On the Asymptotic Properties of Piecewise Contracting Maps

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Abstract

We are interested in the phenomenology of the asymptotic dynamics of piecewise contracting maps. They appear for instance as Poincaré maps in the study of bifurcations of stable heteroclinic cycles of some C^1 (non-equivariant) vector fields on $\mathbb{R}^n$, in the characterization of Cherry flows on compact two-manifolds, or in the characterization of the asymptotic stability of some piecewise continuous vector fields. They can also be found as discrete time models of regulatory networks with thresholds. In either way, they naturally appear in the modelling of some biological systems (neural, genetic and ecological) and in engineering (electro-mechanical and switched arrival-server systems), where the number of variables can be significantly high.

We consider here a wide class of such maps, i.e. Lipschitz contracting when restricted to any piece of a finite and dense union of disjoint open pieces in a compact metric space X. We give sufficient conditions to ensure some general basic properties, such as the periodicity, the total disconnectedness or the zero Lebesgue measure (when $X \subset \mathbb{R}^n$, $n \geq 1$) of the attractor. These conditions show in particular that a non-periodic attractor necessarily contains discontinuities of the map. Under this hypothesis, we obtain numerous examples of attractors, ranging from finite to connected and chaotic, contrasting with the (quasi-)periodic asymptotic behaviours observed so far.

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