Validated Function Calculus for the Rigorous Solution of Differential Equations

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Abstract

A *validated function calculus* is a numerical method for manipulating functions as first-class objects in a rigorous way. It provides a powerful extension of interval methods (for rigorous computation with numbers) allowing computed solutions of the *flow* $\phi(x_0, t)$ of a differential equation to be used in further analysis, such as reachability tests or computation of periodic orbits. A validated function calculus based on *Taylor models* is used in Berz & Makino's tool COSY Infinity (1998), and an abstract, extensible function calculus is implemented in the tool Ariadne (C. et al, 2005–) for rigorous numerics and verification of hybrid systems.

In this talk I will briefly explain the basic ideas of validated function calculus, giving examples of function calculi based on polynomials and Fourier series, and demonstrate their use in the study of ordinary differential equations, including approaches based on the Picard operator and Taylor series. I will demonstrate the solution of delay differential equations, and composite systems via modular decomposition, and outline a method for the analysis of reduced-order systems, which requires the solution of differential inclusions (Zivanovic & C., 2010). Finally, I will describe challenges for further research in the rigorous solution of differential equations, including stiff systems with multiple time-scales, and the simplification problem for enclosure sets.

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