanska-Debowska*[1]

[1]Institute of Mathematics, Lodz University of Technology – 90-924 Łódź, ul. Wólczańska 215, Poland

Abstract

We study the existence of at least one solution to the following systems of resonant boundary value problems

$$(\varphi(x'))' = f(t, x, x'), \quad x'(0) = 0, \quad x'(\infty) = 0,$$

where $f : \mathbb{R}_+ \times \mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}^k$ is continuous,

$$\varphi(s) = (\varphi_1(s_1), \ldots, \varphi_k(s_k)),$$

$s \in \mathbb{R}^k$, and $\varphi_i : \mathbb{R} \to \mathbb{R}$ is an increasing homeomorphism such that $\varphi_i(0) = 0$, $i = 1, \ldots, n$.

We give conditions for the existence of a solution for
this BVP using the generalization of the Miranda Theorem [1]:

Let $M_i > 0$, $i = 1, \ldots, k$, and F be an admissible map from $\prod_{i=1}^k [-M_i, M_i]$ to $\mathbb{R}^k$, i.e. there exist a Banach space E, $\dim E \geq k$, a linear, bounded and surjective map $\varphi : E \to \mathbb{R}^k$ and an R_δ-map Φ from $\prod_{i=1}^k [-M_i, M_i]$ to E such that
$F = \varphi \circ \Phi$.
If for any $i = 1, \ldots, k$ and every $y \in F(x)$, where $|x_i| = M_i$, we have

$$x_i \cdot y_i \geq 0,$$

then there exists x such that $0 \in F(x)$.

[1] K. Szymańska-Dębowska, On a generalization of the Miranda Theorem and its application to boundary value problems, J. Differential Equations, 258 (2015), 2686-2700.

*Speaker