Existence and blowing up character of solutions to Volterra type integral equations related to ordinary differential equations with monotonic nonlinearities

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Abstract

We are going to discuss a nonlinear Abel type integral equation of the

$$u(t) = \int_{-\infty}^{t} (t-s)^{\alpha-1} r(s) g(u(s)) ds \quad (-\infty < t, \quad 0 < \alpha),$$

where $r(s)$ and $g(u)$ (with $g(0) = 0$) are nonnegative nondecreasing functions. For an integer α, it is an integral formulation of the initial value problem for the ordinary differential equation

$$u^{(n)}(t) = r(t)g(u(t)), \;\; u(-\infty) = u'(-\infty) = \cdots = u^{(n-1)}(-\infty) = 0.$$

Such equations arise in the investigation of one-dimensional models of a diffusive medium which can experience explosive behavior. It is easily observed that these equations have trivial solution $u(t) \equiv 0$. However, from physical point of view only positive solutions are interesting. We analyze nonnegative nondecreasing solutions $u(t)$, $t > -\infty$ and show when they starts with some $t_0 > -\infty$, i.e. $u(t) = 0$ for $t \leq t_0$ and $u(t) > 0$ for $t > t_0$. We also provide conditions under which the solution $u(t)$ is blowing up.

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