Symmetry results for a family of Caffarelli-Kohn-Nirenberg inequalities

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Abstract

We investigate optimal functions for the following inequalities:

$$\left(\int_{\mathbb{R}^d} |w(x)|^{2p}\,|x|^{-\gamma}\mathrm{d}x\right)^{\frac{1}{2p}} \leq C_\gamma \left(\int_{\mathbb{R}^d} |\nabla w(x)|^2\,\mathrm{d}x\right)^{\frac{\vartheta}{2}} \left(\int_{\mathbb{R}^d} |w(x)|^{p+1}\,|x|^{-\gamma}\mathrm{d}x\right)^{\frac{1-\vartheta}{p+1}},$$

where $d \geq 3$, $\gamma \in (0,2)$, $p \in (1,(d-\gamma)/(d-2))$, $\vartheta = \vartheta(d,\gamma,p) \in (0,1)$ and $C_\gamma > 0$. For $\gamma = 0$ they coincide with $b(x) := (1+|x|^2)^{-\frac{1}{p-1}}$ [Del Pino, Dolbeault, JMPA (2002)]. If $\gamma \in (0,2)$ they continue to exist, and our main concern is their radial symmetry. Indeed, as soon as optimal functions are radial, they coincide with $b_\gamma(x) := (1+|x|^{2-\gamma})^{-\frac{1}{p-1}}$. However, Schwarz symmetrization techniques fail. We then let $\gamma \to 0$ and exploit known results at $\gamma = 0$. By means of a concentration-compactness analysis we prove that optimal functions converge to b. Afterwards we use a perturbation argument by contradiction, which involves angular derivatives of possibly nonradial optimal functions. Such an argument allows us to prove that optimal functions do coincide with b_γ for γ small. The talk is based on [Dolbeault, M., Nazaret, Symmetry in weighted interpolation inequalities, preprint].

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