A higher-order large-scale regularity theory for random elliptic operators

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Abstract

We develop a large-scale regularity theory of higher order for divergence-form elliptic equations with heterogeneous coefficient fields a in the context of stochastic homogenization. Under the assumptions of stationarity and slightly quantified ergodicity of the ensemble, we derive a $C^{2,\alpha}$-"excess decay" estimate on large scales and a $C^{2,\alpha}$-Liouville principle: For a given a-harmonic function u on a ball B_R, we show that its energy distance to the space of a-harmonic "corrected quadratic polynomials" on some ball B_r has the natural decay in the radius r above some minimal (random) radius r_0. Our Liouville principle states that the space of a-harmonic functions growing at most quadratically has (almost surely) the same dimension as in the constant-coefficient case. The existence of a-harmonic "corrected quadratic polynomials" – and therefore our regularity theory – relies on the existence of second-order correctors for the homogenization problem. By an iterative construction, we are able to establish existence of subquadratically growing second-order correctors.

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