Glimpses on Lipschitz truncations and regularity

Bianca Stroffolini[*][†][1]

[1]Dipartimento di Matematica e Applicazioni "Renato Caccioppoli" – Via Cintia, Monte S. Angelo I-80126 Napoli, Italy, Italy

Abstract

It is well known that in the vectorial case solutions of a nonlinear elliptic system are of class $C^{1,\alpha}(\Omega \setminus \Sigma, \mathbb{R}^N)$ where Σ has n-dimensional Lebesgue measure zero.
A recent approach for proving partial regularity is based on the so called $\mathcal{A}$-harmonic approximation method, that is, obtaining a good approximation of functions$u \in W^{1,2}(B, \mathbb{R}^N)$ with the solution h of a suitable linearized system. The solution u is shown to be *almost* $\mathcal{A}$*-harmonic*:

$$|\frac{1}{|B|}\int_B \mathcal{A}\nabla u \cdot \nabla \xi \, dx| \leq \delta\Big(\frac{1}{|\widetilde{B}|}\int_{\widetilde{B}} |\nabla u| \, dx\Big)\|\nabla \xi\|_{L^\infty} \tag{1}$$

for all $\xi \in C_0^\infty(B, \mathbb{R}^N)$.

The idea now is that the good regularity estimates available for h are inherited by u. Originally, the closeness of the function to its $\mathcal{A}$-harmonic approximation was stated in terms of the L^2-distance and , for nonlinear problems, in terms of the L^p-distance. Based on a refinement of the Lipschitz truncation technique, we have shown that also the distance in terms of the gradients is small.

I will present a suitable version of the $\mathcal{A}$-harmonic approximation lemma in the Orlicz setting.
Next, for parabolic systems, I will state a p-caloric approximation Lemma and related partial regularity results.

[*]Speaker
[†]Corresponding author: bstroffo@unina.it