Harmonic perturbations with delay of periodic separated variables differential equations

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Abstract

In this talk we present recent results jointly obtained with Marco Spadini (University of Florence). We investigated the set of harmonic solutions of T-periodic perturbations of T-periodic separated variables ODEs on manifolds, allowing the perturbing term to contain a finite delay. Namely, given $T > 0$ and a boundaryless smooth manifold $N \subseteq \mathbb{R}^d$, we consider T-periodic solutions of equations of the form

$$\dot{\zeta}(t) = a(t)\Phi(\zeta(t)) + \lambda\Xi\big(t, \zeta(t), \zeta(t-r)\big), \quad \lambda \geq 0, \tag{1}$$

where $r > 0$ is a finite time lag, $a : \mathbb{R} \to \mathbb{R}$ is continuous and T-periodic, $\Phi : N \to \mathbb{R}^d$ and $\Xi : \mathbb{R} \times N \times N \to \mathbb{R}^d$ are given continuous *tangent vector fields* on N, in the sense that $\Phi(\xi)$ belongs to the tangent space $T_\xi N$, for any $\xi \in N$, and Ξ is T-periodic in the first variable and tangent to N in the second one, i.e. $\Xi(t, \xi, \eta) = \Xi(t + T, \xi, \eta) \in T_\xi N$, $(t, \xi, \eta) \in \mathbb{R} \times N \times N$. We assume that the average $\bar{a}$ of a on $[0, T]$ is nonzero, that is: $\bar{a} := \frac{1}{T}\int_0^T a(t)dt \neq 0$.

By applying degree-theoretic methods we obtain a global continuation result for the T-periodic solutions of (1) and we provide sufficient conditions for the existence of branches of T-periodic solutions.

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