Harmonic perturbations with delay of periodic separated variables differential equations

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Abstract

In this talk we present recent results jointly obtained with Marco Spadini (University of Florence). We investigated the set of harmonic solutions of $T$-periodic perturbations of $T$-periodic separated variables ODEs on manifolds, allowing the perturbing term to contain a finite delay. Namely, given $T > 0$ and a boundaryless smooth manifold $N \subseteq \mathbb{R}^d$, we consider $T$-periodic solutions of equations of the form

$$
\dot{\zeta}(t) = a(t)\Phi(\zeta(t)) + \lambda \Xi(t, \zeta(t), \zeta(t-r)), \quad \lambda \geq 0,
$$

where $r > 0$ is a finite time lag, $a : \mathbb{R} \to \mathbb{R}$ is continuous and $T$-periodic, $\Phi : N \to \mathbb{R}^d$ and $\Xi : \mathbb{R} \times N \times N \to \mathbb{R}^d$ are given continuous tangent vector fields on $N$, in the sense that $\Phi(\xi)$ belongs to the tangent space $T_{\xi}N$, for any $\xi \in N$, and $\Xi$ is $T$-periodic in the first variable and tangent to $N$ in the second one, i.e. $\Xi(t, \xi, \eta) = \Xi(t+T, \xi, \eta) \in T_{\xi}N$, $(t, \xi, \eta) \in \mathbb{R} \times N \times N$.

We assume that the average $\bar{a}$ of $a$ on $[0,T]$ is nonzero, that is: $\bar{a} := \frac{1}{T} \int_0^T a(t)dt \neq 0$.

By applying degree-theoretic methods we obtain a global continuation result for the $T$-periodic solutions of (1) and we provide sufficient conditions for the existence of branches of $T$-periodic solutions.

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