
L^∞ -stability of traveling waves to a hyperbolic-elliptic coupled system

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Abstract

In this talk, we discuss L^∞ -stability of traveling waves or shock waves to a hyperbolic-elliptic coupled system of $u_t + uu_x + q_x = 0$, $-q_{xx} + q + u_x = 0$ over $x \in \mathbb{R}$, where u and q are scalar functions. This is called the Hamer model, derived by simplifying a system of a gas dynamics with radiation. S. Kawashima and S. Nishibata(1998) clarified that the system admits traveling waves uniquely up to shifts if $\delta := u_- - u_+$ is positive, and remarkably, the profile is discontinuous if and only if $\delta > \sqrt{2}$ where $u_\pm := \lim_{x \rightarrow \pm\infty} u(t, x)$.

We first show that subcritical shock waves i.e. the case with $\delta < \sqrt{2}$ are stabilized by radiation, while an arbitrary small perturbation could give rise to a blow up of u_x in a finite time for a critical shock wave ($\delta = \sqrt{2}$). Then we show supercritical shock waves ($\delta > \sqrt{2}$) regain stability thanks to the presence of discontinuity and the contribution of convection. These results could be understood as a rationalization of the difference in continuity of traveling waves from the view point of L^∞ -stability, making a stark contrast to the powerful results by D. Serre(2003): all traveling waves are L^1 -stable to arbitrary large perturbations.

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