

---

# Global existence and convergence of smooth solutions to Yang-Mills gradient flow over compact four-manifolds

Paul Feehan<sup>\*1</sup>

<sup>1</sup>Department of Mathematics - Rutgers School of Arts and Sciences – Hill Center - Busch Campus 110  
Frelinghuysen Road Piscataway, NJ 08854-8019, United States

## Abstract

We develop new results on global existence and convergence of solutions to the gradient flow equation for the Yang-Mills energy functional on a principal bundle, with compact Lie structure group, over a closed, four-dimensional, Riemannian, smooth manifold, including the following. If the initial connection is close enough to a local minimum of the Yang-Mills energy functional, in a norm or energy sense, then the Yang-Mills gradient flow exists for all time and converges to a Yang-Mills connection. If the initial connection is allowed to have arbitrary energy but we restrict to the setting of a Hermitian vector bundle over a compact, complex, Hermitian (but not necessarily Kähler) surface and the initial connection has curvature of type  $(1,1)$ , then the Yang-Mills gradient flow exists for all time, though bubble singularities may (and in certain cases must) occur in the limit as time tends to infinity. The Lojasiewicz-Simon gradient inequality plays a crucial role in our approach and we develop two versions of that inequality for the Yang-Mills energy functional.

---

<sup>\*</sup>Speaker